MODELLING AND FILTERING FOR A STOCHASTIC UNCERTAIN SYSTEM IN A COMPLEX SCENARIO

by

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This paper presents a new approach to filter signals for discrete-time physical problems with stochastic uncertain in the presence of random data transmission delays, out-of-order packets and correlated noise. To deal with the packet disorder, the system model synthesizing the transmission delays and out-of-order packets from the plant to the filter is established by utilizing signal reconstruction schemes based on the zero-order-holder and logic zero-order-holder. A robust finite horizon Kalman filter is proposed by augmenting the state-space model and minimizing the error covariance. To further improve the filtering performance, a linear estimation-based delay compensation strategy is proposed by employing the reorganized time-stamped measurements. Moreover, for solving the missing measurement problem whilst reducing the computational costs, an artificial delay compensation approach is established using an one-step prediction approach. Simulation results show the effectiveness of the proposed method.

Key words: *finite horizon filtering, transmission delays, out-of-order packets, variational principle, fractal calculus*

Introduction

With the expansion of the physical device and system functions, network system with universal bus structure, that is, network control systems, then in its complete architecture, distributed mode of operation, relatively independent and well interconnected communication, saving wiring and signaling of reliability, showing all the virtues. Networked systems have gained rapid advances owing to the development of the communication technology and increasing computation power [1, 2] with successful applications in a wide range of significant areas such as cyber-physical systems [3], smart grids [4], and communication networks [5, 6]. In order to meet the requirements of growing computation and large-scale system integration, it is necessary to design a suitable communication platform for improving the capacity of the communication link between physical and computational elements whilst increasing system flexibility and reducing installation and maintenance costs [2, 3, 7]. However, due to the constraints of communication bandwidth, networked systems often experience various network performance problems such as transmission delays, packet losses, out-of-order packets, miss-

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ing/fading measurements, and varied sampling/transmission intervals. Therefore, it is desirable to investigate these scenarios and find effective ways to deal with these problems.

In the literature, state estimation or filtering for networked systems has attracted much interest, in which measurements are transmitted via various communication channels. Zhong *et al.* [8] developed an automatic cross filtering method to improve the signal processing performance. For data transmission over networks, several filtering or noise reduction methods have been proposed to alleviate the influence of the network performance problems mentioned before. To guarantee the optimisation of the parameters for the local filters, Yuan *et al.* [9] dealt with the distributed quantized multi-modal, H_∞, fusion filtering problem using a class of two-time-scale system. Liu *et al.* [10] presented a weighted approach for error cross-covariance matrices, and introduced a distributed Kalman filtering scheme to handle data transmission delay and cross-correlated noise. Zhou and Zhu [11] proposed a robust finite-time state estimation approach for a class of discrete-time networks with Markovian jump parameters. For discrete and distributed delay, Liu and Luo [12] proposed the double delayed feedback method to conceal the time delay for a 2-D coupled electro-optic chaotic system. Moreover, Yang *et al.* [13] constructed an augmented Lyapunov-Krasovskii functional (LKF) and combined an integral inequality convex approach to estimate the proposed LKF. To increase the simultaneous multi-parameter estimation precisions with time-dependent, Xie and Xu [14] investigated the optimal coherent control scheme. The augmented state approaches [15, 16] applied compensation schemes by one-step prediction deal with random delays. The dimension reduction methods are used to deal with the augmented dimension. Such as the fractional calculus or fractal calculus [17] has to be used to reveal the lost information due to the lower dimensional approach. Using random coupling strength and extended Kalman filters, a recursive state estimator was developed, where the gain matrix was determined by optimizing an upper bound matrix [18]. The measurement reorganization approaches [19, 20] are an effective strategy employing the measurement transformation, so that a random delayed system can be transformed into a delay-free one. In order to describe random delays and packet dropouts, a system could be transferred into random variables of a Bernoulli distribution [15, 21] to establish a linear estimator, and a state augmentation strategy was employed. However, the computational costs are too high due to the augmented system models.

Further, filtering approaches have been investigated such as H_∞ filtering, robust filtering, and finite-horizon filtering. Shen *et al.* [22] investigated event-triggered H_∞ filtering of Markov jump systems, which used general transition probabilities by a mode-dependent event-triggered scheme of zero-order-holder (ZOH). The event-based H_∞ filtering [5] is insensitive to uncertainties appearing in system models and/or exogenous input signals, whereas robust filtering [23] was mainly designed for uncertain systems. On the other hand, the finite-horizon filtering [20, 24] was used for obtaining the upper boundary of the steady-state error covariance. Since the actual error covariance of the estimated state is less than the upper boundary, the finite-horizon filter has a better transient performance for the filtering process in networked systems.

In summary, the aforementioned discussion is related to the design of a filter with correlated noise, multi-step random delays or packet dropouts. The previous developments mainly concentrate on the systems with only one or two-step transmission delays. Moreover, the filtering approaches in the presence of transmission delays and correlated noise combined with norm-bounded uncertainty cannot guarantee an appropriate upper boundary for the estimation of error covariances. Motivated by the aforementioned analysis, this paper focuses on designing a robust finite horizon filter for uncertain time-varying systems, the main contributions of this paper are summarized:

- Taking into account of the random transmission delays, the received data packets with time-stamp are investigated. Our theoretical analysis proves that dropping out-of-order packets using signal reconstruction schemes is capable of improving the filtering performance.
- Establish the stochastic uncertain system model employing two signal reconstruction schemes to handle out-of-order packets.
- To deal with the filtering with random transmission delays, a new linear estimation-based compensation strategy is here proposed, where the delayed system is transformed into the equivalent delay-free one by reorganizing time-stamped measurement sequences to reduce computational costs.

Problem formulation

System description

In order to describe stochastic uncertainty for stochastic uncertain systems, the uncertain parameters are converted to a model with multiplicative noise [15, 20, 25]. To alleviate the computational complexity, the measurement equation of the sensor is described by the following uncertain linear discrete-time systems [21, 25]:

$$x(k+1) = (Ak + F_k K_k E_k)x(k) + B_k w_k, \quad k = 1, 2, \cdots \tag{1}$$

$$z(k) = (C_k + \mathcal{H}_k F_k E_k)x(k) + v_k \tag{2}$$

where $x(k) \in \mathbb{R}^r$ is the state of the process to be estimated, $z(k) \in \mathbb{R}^m$ – the measurement output, F_k – the time-varying parametric uncertainty described by scalar multiplicative noise, and $\mathcal{F}_k$, $\mathcal{H}_k$, an E_k are known time-varying matrices, $w_k \in \mathbb{R}$ and $v_k \in \mathbb{R}^m$ – is the process and measurement noise, respectively, which is zero-mean white noise with covariances Q_k and R_k. The $A_k \in \mathbb{R}^{r\times r}$, $B_k \in \mathbb{R}^r$, and $C_k \in \mathbb{R}^{m\times r}$ are known, real, and time-varying matrices with appropriate dimensions. The initial state $x(0)$ with mean μ_0 and covariance P_0 is assumed to be uncorrelated with any noisy signals. Note that the uncertainty F_k satisfies $F_k F_k^T \le I$.

In practical applications, the received signals are influenced by correlated noise [15, 21, 25]. As shown in fig. 1, we assume that the process noise w_k and measurement noise v_k are correlated at the same time instant k, the statistical properties satisfy:

$$\begin{aligned} &E(w_k) = 0, \quad E(v_k) = 0 \\ &E\left[\begin{pmatrix} w_k \\ v_k \end{pmatrix}\begin{pmatrix} w_l^T & (v_l)^T \end{pmatrix}\right] = \begin{pmatrix} Q_k\delta_{k,l} & S_k\delta_{k,l} \\ (S_k)^T\delta_{k,l} & R_k\delta_{k,l} \end{pmatrix} \end{aligned} \tag{3}$$

where $Q_k = Q_k^T$, $R_k = R_k^T$, and $S_k = S_k^T$.

Modelling based on sequence re-ordering

Due to the limited bandwidth during the network transmission, we often meet network congestion. Figure 1 depicts the filtering with different network problems such as transmission delays and out-of-order packets.

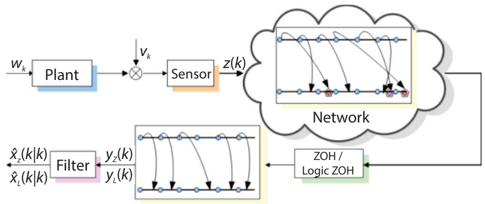

Figure 1. Schematic diagram of filtering with transmission delays and out-of-order packets

Remark 1. Before the time-stamped data packets are transmitted, ZOH stores the most recent data packet, whereas other time-stamped data packets are discarded [22]. In addition, data packets from the previous time instant are used to maintain a reliable communication link [16]. The motivation of using a logic ZOH scheme is to obtain the latest data packet, suggesting that the stored data packets will not be updated until the logic ZOH receives a new signal [5, 26]. Because the latest data packet, before being transmitted, is close to the current actual signal to be estimated, the network-induced packet disorders can be avoided using the logic ZOH [27].

Let the sampling time instant be t, the current time instant is k, the transmission instants are denoted by k_1 and k_2, respectively. Depending on the role of the ZOH and logic ZOH, the transmission delays from the sensor to the processor employing the ZOH and logic ZOH are represented by $\eta(k_1)$ and $\eta(k_2)$, where $0 \leq \eta(k_2) \leq \eta(k_1) \leq N$. The $\tau(k_1) \in \mathbb{N}$ and $\tau(k_2) \in \mathbb{N}$ are the transmission delays at the sampling time instants:

$$k = \tau(k_1) + k_1 = \tau(k_2) + k_2 \tag{4}$$

Since the received latest data packet is approximate to the current data packet, we set $\beta(k) \geq 0$, showing the relation of the received time-stamp k_1 and k_2:

$$k_2 = k_1 + \beta(k) \tag{5}$$

From eqs. (4) and (5):

$$\tau(k_2) = \tau(k_1) - \beta(k) \tag{6}$$

Moreover, at the current time instant k, k_1, and k_2 are re-written:

$$k_1 = k - \tau(k),\ k_2 = k - \tau'(k) \tag{7}$$

where $\tau(k) = \tau(k_1)$ as well as $\tau'(k) = \tau(k) - \beta(k)$.

Remark 2. The received valid data is reorganized by two signal reconstruction schemes. When the ZOH receives the time-stamped data packet $z(k_1)$, the stored signal $y_Z(k)$ is reorganized:

$$y_Z(k) = z[k - \tau(k)] \tag{8}$$

At the same time, when the logic ZOH receives the arriving data packet $z(k_2)$, the stored signal $y_{LZ}(k)$ is modeled:

$$y_{LZ}(k) = z[k - \tau'(k)] \tag{9}$$

where $\tau'(k) = \tau(k) - \beta(k)$. During data transmission from the plant to the filter over a communication network, the filter is able to obtain the knowledge of the data delays and dropout packets [20].

Robust finite horizon filtering

In this section, a robust filtering approach is proposed for stochastic uncertain systems shown in eqs. (1) and (2). As discussed earlier, the received valid data packets models eqs. (8) and (9) are dealt with using two signal reconstruction schemes, *i. e.* ZOH and logic ZOH.

When the time-stamped data packets are transmitted over a communications network, the state estimation is conducted by using the upper boundary of the error covariance. At the current time instant, a linear estimation-based method is proposed to compensate the random transmission delays. A recursive approach based on finite horizon Kalman-like filtering is then introduced to pick up the missing packets.

In the literature, the state estimation or filtering problem with transmission delays is typically solved using the compensation strategy of one-step prediction. Although it leads to more accurate estimation, the computational complexity is increased. Our proposed linear estimation-based method is an approximate state estimation method, and hence its estimation accuracy is similar to that of the standard one-step prediction approach. However, the proposed strategy can significantly reduce the computational costs and improve the estimation efficiency.

Filtering for ZOH scheme

At sampling time instant k, the stored signal $y_Z(k)$ refers to the most recent data packet $z[k-\tau(k)]$ for the ZOH. Let $t=k-\tau(k)$ from eqs. (2) and (8), the stored signal is reorganized:

$$y_Z(k)=z(t)=(C_t+\mathcal{H}_t F_t E_t)x(t)+v_t \tag{10}$$

Based on the reorganized time-stamped data packets and the projection equation, the state estimation is designed by using recursive scheme:

$$\hat{x}_Z(t|t)=proj\{x(t)|y_Z(k),\ y_Z(k-1),\cdots,\ y_Z(0)\}=\hat{x}_Z(t|t-1)+K_{Z,t}\left[z(t)-\hat{C}_{Z,t}\hat{x}_Z(t|t-1)\right] \tag{11}$$

$$\begin{aligned}\hat{x}_Z(t+1|t)&=proj\{x(t+1)|y_Z(k),\ y_Z(k-1),\cdots,\ y_Z(0)\}=\\&=\hat{A}_{Z,t}\hat{x}_Z(t|t-1)+L_{Z,t}\left[z(t)-\hat{C}_{Z,t}\hat{x}_Z(t|t-1)\right]\end{aligned} \tag{12}$$

where $\hat{x}_Z(t|t)$ is the filter, and $\hat{x}_Z(t+1|t)$ – the predictor of state $x(t)$ with time-stamp t before being transmitted, meanwhile, $y_Z(k)$ – the stored signal, and the filter parameters satisfy $K_{Z,t}$, $L_{Z,t}\in\mathbb{R}^{r\times m}$, $\hat{A}_{Z,t}\in\mathbb{R}^{r\times r}$, and $\hat{C}_{Z,t}\in\mathbb{R}^{m\times r}$.

Augmented state vectors

To derive the filter parameters $K_{Z,t}$, $L_{Z,t}$, $\hat{A}_{Z,t}$, and $\hat{C}_{Z,t}$ from the state estimation shown in eqs. (11) and (12), minimize the covariance estimation is developed. From the finite-horizon Kalman filtering theory, the prediction error $\tilde{e}_Z(t)=x(t)-\hat{x}_Z(t|t-1)$ whilst the filtering error $e_Z(t)=x(t)-\hat{x}_Z(t|t)$, such that the corresponding estimations $\hat{x}_Z(t|t-1)$ and $\hat{x}_Z(t|t)$ are represented by augmented vectors:

$$\tilde{\Psi}_Z(t)=\begin{bmatrix}\tilde{e}_Z(t)\\ \hat{x}_Z(t|t-1)\end{bmatrix},\quad \Psi_Z(t)=\begin{bmatrix}e_Z(t)\\ \hat{x}_Z(t|t)\end{bmatrix} \tag{13}$$

Furthermore, $\tilde{\Psi}_Z(t)$ is used to derive the augmented vectors $\Psi_Z(t)$ and $\tilde{\Psi}_Z(t+1)$. Combining eqs. (1), (10), and (11)-(13), the augmented estimation vectors with transmission delays $\tau(k)\le N$ and $t=k-\tau(k)$ are represented:

$$\Psi_Z(t)=\left(A_{Z,t1}+H_{Z,t1}F_tE_{Z,t1}\right)\tilde{\Psi}_Z(t)+D_{Z,t1}v_t \tag{14}$$

$$\tilde{\Psi}_Z(t+1)=\left(A_{Z,t2}+H_{Z,t2}F_tE_{Z,t2}\right)\tilde{\Psi}_Z(t)+B_{Z,t2}w_t+D_{Z,t2}v_t \tag{15}$$

where

$$\begin{aligned}&A_{Z,t1}=\begin{bmatrix}I-K_{Z,t}C_t & K_{Z,t}\left(\hat{C}_{Z,t}-C_t\right)\\ K_{Z,t}C_t & I+K_{Z,t}\left(C_t-\hat{C}_{Z,t}\right)\end{bmatrix},H_{Z,t1}=\begin{bmatrix}-K_{Z,t}\mathcal{H}_t\\ K_{Z,t}\mathcal{H}_t\end{bmatrix},E_{Z,t1}=E_{Z,t2}=[E_t\ \ E_t],D_{Z,t1}=\begin{bmatrix}-K_{Z,t}\\ K_{Z,t}\end{bmatrix}\\&A_{Z,t2}=\begin{bmatrix}A_t-L_{Z,t}C_t & A_t-\hat{A}_{Z,t}+L_{Z,t}\left(\hat{C}_{Z,t}-C_t\right)\\ L_{Z,t}C_t & \hat{A}_{Z,t}+L_{Z,t}\left(C_t-\hat{C}_{Z,t}\right)\end{bmatrix},H_{Z,t2}=\begin{bmatrix}\mathcal{F}_t-L_{Z,t}\mathcal{H}_t\\ L_{Z,t}\mathcal{H}_t\end{bmatrix},B_{Z,t2}=\begin{bmatrix}B_t\\ 0\end{bmatrix},D_{Z,t2}=\begin{bmatrix}-L_{Z,t}\\ L_{Z,t}\end{bmatrix}\end{aligned} \tag{16}$$

From eqs. (14) and (15), the covariance matrices are defined based on the augmented estimation vectors $\tilde{\Psi}_Z(t)$, and $\Psi_Z(t)$. Let $\sum_Z(t)=E[\tilde{\Psi}_Z(t)\,\tilde{\Psi}_Z^T(t)]$ and $\Theta_Z(t)=E[\Psi_Z(t)\Psi_Z^T(t)]$, the filtering covariance matrices are calculated according to the projection equation and correlation for noise:

$$\Sigma_Z(t)=\begin{bmatrix}\bar{\Sigma}_Z(t) & 0\\ 0 & P(t)-\bar{\Sigma}_Z(t)\end{bmatrix} \tag{17}$$

where $\bar{\sum}(t) = E[\tilde{e}_Z(t)\,\tilde{e}_Z^T(t)]$ and $P(t) = E[x(t)x^T(t)]$.

Therefore, in eqs. (14)-(17), the Riccati-like equations of error covariance matrices are evolved:

$$\tilde{\Theta}_Z(t)=\left(A_{Z,t1}+H_{Z,t1}F_tE_{Z,t1}\right)\tilde{\Sigma}_Z(t)\left(A_{Z,t1}+H_{Z,t1}F_tE_{Z,t1}\right)^T+D_{Z,t1}R_tD_{Z,t1}^T \tag{18}$$

and

$$\begin{aligned}\tilde{\Sigma}_Z(t+1)&=\left(A_{Z,t2}+H_{Z,t2}F_tE_{Z,t2}\right)\tilde{\Sigma}_Z(t)\left(A_{Z,t2}+H_{Z,t2}F_tE_{Z,t2}\right)^T+\\&+B_{Z,t2}Q_tB_{Z,t2}^T+D_{Z,t2}R_tD_{Z,t2}^T+B_{Z,t2}S_tD_{Z,t2}^T+D_{Z,t2}S_t^TB_{Z,t2}^T\end{aligned} \tag{19}$$

Note that the error covariance matrices are composed of the augmented estimation vectors, correlated noise, filtering and prediction equations.

Upper boundary for estimation covariance

The objective of the robust finite horizon Kalman-like filtering is to obtain a guaranteed upper boundary utilizing the minimum error covariance. Therefore, the guaranteed upper boundaries for filtering and prediction covariance matrices are chosen to derive the filter parameters in this section. The *Lemmas 1 and 2* referring to [20] are introduced to deduce the upper boundary.

Theorem 1. For $(A + HFE)X(A + HFE)^T$, according to *Lemmas 1 and 2*, if there exists a positive scalar α and a symmetric positive definite matrix X, from eqs. (18) and (19), α_t is a positive scalar and $\sum_Z(t)$ is a symmetric positive definite matrix, which satisfy $\alpha_t^{-1}I - E_{Z,t2}\sum_Z(t)E_{Z,t2}^T > 0$. Then $\tilde{\sum}_Z(t) \le \sum_Z(t)$ and $\tilde{\Theta}_Z(t) \le \Theta_Z(t)$, derived from $\tilde{\sum}_Z(t)$. Therefore, the upper boundaries $\Theta_Z(t)$ and $\sum_Z(t + 1)$ are the solutions of the recursive equations:

$$\begin{aligned}\Theta_Z(t)&=A_{Z,t1}\Sigma_Z(t)A_{Z,t1}^T+\alpha_t^{-1}H_{Z,t1}H_{Z,t1}^T+D_{Z,t1}R_tD_{Z,t1}^T+\\&+A_{Z,t1}\Sigma_Z(t)E_{Z,t1}^T\left(\alpha_t^{-1}I-E_{Z,t1}\Sigma_Z(t)E_{Z,t1}^T\right)^{-1}E_{Z,t1}\Sigma_Z(t)A_{Z,t1}^T\end{aligned} \tag{20}$$

and

$$\begin{aligned}\Sigma_Z(t+1)&=A_{Z,t2}\Sigma_Z(t)A_{Z,t2}^T+\alpha_t^{-1}H_{Z,t2}H_{Z,t2}^T+B_{Z,t2}Q_tB_{Z,t2}^T+D_{Z,t2}R_tD_{Z,t2}^T+B_{Z,t2}S_tD_{Z,t2}^T+\\&+D_{Z,t2}S_t^TB_{Z,t2}^T+A_{Z,t2}\Sigma_Z(t)E_{Z,t2}^T\left[\alpha_t^{-1}I-E_{Z,t2}\Sigma_Z(t)E_{Z,t2}^T\right]^{-1}E_{Z,t2}\Sigma_Z(t)A_{Z,t2}^T\end{aligned} \tag{21}$$

Proof. The proof is similar to that reported in [20].

Note that the inequality $\alpha^{-1}I - EXE^T > 0$ and an arbitrary positive constant $\alpha > 0$, both of which are involved in the determination of the upper boundary of the filtering error covariance. Therefore, constant α is critical to defining the upper boundary for filtering. For retaining the filter parameters, the filtering and prediction error covariances are minimized. Based on *Theorem 1* and the Kalman-like filtering, the upper boundary of the error covariance matrices can be defined:

$$E\left[e_Z(t)e_Z^T(t)\right]=[I\,0]\tilde{\Theta}_Z(t)\begin{bmatrix}I\\0\end{bmatrix}\le[I\,0]\Theta_Z(t)\begin{bmatrix}I\\0\end{bmatrix}=\bar{\Theta}_Z(t) \tag{22}$$

and

$$E\left[\tilde{e}_Z(t+1)\tilde{e}_Z^T(t+1)\right]=[I\,0]\tilde{\Sigma}_Z(t+1)\begin{bmatrix}I\\0\end{bmatrix}\le[I\,0]\Sigma_Z(t+1)\begin{bmatrix}I\\0\end{bmatrix}=\bar{\Sigma}_Z(t+1) \tag{23}$$

where $\Theta_Z(t)$ and $\sum_Z(t + 1)$ are derived using eqs. (20) and (21), respectively.

Theorem 2. Introduces the solutions of $\bar{\Theta}_Z(t)$ and $\bar{\Sigma}_Z(t+1)$, for time-stamp $t = k - \tau(k)$, we let $\alpha_t > 0$ a sequence of positive scalars. If the following discrete-time Riccati-like recursive equations:

$$\bar{\Theta}_Z(t) = \bar{\Sigma}_Z(t) + \bar{\Sigma}_Z(t) E_t^T \tilde{M}_{Z,t}^{-1} E_t \bar{\Sigma}_Z(t) - \Lambda_Z(t) \Xi_Z^{-1}(t) \Lambda_Z^T(t) \tag{24}$$

$$\bar{\Sigma}_Z(t+1) = A_t \bar{\Sigma}_Z(t) \left[I + E_t^T M_{Z,t}^{-1} E_t \bar{\Sigma}_Z(t) \right] A_t^T - \Delta_Z(t) \Xi_Z^{-1}(t) \Delta_Z^T(t) + B_t Q_t B_t^T + \alpha_t^{-1} \mathcal{F}_t \mathcal{F}_t^T \tag{25}$$

$$P(t+1) = A_t \left[P^{-1}(t) - \alpha_t E_t^T E_t \right]^{-1} A_t^T + \alpha_t^{-1} \mathcal{F}_t \mathcal{F}_t^T + B_t Q_t B_t^T \tag{26}$$

where

$$\Lambda_Z(t) = \left[I + \bar{\Sigma}_Z(t) E_t^T M_{Z,t}^{-1} E_t \right] \bar{\Sigma}_Z(t) C_t^T \quad \text{and}$$

$$\Delta_Z(t) = A_t \bar{\Sigma}_Z(t) \left[I + E_t^T M_{Z,t}^{-1} E_t \bar{\Sigma}_Z(t) \right] C_t^T + \alpha_t^{-1} \mathcal{F}_t \mathcal{H}_t^T + B_t S_t$$

satisfy

$$P^{-1}(t) - \alpha_t E_t^T E_t > 0 \quad \text{and} \quad M_{Z,t} = \alpha_t^{-1} I - E_t \bar{\Sigma}_Z(t) E_t^T > 0$$

then $\tilde{\Theta}_Z(t)$, $\tilde{\Sigma}_Z(t+1)$ and $P(t+1)$ are the positive definite solutions. The Kalman-like filter parameters can be derived from eqs. (11) and (12):

$$\hat{C}_{Z,t} = C_t \left[I + \bar{\Sigma}_Z(t) E_t^T M_{Z,t}^{-1} E_t \right] \tag{27}$$

$$K_{Z,t} = \Lambda_Z(t) \Xi_Z^{-1}(t) \tag{28}$$

$$\hat{A}_{Z,t} = A\left[I + \bar{\Sigma}_Z(t) E_t^T M_{Z,t}^{-1} E_t \right]_t \tag{29}$$

$$L_{Z,t} = \Delta_Z(t) \Xi_Z^{-1}(t) \tag{30}$$

where

$$\Xi_Z(t) = C_t \bar{\Sigma}_Z(t) \left[I + E_t^T M_{Z,t}^{-1} E_t \bar{\Sigma}_Z(t) \right] C_t^T + \alpha_t^{-1} \mathcal{H}_t \mathcal{H}_t^T + R_t \quad \text{and} \quad \tilde{M}_{Z,t} = \alpha_t^{-1} I - E_t P(t) E_t^T$$

Proof. The proof of this theorem is similar to the derivation shown in [26].

Delay compensation scheme

Note that the output $z(t)$ is stored at time instant, k, with transmission delay $\tau(k)$. The received output, $y(k)$, is used for the state estimation of $\hat{x}(k|t)$. It is worth mentioning that the transmission delays deteriorate the system performance, and our linear estimation-based delay compensation method is proposed to reduce the computational complexity and alleviate the consequence of transmission delays. Suppose that the current time instant is k and the received data packet is $z(t)$. Meanwhile, the estimated state $\hat{x}(t|t)$ is yielded from eq. (11). In order to estimate the state $\hat{x}(k|t)$, the state prediction value $\hat{x}(t+1|t)$ is used for the linear compensation. Depending on the largest delay N and the current transmission delay $\tau(k)$, $\hat{x}(k|t)$is obtained:

$$\hat{x}(k|t) = \left[1 - \frac{\tau(k)-1}{N} \right] \hat{x}(t+1|t) \tag{31}$$

*Remark 3***.** The proposed linear estimation-based method is for approximate state estimation with delay-free. Using the ZOH scheme, the estimated state for the most recent data packet $\hat{x}(t|t)$ is stored with time-stamp before being transmitted. Therefore, the estimated state $\hat{x}(k|k)$ can be computed:

$$\hat{x}(k|k)=\left[\prod_{\tau=1}^{\tau(k)}A(k-\tau)\right]\hat{x}\left[k-\tau(k)|k-\tau(k)\right]$$

where the stored signal at time instant k is $\hat{x}[k-\tau(k)\mid k-\tau(k)]$. On the other hand, if there are no arriving signals from the sensor at k, the estimated state $\hat{x}(k|k)$ will be compensated by one-step prediction of $\hat{x}(k-1|\,k-1)$ [16]. The computational complexity of the aforementioned one-step prediction and the linear estimation-based delay compensation is $O(n^3)$ and $O(n^2)$, respectively, where n expresses the dimension of the state. It is noteworthy that the proposed strategy is able to suppress the computational costs whilst improving the estimation efficiency.

On the other hand, at time instant $k+1$, the arriving data is $y_Z(k+1)$ with transmission delay $\tau(k+1)$. Let $s=k+1-\tau(k+1)$, then the received output is $z(s)$ before being transmitted. Based on the disorder packets, three cases can be considered when we design the filter:

Case 1. For $s=t$ or $s=t+1$, the estimation $\hat{x}_Z(s|s)$ is derived from eq. (11), and the filter parameters are computed by the iterative equations based on *Theorem 2*.

Case 2. For $s>t+1$, the estimation $\hat{x}_Z(s|s)$ will be compensated by one-step prediction of $\hat{x}_Z(t+1|t+1)$ derived from eqs. (11) and (12) with the artificial delay $\tau^{st}(k)=s-t>1$, and the reorganized state estimate sequence:

$$\left\{\hat{x}_Z(t+1|t+1),\cdots,\hat{x}_Z\left[t+\tau^{st}(k)|t+\tau^{st}(k)\right]\right\}$$

For the given systems shown in eqs. (1) and (2), the compensated state and filter parameters are computed using the recursive system shown in eqs. (24)-(30).

Case 3. For $s<t$, the estimated state $\hat{x}_Z(s|s)$ is obtained from $\hat{x}_Z(t-s|t-s)$ shownn in eq. (11). Thus, the filter parameters are updated using the recursive form based on *Theorem 2*.

As mentioned before, the solution of $\hat{x}_Z[k|k-\tau(k)]$ applies to the linear estimation-based compensation method derived from eq. (31).

Filtering for logic ZOH

Here the robust finite horizon Kalman-like filtering is designed based on the standard logic ZOH scheme. At the current sampling time instant k, the received data packet is $z[k-\tau'(k)]$, before being transmitted, with transmission delay $\tau'(k)$. As mentioned before, $\tau'(k)=\tau(k)-\beta(k)$ and $\beta(k)>0$. Let $r=k-\tau'(k)$, the stored measurement is then reorganized using eqs. (2) and (9):

$$y_{LZ}(k)=z(r)=(C_r+\mathcal{H}_rF_rE_r)x(r)+v_r \tag{32}$$

Suppose that the filter can be used to estimate the optimal state $\hat{x}_{LZ}(r|r)$, depending on the stored data $\{y_{LZ}(0),..., y_{LZ}(k-1), y_{LZ}(k)\}$.

The largest transmission delay does not exceed N steps, furthermore, $\tau'(k)\le\tau(k)$ as the received valid data packet is close to the current signal. The logic ZOH is used to choose time-stamped signals with $\tau'(k)$-step transmission delays, *i. e.* $r=k-\tau'(k)$, and the structure of the robust Kalman-like filtering is proposed:

$$\hat{x}_{LZ}(r|r)=\hat{x}_{LZ}(r|r-1)+K_{LZ,r}\left[z(r)-\hat{C}_{LZ,r}\hat{x}_{LZ}(r|r-1)\right] \tag{33}$$

$$\hat{x}_{LZ}(r+1|r)=\hat{A}_{LZ,r}\hat{x}_{LZ}(r|r-1)+L_{LZ,r}\left[z(r)-\hat{C}_{LZ,r}\hat{x}_{LZ}(r|r-1)\right] \tag{34}$$

where $\hat{C}_{LZ,r}$, $K_{LZ,r}$, $\hat{A}_{LZ,r}$, and $L_{LZ,r}$, are the filter parameters.

To obtain the filter parameters and the upper boundary from the estimation covariance matrices, *Theorem 3* presents the solution of the proposed filtering method. The subscript Z represented by the ZOH is replaced by the logic ZOH denoted by LZ, and the received time stamp is expressed as r instead of t .

Theorem 3. At current time instant k, for the measurement $y_{LZ}(k)$ with transmission delay $\tau'(k)$, $z(r)$ with time-stamp $r = k - \tau'(k)$ is received. Let α_r be a positive scalar, then $\bar{\sum}(r)$ and $P(r)$ are the positive definite solutions for discrete-time Riccati-like iterations:

$$\bar{\Theta}_{LZ}(r) = \bar{\Sigma}_{LZ}(r) + \bar{\Sigma}_{LZ}(r) E_r^T \tilde{M}_{LZ,r}^{-1} E_r \bar{\Sigma}_{LZ}(r) - \Lambda_{LZ}(r) \Xi_{LZ}^{-1}(r) \Lambda_{LZ}^T(r) \tag{35}$$

$$\bar{\Sigma}_{LZ}(r+1) = A_r \bar{\Sigma}_{LZ}(r)\left[I + E_r^T M_{LZ,r}^{-1} E_r \bar{\Sigma}_{LZ}(r)\right] A_r^T - \Delta_{LZ}(r) \Xi_{LZ}^{-1}(r) \Delta_{LZ}^T(r) + B_r Q_r B_r^T + \alpha_r^{-1} \mathcal{F}_r \mathcal{F}_r^T \tag{36}$$

$$P(r+1) = A_r \left[P^{-1}(r) - \alpha_r E_r^T E_r\right]^{-1} A_r^T + \alpha_r^{-1} \mathcal{F}_r \mathcal{F}_r^T + B_r Q_r B_r^T \tag{37}$$

where

$$\Lambda_{LZ}(r) = \bar{\Sigma}_{LZ}(r) C_r^T + \bar{\Sigma}_{LZ}(r) E_r^T M_{LZ,r}^{-1} E_r \bar{\Sigma}_{LZ}(r) C_r^T$$

$$\Delta_{LZ}(r) = A_r \bar{\Sigma}_{LZ}(r)\left[I + E_r^T M_{LZ,r}^{-1} E_r \bar{\Sigma}_{LZ}(r)\right] C_r^T + \alpha_r^{-1} \mathcal{F}_r \mathcal{H}_r^T + B_r S_r$$

satisfying

$$P^{-1}(r) - \alpha_r E_r^T E_r > 0 \text{ and } M_{LZ,r} = \alpha_r^{-1} I - E_r \bar{\Sigma}_{LZ}(r) E_r^T > 0$$

Then, the Kalman-like filtering shown in eqs. (33) and (34) has the parameters:

$$\hat{C}_{LZ,r} = C_r\left[I + \bar{\Sigma}_{LZ}(r) E_r^T M_{LZ,r}^{-1} E_r\right] \tag{38}$$

$$K_{LZ,r} = \Lambda_{LZ}(r) \Xi_{LZ}^{-1}(r) \tag{39}$$

$$\hat{A}_{LZ,r} = A_r\left[I + \bar{\Sigma}_{LZ}(r) E_r^T M_{LZ,r}^{-1} E_r\right] \tag{40}$$

$$L_{LZ,r} = \Delta_{LZ}(r) \Xi_{LZ}^{-1}(r) \tag{41}$$

where

$$\Xi_{LZ}(r) = C_r \bar{\Sigma}_{LZ}(r)\left[I + E_r^T M_{LZ,r}^{-1} E_r \bar{\Sigma}_{LZ}(r)\right] C_r^T + \alpha_r^{-1} \mathcal{H}_r \mathcal{H}_r^T + R_r \text{ and } \tilde{M}_{LZ,r} = \alpha_r^{-1} I - E_r P(r) E_r^T$$

Proof. Different from the ZOH scheme, the received measurement $y_{LZ}(k)$ is represented by the valid arriving data $z(r)$ with $r = k - \tau'(k)$, and $\tau'(k) = \tau(k) - \beta(k)$. Accordingly, the filter parameters $\hat{C}_{LZ,r}$, $K_{LZ,r}$, $\hat{A}_{LZ,r}$, and $L_{LZ,r}$ are calculated by different signal reconstruction schemes. Subsequently, the proof is similar to that of *Theorem 2*.

The obtained state estimation $\hat{x}(r|r)$ is used for compensating $\hat{x}(k|r)$ at the current time instant k. For the logic ZOH, we assume that the largest transmission delay is N, and the filter receives the ACK data packet $z(r)$ with transmission delay $\tau'(k)$ at time k. Similar to the linear estimation-based compensation approach, the estimated state $\hat{x}_{LZ}(r|r)$ derived from eq. (33) is used for compensating the predicted state $\hat{x}_{LZ}(k|r)$, which is represented:

$$\hat{x}_{LZ}(k \mid r) = \left[1 - \frac{\tau'(k) - 1}{N}\right] \hat{x}_{LZ}(r+1 \mid r) \tag{42}$$

Remark 4. The logic ZOH has the transmission delay as $\tau'(k) \le \tau(k)$. Different from the ZOH, based on the linear compensation strategy, the estimated state $\hat{x}(k|r)$ conforms to the inequality $\hat{x}_Z(k|r) \le \hat{x}_{LZ}(k|r)$. In this case, the signal reconstruction scheme using logic ZOH can discard out-of-order packets and improve the estimation performance for the networked systems.

For the next sampling time $k + 1$, the arriving data is $y_{LZ}(k + 1)$ with delay $\tau'(k + 1)$. Meanwhile, the received output is $z(s)$ assuming $s = k + 1 - \tau'(k + 1)$. Since the disordered packets are discarded, we shall have $s \ge r$. The designed filter $\hat{x}_{LZ}(s|s)$ has two cases:

Case 1. For $s = r$ or $s = r + 1$, the state estimation $\hat{x}_{LZ}(s|s)$ is derived from eq. (33), and the filter parameters are calculated by the recursive equations based on eqs. (33) and (34), and *Theorem 3*.

Case 2. For $s > r + 1$, the estimated state $\hat{x}_{LZ}(s|s)$ will be compensated by $\tau'(k)$-step prediction, which is calculated from eqs. (33) and (34) with the artificial delay $\tau^{sr}(k) = s - r > 1$. Thus, based on $\hat{x}_{LZ}(r + 1|r + 1)$, the reorganized state estimation sequence is compensated by one-step:

$$\left\{\hat{x}_{LZ}\left(r+1\,|\,r+1\right),\cdots,\hat{x}_{LZ}\left[r+\tau^{sr}\left(k\right)|\,r+\tau^{sr}\left(k\right)\right]\right\} \tag{43}$$

For the given systems shown in eqs. (1) and (2), the missing states are compensated and the filter parameters are computed by the recursions shown in eqs. (35)-(41). The proposed approach can be summarized in *Algorithm I* using the logic ZOH scheme.

***Algorithm I*. Algorithm of the robust finite horizon filtering using the logic ZOH**

Input: The initial state $x(0)$, the estimated state $\hat{x}_{LZ}(0|-1) = \mu_0$ with variance P_0, the upper boundary of the prediction error covariance $\bar{\sum}_{LZ}(0)$ and the positive scalar α_0, the sample time *iter*

Output: The filtering error covariance $\bar{\sum}_{LZ}(k + 1)$ and $P(k + 1)$

```
for k = 1 to iter
    set r = k – τ′(k),     //   τ′(k)-step delays
    if τ^sr(k) = 0 or τ^sr(k) = 1
        calculate Θ̄_LZ(r) using eq. (35),    //   Θ̄_LZ(r) is the upper boundary/
        solve Ĉ_LZ,t, K_LZ,t, Â_LZ,t, and L_Z,t, given in eqs. (38)-(41), //  they are derived from Σ̄_LZ(r)
        compute x̂_LZ(r|r) in eq. (33), and x̂_LZ(r + 1|r) in eq. (34), // design filter and predictor
        solve Σ̄_LZ(r | 1) using eq. (36), and P(r + 1) using eq. (37), // filtering and state covariance
    else if τ^sr(k) > 1
        x̂_LZ(s|s) is compensated by x̂_LZ(r + 1|r + 1) using eqs. (33) and (34)
    end if
    calculate x̂_LZ(k|r) by eq. (42),   //   the compensation of the estimation-based state
    define s = k + 1 – τ′(k) + 1;   //   τ′(k + 1)-step delays
    set τ^sr(k) = s – r ;   //   the artificial delay
end
```

Remark 5. A natural way of handling the finite-horizon Kalman filtering is to augment the system states [20, 23]. However, as seen from *Theorem 1*, such a state augmentation approach leads to a significant increase of the system dimension and thereby brings heavy computational burden to the recursive filtering algorithm. Compared to the state augmentation method, our proposed finite horizon Kalman-like recursive filter employing the signal re-ordering scheme shown in eqs. (11), (12), (33), and (34) does not suffer from the expensive computation problem. The filter parameters and the upper boundary of minimizing the error covariance matrices have been discussed in [20, 23]. In our work, *Theorems 2 and 3* lead to the filter gain matrices K and L for obtaining the upper boundary and improving the estimation accuracy of the stochastic uncertain systems. In addition, as pointed out in the linear estimation-based delay compensation method, the proposed filter avoids using the traditional one-step prediction approach so that it has satisfactory computational efficiency to handle random transmission delays. Therefore, the proposed finite horizon filtering approach is suitable for addressing the issue of the distributed state estimation.

Simulation results

In this section, numerical evaluation is conducted to illustrate the effectiveness of the proposed robust finite horizon Kalman-like filtering approach.

The target tracking systems with intermittent measurements are a class of stochastic uncertain systems presented [15, 20, 25]:

$$x(k+1)=\left(\begin{bmatrix}0.9 & T & \frac{T^2}{2}\\ 0 & 0.9 & T\\ 0 & 0 & 0.9\end{bmatrix}+\mathcal{F}_k F_k E_k\right)x(k)+\begin{bmatrix}\frac{T^2}{2}\\ T\\ 1\end{bmatrix}w_k\,,\quad k=1,2,\cdots \tag{44}$$

$$z(k)=\left(C_k+\mathcal{H}_k F_k E_k\right)x(k)+v_k \tag{45}$$

$$w_k=\eta_k \tag{46}$$

$$v_k=\zeta w_k \tag{47}$$

where we set $T = 0.1$ second as the sample period. The upper limit of the transmission delay is assumed to be $N = 5$, and the time-varying parametric uncertainty satisfies $F_k = \sin(0.6k)$. The state $x(k) = (s_k, \dot{s}_k, \ddot{s}_k)^T$ is composed of position, velocity and acceleration, respectively, of the target at time instant kT, $\eta_k \in \mathbb{R}$ is the zero mean white noise with variance $\sigma_\eta^2 = 0.09$, which is uncorrelated with the other signals. Set the state transition matrices as $\mathcal{F}_k = [0.1\ 0.1\ 0.1]^T$, $E_k = [0.02\ 0.02\ 0.02]$, $C_k = [0.6\ 0.8\ 1]$, and the measurement matrix is $\mathcal{H}_k = 0.8$. Taking into account the correlation between the process noise and the measurement noise, the variable ζ shown in eq. (47) determines the correlated strength and here $\zeta = 2$. Without loss of generality, the process noise w_k shown in eq. (46) with unity covariance Q_k. Meanwhile, the covariances are denoted as $R_k = \zeta Q_k \zeta^T$ and $S_k = Q_k \zeta^T$, given in eq. (47), respectively.

Set the initial values to be $\alpha_k = 3$, $\hat{x}(0|0) = \mu_0 = E[x(0)] = (1\ 1\ 1)^T$ and $P(0|0) = 0.01I_3$. The proposed method is evaluated using 300 sampling points, and the results are obtained, based on 100 Monte-Carlo simulations.

Figures 2(a)-2(d) show the traces of the filtering error covariances and the estimated state with delay-free, which are calculated using the proposed filtering method and the improved robust finite-horizon Kalman filtering (IRFHKF) introduced in [20]. To compare the estimation proposed method is very close to the actual state and performance of the both methods, we observe that the upper boundary of the filtering error covariance employing the proposed filter is remarkably lower than that of the IRFHKF method. The dynamic tracking trajectory of the proposed method is very close to the actual state.

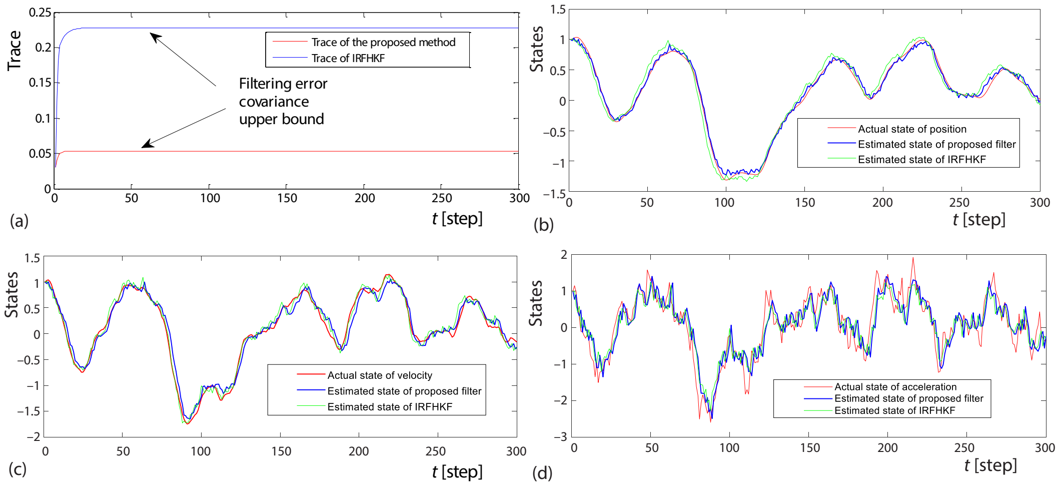

Figure 2. Comparison of the proposed method and IRFHKF method; (a) upper boundary of filtering error covariance, (b) comparison of the estimated state for position, (c) comparison of the estimated state for velocity, and (d) comparison of the estimated state for acceleration

Table 1. Comparison of the filtering error covariances

Method	Position	Velocity	Acceleration	Trace
Proposed method	0.0096~0.0150	0.0016~0.0100	0.0100~0.0372	0.0300~0.0533
IRFHKF	0.0100~0.0267	0.0100~0.0235	0.0100~0.1779	0.0300~0.2280

The range of the performance index from the minimum to the maximum is presented in tab. 1. Since the upper boundary of the error covariance by the proposed method is less than IRFHKF, the proposed filtering method has relatively small errors for the estimated state.

To further illustrate the effectiveness of the proposed method, the corresponding filtering results of the estimated states are shown in fig. 3, which are obtained by eqs. (11) and (12) in *Theorem 2* for ZOH and eqs. (33) and (34) in *Theorem 3* for logic ZOH, respectively.

Comparing IRFHKF, ZOH and logic ZOH schemes, figs. 3(a)-3(c) show that the proposed system can handle the packet loss and transmission delay. Furthermore, taking into account the network-induced random transmission delays, the logic ZOH scheme has better performance for target tacking and computational efficiency using the linear estimation-based compensation strategy.

Conclusion

With the aid of a linear delay compensation strategy, the state estimation problem has been investigated for discrete-time stochastic uncertain systems with correlated noise. Considering networked-induced transmission delays and out-of-order packets, a new system model has been established using the ZOH and logic ZOH schemes, respectively, to deal with packet disorders caused by sequence re-ordering. Based on the established model, a robust finite horizon Kalman-like filter has been designed to guarantee an optimized upper boundary. For random transmission delays, a linear estimation-based delay compensation scheme has been proposed to improve the filter performance whilst reducing the computational complexity. Compared with the ZOH and the logic ZOH schemes, the proposed modelling and filtering strategies have superior performance to drop packet disorders and reduce the upper boundary

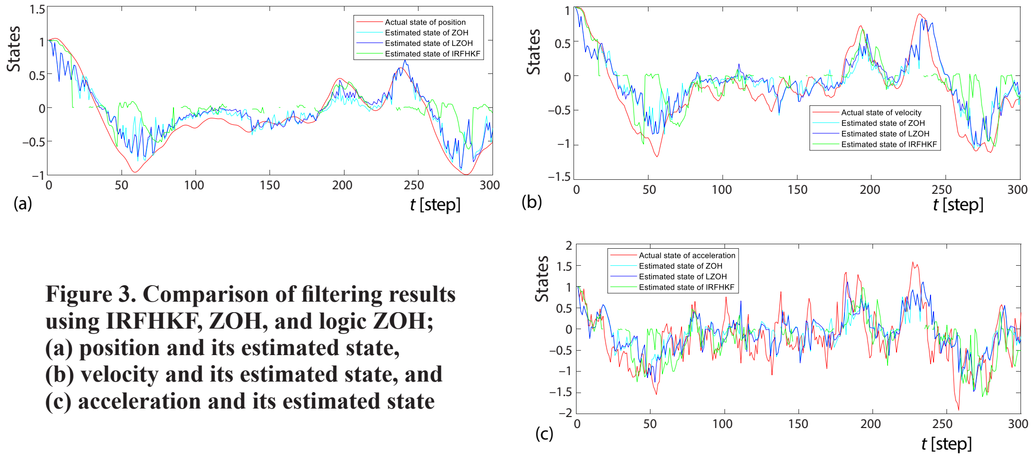

Figure 3. Comparison of filtering results using IRFHKF, ZOH, and logic ZOH; (a) position and its estimated state, (b) velocity and its estimated state, and (c) acceleration and its estimated state

of the estimation error covariances. A target tracking system and numerical simulations have been performed to demonstrate the effectiveness of the proposed approach. The state estimation problem can be also modelled by the variational principle, and the stochastic uncertain can be effectively described by the fractal calculus [17], which we will discuss in a forthcoming paper.

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Nomenclature

$E(\cdot)$ – mathematical expectation operator
M^{-1} – inverse of the positive-definite matrix M
M – real symmetric positive-definite matrix (> 0)
$\mathbb{R}^r$ – r-dimensional Euclidean space
$\mathbb{R}^{r\times r}$ – $r\times r$-dimensional Euclidean space
$tr(M)$ – trace of matrix M

Greek symbol

δ_{k-l} – Kronecker function, *i. e.* $\delta_{k-l} = 1$ if $k = l$, otherwise $\delta_{k-l} = 0$ if $k \neq 1$

Superscript

T – transpose

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