A FRACTAL RESISTANCE-CAPACITANCE CIRCUIT MODEL FOR THE CURRENT FLOWING IN POROUS MEDIA

by

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The current flows in a porous media is a very complex nature phenomenon, and it is very difficult to establish its mathematical model with the traditional definition of derivative. In this paper, the fractal resistance-capacitance circuit of porous media is successfully established based on He's fractal derivative, and the two-scale transform is adopted to solve the fractal circuit. In this fractal resistance-capacitance circuit circuit, the fractal dimension represents the effective porosity of the two plates of the capacitor, and the influence of its value on the fractal resistance-capacitance circuit circuit is also elaborated.

Key words: *He's fractal derivative, fractal resistance-capacitance circuit, porous media, two-scale transform method*

Introduction

The micro-pore structure of the porous media, *e. g.* natural rock and man-made material, is always very complex. These porous materials have very strong micro-heterogeneity [1-4], and the flow of electric current through these porous conductors is a very complex process, see fig. 1.

The transport properties of current in porous materials are affected by structural factors, such as tortuosity, pore size distribution, connectivity, porosity and so on. The mathematical model is so complex that the classical definition of the derivative is completely invalid [5, 6]. Professor He [7-15] proposed the definition of fractal derivative in 2014, which is a very powerful tool to establishment of a complex mathematical model in a fractal space or a porous medium.

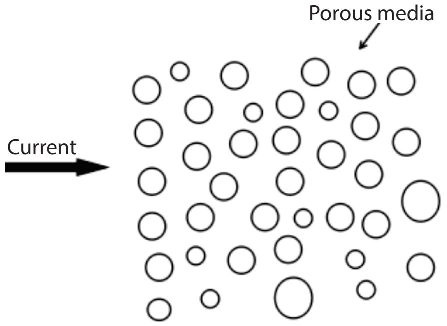

Figure 1. Electric current flow in porous conductor

He's fractal derivative is defined [16]:

$$\frac{\partial \Xi}{\partial \tau^{\mu}}(\tau_0) = \Gamma(1+\mu) \lim_{\tau-\tau_0=\Delta\tau\to\alpha} \frac{\Xi(\tau)-\Xi(\tau_0)}{(\tau-\tau_0)^{\mu}}$$

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In this paper, we mainly use He's fractal derivative to establish the fractal resistance-capacitance (R-C) circuit model for the current flowing in porous media. The two-scale transform is adopted to solve the fractal R-C circuit, and the physical meaning of fractal order is explained in detail.

The two-scale transform

The two-scale transform is a new transform method, which is an extension of He's fractional complex transform [17]. It was proposed by He [18] successfully used to solve many fractal problems.

Consider a fractal equation:

$$\frac{\mathrm{D}\varphi}{\mathrm{D}t^{\alpha}} + F\left(\varphi\right) = 0 \tag{1}$$

In order to use the two-scale transform, we assume:

$$T = t^{\gamma} \tag{2}$$

where t is for the small scale and T for large-scale, α – the two-scale dimension. Therefore, the eq. (1) can be converted into its traditional partner:

$$\frac{D\varphi}{DT} + F(\varphi) = 0 \tag{3}$$

Then the eq. (3) can be solve by many classical methods, such as the homotopy perturbation method [19-21], the variational iteration method [22-24], Taylor series method [25, 26], the exp-function method [27] and so on [28-35].

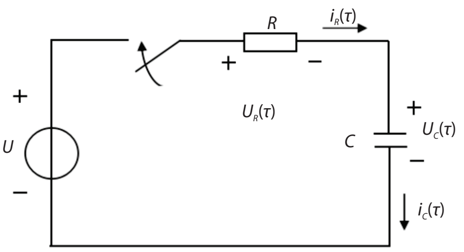

Figure 2. The classical zero state R-C circuit

The fractal R-C circuit

The R-C circuit is a voltage divider consisting of a resistance, R, and a capacitance, C. As a very important circuit in the circuit, the R-C circuit is usually used as signal transmission circuit in electronic circuit. According to different needs, it can realize coupling, phase shifting, filtering and other functions in the circuit.

As shown in fig. 2, the classical zero state R-C circuit can be described:

$$RC\frac{\mathrm{d}U_C\left(\tau\right)}{\mathrm{d}\tau} + U_C\left(\tau\right) = U \tag{4}$$

Subject to the initial condition:

$$U_C\left(0\right) = 0$$

However, when the capacitor is a porous medium, the aforementioned equation cannot describe the effect of porous property on the R-C circuit, so we need to establish a new fractal R-C circuit model:

$$RC_{\gamma}\frac{\mathrm{d}U_{\gamma,C}\left(\tau\right)}{\mathrm{d}\tau^{\gamma}} + U_{\gamma,C}\left(\tau\right) = U,\ 0 < \gamma < 1 \tag{5}$$

Taking the two-scale transform:

$$T = t^{\gamma} \tag{6}$$

Equation (5) can be converted into the form:

$$RC_\gamma \frac{\mathrm{d}U_C(T)}{\mathrm{d}T} + U_C(T) = U \tag{7}$$

The solution of the eq. (7) is given:

$$U_C(T) = U\left(1 - \mathrm{e}^{-\frac{T}{\sigma}}\right) \tag{8}$$

where $\sigma = RC_\gamma$, which is called the time constant.

Correspondingly:

$$i_C(T) = C\frac{\mathrm{d}U_C(T)}{\mathrm{d}T} = \frac{U}{R}\mathrm{e}^{-\frac{T}{\sigma}} \tag{9}$$

So, the solution of the eq. (7) is obtained by using eqs. (6) and (8):

$$U_{\gamma,C}(\tau,\gamma) = U\left(1 - \mathrm{e}^{-\frac{\tau\gamma}{\sigma}}\right) \tag{10}$$

Correspondingly, there is:

$$i_{\gamma,C}(\tau,\gamma) = \frac{U}{R}\mathrm{e}^{-\frac{\tau\gamma}{\sigma}} \tag{11}$$

Letting $U = 1$, $\sigma = 6$, we plot the behavior of $i_{\gamma,C}(\tau, \gamma)$ with $\gamma = 0.1, 0.2, 0.4, 0.7, 0.9$, respectively in fig. 3.

Figure 3 shows that the value of current $i_{\gamma,C}(\tau, \gamma)$ decreases as the value of γ increases, where the fractal dimension γ represents the effective porosity of the two plates of the capacitor that illustrated in fig. 4.

Then we get the curves of the $U_{\gamma,C}(\tau, \gamma)$ with $\gamma = 0.1, 0.2, 0.4, 0.7, 0.9$, respectively in fig. 5. Obviously, the charging speed of the ca-

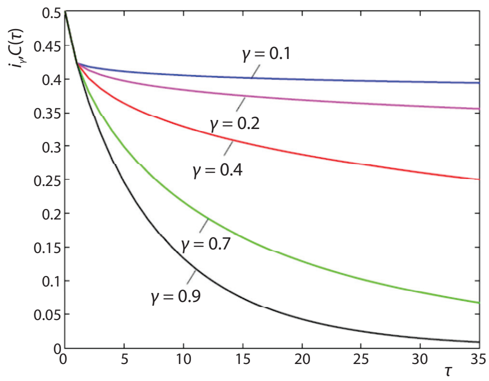

Figure 3. The curves of the $i_{\gamma,C}(\tau, \gamma)$ with $\gamma = 0.1, 0.2, 0.4, 0.7, 0.9$

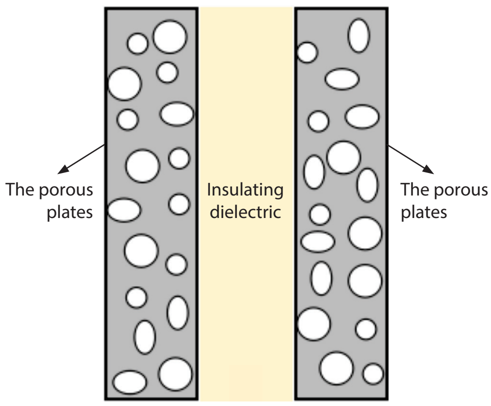

Figure 4. The fractal capacitor with porous plates

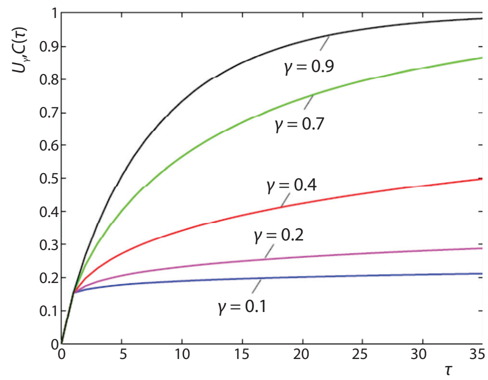

Figure 5. The curves of the $U_{\gamma,C}(\tau, \gamma)$ with $\gamma = 0.1, 0.2, 0.4, 0.7, 0.9$

pacitor increases with the increase of γ, which is related to the time constant σ. Now we recall the definition of capacitance:

$$C_\gamma = \frac{\varepsilon S_\gamma}{4k\pi d} \tag{12}$$

where ε is the relative permittivity, k – the electrostatic force constant, S_γ is the positive area of the two plates, and d – the distance between the two plates.

With the increase of γ, the effective facing area S_γ decreases, that is the decrease of C_γ, which leads to the decrease of the time constant σ.

Conclusion

This paper proposes a fractal R-C circuit by using He's fractal derivative. The two-scale transform is adopted to solve the fractal R-C circuit. The fractal dimension γ that means the porosity of the two plates of capacitor is discussed and its effect on the circuit is studied. The obtained results in this paper are expected to shed a bright light on practical applications of the circuit theory.

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