DIRECT ALGEBRAIC METHOD FOR SOLVING FRACTIONAL FOKAS EQUATION

by

Yi TIAN a and Jun LIU b,c∗

a College of Data Science and Application, Inner Mongolia University of Technology, Hohhot 010080, China
b School of Materials Science and Engineering, Inner Mongolia University of Technology, Hohhot 010051, China
c Inner Mongolia Key Laboratory of Graphite and Graphene for Energy Storage and Coating, Hohhot 010051, China

Fractional Fokas equation is studied, its exact solution is obtained by the direct algebraic method. The solution process is elucidated step by step, and the fractional complex transform and the characteristic set algorithm are emphasized.

Key words: Direct algebraic method, characteristic set algorithm, space-time fractional Fokas equation

Introduction

In recent decades, the nonlinear fractional partial differential equations have attracted much attention due to their wide applications to various complex phenomena arising in elasticity, plasma physics, solid state physics, gas dynamics, material and others[1-10]. Searching for their exact solutions is an important topic in both mathematics and engineering. A wealth of methods have been developed for this purpose, for examples, the homotopy perturbation method [11-17], variational iteration method [18-20], the exp-function method[21-25], He-Laplace method[26-28], the symmetry reduction method[29-30], the reproducing kernel method[32,33], and others[34-36].

In this paper, solitary wave solutions of space-time fractional Fokas equation[37] are considered. The direct algebraic method[38,39] is used to solve the equation, which leads to a large system of algebraic equations, the characteristic set algorithm[40-42] is adopted to solve the algebraic equations.

The direct algebraic method

There are many types fractional derivative in literature, for example the Jumaries’s modification of the Riemann-Liouville derivatives of fractional-order α is defined by the following expression [1]:

\[ D_0^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi \quad 0 < \alpha < 1, \]

where \( f(t) \) is a real and continuous function defined on \( R \).

The He’s fractional derivative defined as [43,44]:

\[ D_0^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t (s-t)^{n-\alpha-1}(f_n(s)-f(s))ds \quad 0 < \alpha \leq 1, \]

We outline the main steps of the direct algebraic method [38,39] with modified Riemann-Liouville derivative for finding exact travelling solutions of fractional-order nonlinear partial differential equations.

Consider the fractional-order nonlinear partial differential equation in the following

Corresponding author; e-mail: clxylj@163.com, ttxsun@163.com
form:

\[ Q(u, D^\alpha u, D^\beta u, D^\gamma u, \ldots, D^{2\alpha} u, D^{2\beta} u, D^{2\gamma} u, \ldots) = 0, \]  

(1)

where \( Q \) is a polynomial of \( u \) and its fractional derivatives.

**Step-1.** The fractional complex transform\([45,46]\) is used

\[
\begin{align*}
\xi &= \frac{ct^\alpha}{\Gamma(1+\alpha)} + \frac{k_1x_1^\alpha}{\Gamma(1+\alpha)} + \frac{k_2x_2^\alpha}{\Gamma(1+\alpha)} + \cdots + \frac{k_nx_n^\alpha}{\Gamma(1+\alpha)}, \\
b_i &= \sum_{i=0}^{N} b_i Q(\xi)^i,
\end{align*}
\]

(2)

where \( c, k_1, k_2, \ldots, k_n \) are arbitrary constants, the Eq.(2) transform Eq.(1) into an ordinary differential equation:

\[ \tilde{Q}(u, c\mu, k_1\mu, k_2\mu, \ldots, c^n\mu^\alpha, k_1^n\mu^\alpha, k_2^n\mu^\alpha, \ldots) = 0, \]  

(3)

**Step-2.** We look for exact solution of Eq. (3) in the form

\[ u(\xi) = \sum_{i=0}^{N} b_i Q(\xi)^i, \quad b_N \neq 0, \]

(4)

where \( b_i (0 \leq i \leq N) \) are constants to be determined, and \( Q(\xi) \) satisfies the ODE in the form \([39]\):

\[ Q(\xi) = \ln(A)(\alpha + \beta Q(\xi) + \sigma Q(\xi)^2), \quad A \neq 0,1, \]  

(5)

**Step-3.** By balancing the highest order derivative terms with the nonlinear terms of the highest order in Eq.(3), we can evaluated the value of the positive integer \( N \).

**Step-4.** By substituting Eq.(4) along with Eq.(5) into Eq.(3) and equating all the coefficients of same power of \( Q(\xi) \) to zero, we obtained a system of algebraic equations. The obtaining system can be solved to find the value of \( c, k_1, k_2, \ldots, k_n, b_i (0 \leq i \leq N) \), substituting these terms into Eq.(4), the determination of solutions of Eq.(1) will be completed.

**Exact solutions of space-time fractional Fokas equation**

Consider the following space-time fractional Fokas equation\([37]\) which could be used to describe various physical phenomena such as fluid mechanics, water wave theory, ocean dynamics and many others.

\[
4^{\frac{3\alpha}{2}}u - \frac{\partial^{4\alpha}u}{\partial x_1^{4\alpha}} + \frac{\partial^{4\alpha}u}{\partial x_2^{4\alpha}} + \frac{\partial^{4\alpha}u}{\partial x_3^{4\alpha}} + \frac{12\partial^{2\alpha}u\partial^{2\alpha}u}{\partial x_1^{2\alpha}\partial x_2^{2\alpha}} + \frac{12\partial^{2\alpha}u\partial^{2\alpha}u}{\partial x_1^{2\alpha}\partial x_3^{2\alpha}} + \frac{6\partial^{3\alpha}u}{\partial x_1^{3\alpha}\partial x_2^{3\alpha}} = 0, 
\]

(6)

The fractional complex transform is

\[
\begin{align*}
\xi &= \frac{ct^\alpha}{\Gamma(1+\alpha)} + \frac{k_1x_1^\alpha}{\Gamma(1+\alpha)} + \frac{k_2x_2^\alpha}{\Gamma(1+\alpha)} + \frac{l_1y_1^\alpha}{\Gamma(1+\alpha)} + \frac{l_2y_2^\alpha}{\Gamma(1+\alpha)}, \\
b_i &= \sum_{i=0}^{N} b_i Q(\xi)^i,
\end{align*}
\]

(7)

where \( c, k_1, k_2, \ldots, k_n \) are arbitrary constants, and \( c, k_1, k_2, \ldots, k_n \neq 0 \). Using the wave variable (7), Eq. (6) becomes

\[ 4ck_1u'' - k_1^3k_2u^{(4)} + k_1k_2^3u^{(4)} + 12k_1k_2^2(u')^2 + 12k_1k_2^2u'' - 6l_1l_2u'' = 0, \]

(8)

Integrating Eq.(8) twice with respect to \( \xi \) and setting the integration constant as zero, we get:

\[ (4ck_1 - 6l_1l_2)u + 6k_1k_2u^2 + k_1k_2(-k_1^2 + k_2^2)u'' = 0, \]

(9)

Suppose that the solution of Eq. (9) can be expressed:

\[ u(\xi) = \sum_{i=0}^{N} b_i Q(\xi)^i, \]

(10)
where \( b_i (0 \leq i \leq N) \) are constants to be determined, such that \( b_N \neq 0 \).

Consider the homogeneous balance between the highest order derivative \( u^{(3)} \) and nonlinear term \( uu' \) appearing in (9), we have \( N = 2 \), we then suppose that Eq. (9) has the following solutions:

\[
u(\xi) = b_0 + b_1 Q(\xi) + b_2 Q(\xi)^2, \quad b_2 \neq 0,
\]

Substituting Eq. (11) along with Eq. (5) into Eq. (9) and collecting all the terms with the same power of \( Q(\xi) \) together, equating each coefficient to zero, yields a set of algebraic equations, which is large and difficult to solve, with the aid of the characteristic set algorithm [40-41], we can distinguish the different cases namely:

**Case (1)**

\[
\begin{align*}
b_2 &= \frac{(4ck_1 - 6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A))^2}{16k_1^2k_2^2(k_1^2 - k_2^2)\alpha^2 \ln(A)^2}, \\
b_1 &= \frac{-4ck_1(6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A))}{4k_1k_2}, \\
b_0 &= \frac{-4ck_1 + 6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A)^2}{4k_1k_2}, \\
c &= \frac{6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A)^2}{4k_1}.
\end{align*}
\]

**Case (2)**

\[
\begin{align*}
b_2 &= \frac{(4ck_1 - 6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A))^2}{16k_1^2k_2^2(k_1^2 - k_2^2)\alpha^2 \ln(A)^2}, \\
b_1 &= \frac{4ck_1(6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A))}{4k_1k_2}, \\
b_0 &= \frac{4ck_1 - 6d_1z + 3k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A)^2}{12k_1k_2}, \\
c &= \frac{1}{4}(\frac{6d_1z}{k_1} + k_1(-k_1^2 + k_2^2)(\beta^2 - 4\alpha\sigma) \ln(A)^2).
\end{align*}
\]

**Case (3)**

\[
\begin{align*}
b_2 &= (k_1^2 - k_2^2)\sigma^2 \ln(A)^2, \\
b_1 &= (k_1^2 - k_2^2)\beta\sigma \ln(A)^2, \\
b_0 &= 0, \\
c &= \frac{6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A)^2}{4k_1}, \quad \alpha = 0.
\end{align*}
\]

**Case (4)**

\[
\begin{align*}
b_2 &= (k_1^2 - k_2^2)\sigma^2 \ln(A)^2, \\
b_1 &= (k_1^2 - k_2^2)\beta\sigma \ln(A)^2, \\
b_0 &= \frac{-2ck_1 + 3d_1z}{3k_1k_2}, \\
c &= \frac{6d_1z + k_1k_2(-k_1^2 + k_2^2)\beta^2 \ln(A)^2}{4k_1}, \quad \alpha = 0.
\end{align*}
\]

For the sake of simplicity, we consider only the solution with respect to Case (1), the other solutions can be obtained in a similar way.

(1) when \( \beta^2 - 4\alpha\sigma < 0 \) and \( \sigma \neq 0 \)

\[
\begin{align*}
u_1 &= \frac{1}{4} \left[ \frac{F}{\alpha^2 \sigma^2 \ln^2(A)(k_1^2 - k_2^2)} - \frac{2\beta U}{\alpha \sigma} + 4 \right], \\
u_2 &= \frac{1}{4} \left[ \frac{FV^2}{\alpha^2 \sigma^2 \ln^2(A)(k_1^2 - k_2^2)} - \frac{2\beta V}{\alpha \sigma} + 4 \right],
\end{align*}
\]

\[
\begin{align*}
u_{s+1} &= \frac{1}{4} \left[ \frac{F(W)^2}{\alpha^2 \sigma^2 \ln^2(A)(k_1^2 - k_2^2)} + \frac{2\beta(W)}{\alpha \sigma} + 4 \right], \\
u_{s+2} &= \frac{1}{4} \left[ \frac{F(W)^2}{\alpha^2 \sigma^2 \ln^2(A)(k_1^2 - k_2^2)} - \frac{2\beta(W)}{\alpha \sigma} + 4 \right],
\end{align*}
\]

\[
\begin{align*}
u_{s+3} &= \frac{1}{4} \left[ \frac{F(W)^2}{\alpha^2 \sigma^2 \ln^2(A)(k_1^2 - k_2^2)} + \frac{2\beta(W)}{\alpha \sigma} + 4 \right],
\end{align*}
\]
\[
\begin{align*}
\mathbf{u}_{t+1} &= \frac{1}{4} F \left( \frac{F(X1)^2}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta(X1)}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_{t+2} &= \frac{1}{4} F \left( \frac{F(X2)^2}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta(X2)}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_3 &= F \left( \frac{FY^2}{16 \alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{\beta Y}{4 \alpha \sigma} + 1 \right),
\end{align*}
\]
where
\[
M = 4 \alpha \sigma - \beta^2, \quad F = \frac{\beta^2 k_i k_j \ln^2(A)(k_i^2 - k_j^2) - 4ck_i + 6l_j}{4k_i k_j}, \quad U = \beta - \sqrt{M \tan \left( \frac{M \xi}{2} \right)},
\]
\[
\begin{align*}
V &= \beta + \sqrt{M \cot \left( \frac{M \xi}{2} \right)}, \\
W_1 &= -\beta + \sqrt{Mpq \sec \left( \sqrt{M \xi} \right)} + \sqrt{M \tan \left( \frac{M \xi}{2} \right)}, \\
W_2 &= \beta + \sqrt{Mpq \sec \left( \sqrt{M \xi} \right)} - \sqrt{M \tan \left( \frac{M \xi}{2} \right)}, \\
X_1 &= \beta + \sqrt{M \cot \left( \sqrt{M \xi} \right)} - \sqrt{M \cot \left( \sqrt{M \xi} \right) \sqrt{Mpq}}, \\
X_2 &= \beta + \sqrt{M \cot \left( \sqrt{M \xi} \right)} + \sqrt{M \cot \left( \sqrt{M \xi} \right) \sqrt{Mpq}}, \\
Y &= 2\beta + \sqrt{M \cot \left( \frac{M \xi}{4} \right)} - \sqrt{M \tan \left( \frac{M \xi}{4} \right)}.
\end{align*}
\]

(2) when \( \beta^2 - 4\alpha \sigma > 0 \) and \( \sigma \neq 0 \)
\[
\begin{align*}
\mathbf{u}_6 &= \frac{1}{4} F \left( \frac{F(U^2)}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta U}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_7 &= \frac{1}{4} F \left( \frac{F(V^2)}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta V}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_8 &= \frac{1}{4} F \left( \frac{F(W_1^2)}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta(W_1)}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_9 &= \frac{1}{4} F \left( \frac{F(W_2^2)}{\alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{2 \beta(W_2)}{\alpha \sigma} + 4 \right), \\
\mathbf{u}_{10} &= F \left( \frac{FY^2}{16 \alpha \sigma^2 \ln^2(A)(k_i^2 - k_j^2)} - \frac{\beta Y}{4 \alpha \sigma} + 1 \right),
\end{align*}
\]
where
\[
N = \beta^2 - 4\alpha \sigma, \quad F = \frac{\beta^2 k_i k_j \ln^2(A)(k_i^2 - k_j^2) - 4ck_i + 6l_j}{4k_i k_j}, \quad U = \beta + \sqrt{N \cot \left( \frac{N \xi\sqrt{N}}{2} \right)},
\]
\[
\begin{align*}
V &= \beta + \sqrt{N \cot \left( \frac{N \xi\sqrt{N}}{2} \right)}, \\
W_1 &= \beta - iN \sqrt{pq \sec \left( \xi \sqrt{N} \right) + \sqrt{N \tan \left( \xi \sqrt{N} \right)}, \\
W_2 &= \beta + iN \sqrt{pq \sec \left( \xi \sqrt{N} \right) + \sqrt{N \tan \left( \xi \sqrt{N} \right)}}, \\
X_1 &= \beta + \sqrt{N \cot \left( \xi \sqrt{N} \right) - \sqrt{N \sqrt{pq \csc \left( \xi \sqrt{N} \right)}, \\
X_2 &= \beta + \sqrt{N \cot \left( \xi \sqrt{N} \right) + \sqrt{N \sqrt{pq \csc \left( \xi \sqrt{N} \right)}, \\
Y &= 2\beta + \sqrt{N \cot \left( \frac{\xi \sqrt{N}}{4} \right) + \sqrt{N \tan \left( \frac{\xi \sqrt{N}}{4} \right)}.
\end{align*}
\]

(3) when \( \alpha \sigma > 0 \) and \( \beta = 0 \)
\[ u_1 = \frac{F^2 \tan \left( \frac{\sqrt{\alpha} \sigma}{2} \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad u_{12} = \frac{F^2 \cot \left( \frac{\sqrt{\alpha} \sigma}{2} \right)}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \]
\[ u_{15} = \frac{F^2 \left( \cot \left( \frac{\sqrt{\alpha} \sigma}{2} \right) - \tan \left( \frac{\sqrt{\alpha} \sigma}{2} \right) \right)^2}{4 \alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \text{and} \quad F = \frac{-4ck_1 + 6ll_2}{4k,k_2}. \]

(4) when $\alpha \sigma < 0$ and $\beta = 0$

\[ u_{16} = F - \frac{F^2 \tan \left( \frac{\sqrt{\alpha} \sigma}{2} \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)}, \quad u_{17} = F - \frac{F^2 \cot \left( \frac{\sqrt{\alpha} \sigma}{2} \right)}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)}, \]
\[ u_{18} = F - \frac{F^2 \left( \sqrt{-\frac{\alpha \sigma}{\rho}} \sech \left( 2 \frac{\sqrt{\alpha} \sigma}{2} \right) \pm i \sqrt{-\frac{\alpha \sigma}{\rho}} \tan \left( \frac{2 \sqrt{\alpha} \sigma}{2} \right) \right)^2}{\alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)}, \]
\[ u_{19} = \frac{F^2 \left( \cot \left( \frac{\sqrt{\alpha} \sigma}{2} \right) - \tan \left( \frac{\sqrt{\alpha} \sigma}{2} \right) \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \text{and} \quad F = \frac{-4ck_1 + 6ll_2}{4k,k_2}. \]

(5) when $\beta = 0$ and $\sigma = \alpha$

\[ u_{21} = \frac{F^2 \tan \left( \alpha \sqrt{\sigma} \xi \right)^2}{\alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad u_{22} = \frac{F^2 \cot \left( \alpha \sqrt{\sigma} \xi \right)}{\alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \]
\[ u_{23-1,2} = \frac{F^2 \left( \tan \left( 2 \alpha \sqrt{\sigma} \xi \right) \pm \sqrt{pq} \sec \left( 2 \alpha \sqrt{\sigma} \xi \right) \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad u_{24-1,2} = \frac{F^2 \left( \cot \left( 2 \alpha \sqrt{\sigma} \xi \right) \pm \sqrt{pq} \csc \left( 2 \alpha \sqrt{\sigma} \xi \right) \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \]
\[ u_{25} = \frac{F^2 \left( \cot \left( \frac{\alpha \sqrt{\sigma} \xi}{2} \right) - \tan \left( \frac{\alpha \sqrt{\sigma} \xi}{2} \right) \right)^2}{4 \alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \text{and} \quad F = \frac{-4ck_1 + 6ll_2}{4k,k_2}. \]

(6) when $\beta = 0$ and $\sigma = -\alpha$

\[ u_{26} = \frac{F^2 \tan \left( \alpha \sqrt{\sigma} \xi \right)^2}{\alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad u_{27} = \frac{F^2 \cot \left( \alpha \sqrt{\sigma} \xi \right)}{\alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \]
\[ u_{28-1,2} = \frac{F^2 \left( \sqrt{pq} \sec \left( 2 \alpha \sqrt{\sigma} \xi \right) \pm i \tan \left( 2 \alpha \sqrt{\sigma} \xi \right) \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad u_{29-1,2} = \frac{F^2 \left( \cot \left( 2 \alpha \sqrt{\sigma} \xi \right) \pm \sqrt{pq} \csc \left( 2 \alpha \sqrt{\sigma} \xi \right) \right)^2}{\alpha \sigma \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \]
\[
\begin{align*}
 u_{30} &= \frac{F^2 \left( \coth \left( \frac{\alpha \xi}{2} \right) + \tanh \left( \frac{\alpha \xi}{2} \right) \right)^2}{4 \alpha^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + F, \quad \text{and} \quad F = -4ck_1 + 6l_1 l_2 \cdot \\
(7) \text{ when } \beta^2 &= 4\alpha \sigma \\
 u_{31} &= F \left( -\frac{4}{\beta \xi \ln(A)} + \frac{4F(\beta \xi \ln(A) + 2)^2}{\beta^4 \xi^2 \ln^4(A) \left( k_1^2 - k_2^2 \right) - 1} \right), \\
\text{where } F &= \frac{\beta^2 k_1 k_2 \ln^2(A) \left( k_1^2 - k_2^2 \right) - 4ck_1 + 6l_1 l_2}{4k_1 k_2}.
\end{align*}
\]

\[
(8) \text{ when } \beta = k, \alpha = mk (m \neq 0) \text{ and } \sigma = 0 \\
u_{32} = \frac{F^2 \left( A^{k_2} - m \right)^2}{k^2 m^2 \ln^2(A) \left( k_1^2 - k_2^2 \right)} + \frac{F \left( A^{k_1} - m \right)}{m} + F, \\
\text{where } F &= \frac{k^2 k_1 k_2 \ln^2(A) \left( k_1^2 - k_2^2 \right) - 4ck_1 + 6l_1 l_2}{4k_1 k_2}.
\]

\[
(9) \text{ when } \beta = \sigma = 0 \\
u_{33} = \frac{F^2 \xi^2}{k_1^2 - k_2^2} + F, \\
\text{where } F &= \frac{-4ck_1 + 6l_1 l_2}{4k_1 k_2}.
\]

Remark 1. The generalized hyperbolic and triangular functions are defined as [38,39]
\[
\sinh(a) = \frac{pA^a - qA^{-a}}{2}, \quad \cosh(a) = \frac{pA^a + qA^{-a}}{2}, \quad \tanh(a) = \frac{pA^a - qA^{-a}}{pA^a + qA^{-a}}. \\
\coth(a) = \frac{pA^a + qA^{-a}}{pA^a - qA^{-a}}, \\
\sech(a) = \frac{2}{pA^a + qA^{-a}}, \quad \csch(a) = \frac{2}{pA^a - qA^{-a}}, \quad \sin(a) = \frac{pA^{ia} - qA^{-ia}}{2i}, \quad \cos(a) = \frac{pA^{ia} + qA^{-ia}}{2i}. \\
\tan(a) = \frac{pA^{ia} - qA^{-ia}}{pA^{ia} + qA^{-ia}}, \quad \cot(a) = \frac{pA^{ia} + qA^{-ia}}{pA^{ia} - qA^{-ia}}, \quad \sec(a) = \frac{2}{pA^{ia} + qA^{-ia}}, \quad \csc(a) = \frac{2i}{pA^{ia} - qA^{-ia}},
\]
where \( \xi \) is an independent variable and \( p, q > 0 \).

Conclusions

In this paper, we use the direct algebraic method combined with characteristic set algorithm to solve the space-time fractional Fokas equation, a abundant of exact solutions are obtained, to the best of our knowledge, the solutions we obtained have not been reported in literature.

Acknowledgments

The work is supported by National Natural Science Foundation of China (Grant No. 61862048), the Natural Science Foundation of Inner Mongolia (2019MS05068), and the Scientific Research Project Foundation of Inner Mongolia University of Technology (ZZZ201820).

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Paper submitted: March 6, 2020
Paper revised: July 8, 2020
Paper accepted: July 8, 2020