This paper presents numerical simulations for a magnetohydrodynamic convective process in curved channels. The worked suspension consists of water as a based hybrid nanofluid and two types of the nanoparticles, namely, Cu and Al₂O₃. Two systems of the governing equations are formulated for the hybrid nanofluid and dusty phases. The hybrid nanofluid system is modeled in view of lubrication approach. The governing equations are mapped to a regular computational domain then they solved numerically using the fourth order Runge-Kutta method. The obtained findings revealed that the growing in the Hartmann number causes a reduction in both of the hybrid nanofluid and dusty velocities while the mixture temperature is enhanced. Also, the temperature distributions are supported when either the Grashof number or the amplitude ratio is altered.

**Keywords:** MHD, Peristaltic motion, dusty particles, hybrid nanofluid, curved channels.

**Nomenclature**

- $a$ Dimensional wave amplitude
- $c$ Wave speed
- $c_p$ Specific heat at constant pressure
- $C$ Concentration
- $d_1$ Thickness of the channel
- $D_s$ Mass concentration of the dust particle
- $Ec$ Eckert number
- $g$ Gravity acceleration
- $Gr$ Thermal buoyancy parameter
- $Ha$ Hartmann number
Curvature parameter
Pressure
Prandtl number
Radius parameter
Reynolds number
Time
Temperature

Velocity components in the R and X direction

Dimensional Cartesian coordinates

Dimensionless Cartesian coordinates

Greek symbols

$\alpha$ Coefficient of linear thermal expansion

$\alpha_d$ Dust parameter depending on the relaxation time of the particles and the buoyancy force

$\beta$ Coefficient of thermal expansion

$\lambda$ Wave length

$\delta$ Wave number

$\varepsilon$ Amplitude ratio

$\tau_m$ Momentum relaxation time

$\tau_t$ Thermal relaxation time

$\theta$ Dimensionless temperature

$\mu$ Dynamic viscosity

$\nu$ Kinematic viscosity

$\kappa$ Thermal conductivity

$\rho$ Density

$\phi$ Nanoparticle volume fraction

$\psi$ Stream function
1. Introduction

Various practical applications can be found for the peristalsis flow inside irregular domains. These applications include, for example, movement of the food in the intestine tract, the urine passage from a kidney to the bladder, blood flow in small veins and arteries of the blood circulation, transferring the ovum in the Fallopian tube and the movement of sperm in the channels. Also, there are many applications of peristaltic motion in the biomedical devices, such as blood pumps and heart lung machines. In the related context, Latham [1], Mishra and Pandey [2] proposed the physiology of the gastrointestinal tract as an example for the peristalsis flow. Also, the peristaltic flows in curved channels were handled by several studies [3-11]. Sato et al. [3] discussed the peristaltic flow in a curved channel. Ali et al. [4] studied the peristaltic flow in a curved channel with a long wavelength approximation. Ali et al. [5, 6] examined effects of the heat transfer and fluid flow of a non-Newtonian third grade fluid in a curved channel. Hayat et al. [7] discussed the Newtonian fluid peristaltic flow, heat and mass transfer in a curved channel with compliant walls. The investigation in [7] was extended by Hayat et al. [8] and Hina et al. [9] to include case of a third grade fluid. Hina et al. [9] considered case of the peristaltic motion in curved channels contain compliant walls using Johnson-Segalman fluid. Hina et al. [10] and Hina et al. [11] examined the double diffusive together with impacts of the wall properties on the peristaltic flow of Johnson-Segalman and the peristaltic flow of pseudoplastic fluid, respectively.

Recently, the hybrid nanofluid which is considered as a novel type of the nanofluids received a major attention from the researchers due to their applications in various fields. The traditional nanofluid mixture is introduced for the first time by Choi [12] who studied the pure fluids with suspended nanoparticles. His results indicated to the substantial augmentation of the heat transported in suspensions of copper or aluminum nanoparticles in water or other liquids. Additionally, the transport governing equations in which the Brownian motion and thermophores impacts were assumed have been presented by Buongiorno [13]. Using these types of mixtures, the peristaltic flows inside the curved channels are still few. Hina et al. [14] discussed numerically the peristaltic flow of a nanofluid in a curved channel. Ayub et al. [15] studied the mixed convection in the presence of a thermal radiation and a chemical reaction, analytically. The results indicated that the heat transfer rate decreases with the increase in thermophoresis parameter. Narla et al. [16] studied the peristaltic transport of a Jeffrey nanofluid in a curved channel and examined effects of various parameters on the fluid flow and the temperature distributions. Noreen et al. [17] discussed the induced magnetic field effects on the peristaltic flow in a curved channel. They found that an increase in the Brownian motion and thermophoresis parameters causes an increase in the temperature profiles. Hayat et al. [18] studied the peristaltic motion of a copper-water based nanoliquid with the thermal slip conditions. Hayat et al. [19] studied the MHD peristaltic flow of Sisko nanofluids with the Joule heating effects. They used a numerical treatment for the governing equations. They found that the increasing values
of the curvature parameter results in symmetric behaviors at the centerline of the channel for the velocity, temperature and concentration distributions. Tanveer et al. [20] studied the peristaltic motion of a Sisko fluid with homogeneous-heterogeneous reaction effects. The results revealed that the lower velocity, temperature and concentration profiles are obtained in case of the higher bending. Tanveer et al. [21] studied the peristaltic flow of Eyring-Powell nanofluids in a curved channel with compliant walls. They found that the Eyring-Powell parameters tend to decrease the velocity and temperature of the nanofluid while the concentration bears a dual response. Hayat et al. [22] discussed the peristalsis of MHD Jeffery nanofluids in a curved channel with a porous medium.

In a related context, there are several practical applications in atmospheric, engineering and physiological fields can be found for the dusty mixture such as conveying of powdered materials, purification of crude oil, environmental pollutants, dust in gas cooling systems, petroleum industry. Rudinger [23] presented a valuable book that includes many other applications of the dusty mixture. Farbar and Morley [24] used a circular tube as a flow domain to examine the heat transfer convective flow of the gas-solid mixtures. The dusty model for the laminar flow was proposed by Saffman [25]. Various works [26–33] extended the dusty fluid topic with different physical circumstance. Recently, many researchers attempt to generalize these topics in case of the nanofluids. Siddiqa et al. [34] conducted an analysis of a two-phase natural convection flow of dusty nanofluid along a vertical wavy surface. They found that presence of the dust particles have a notable influence on the temperature distribution as the isotherms get stronger for the dusty water. Begum et al. [35] studied the gyrotactic bioconvection of the dusty nanofluid along an isothermally heated vertical wall. They applied a numerical treatment for the mathematical model using the two-point implicit finite difference method. Gireeshala et al. [36], Gireesha et al. [37] studied the Hall effect on a two-phase transient flow with stretching sheet using KVL model and irregular heat generation/consumption, respectively. The references [38-39] handle interesting studies in these topics.

In all of the aforementioned works, the magnetic peristaltic motion of the dusty hybrid nanofluids in complex channels is ignored. Therefore, this paper aims to present comprehensive simulations for the magnetohydrodynamic convective peristaltic flow of a hybrid nanofluid within a curved channel. Water is assumed as a base fluid while hybrid nanoparticles of copper and alumina are used as well as the one phase model is applied to simulate this situation. During the formulation of the problem the approximation of the low Reynolds number and the long wave length are taken into account. Also, one of the objectives of this study is to express the pressure distributions in the flow domain and examining effects of the dusty and geometry parameters on the hybrid nanofluid flow and temperature distributions.

2. Discerption of the problem and mathematical formulation

The time-dependent 2D peristaltic flows of dusty hybrid nanofluids within curved channels are considered. The physical model of the channel and the coordinates system are depicted in Figure 1. In order to formulate the governing systems of the problem, the following assumptions are considered:
The X-axis is considered along the wall of the channel while the R-axis is considered normal to the channel surface.

Width of the channel is $2d_1$ surrounded in a circle of a radius $R'$ and a center $O$.

The mathematical formulation of the irregular boundaries of the channel is $\bar{\eta} = \pm \left[d_1 + a \sin(X - ct) \frac{2\pi}{\lambda}\right]$; where $c, a$ and $\lambda$ are the speed, amplitude and length of the wave.

The components of the suspension are water as a based nanofluid and hybrid of Cu and Al$_2$O$_3$ nanoparticles.

Table 1 shows the values of the thermophysical properties of the mixture components.

A thermal equilibrium model is considered between the base fluid and the hybrid nanoparticles.

The magnetic field, Joule heating and viscous dissipation impacts are taken into account.

The linear Boussinesq approximation is applied for the suspension density.

Uniform sizes of the dusty particles are assumed and they distribute equally in the mixture.

Table 1: Values of the base fluid and nanoparticles thermophysical properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>H$_2$O</th>
<th>Al$_2$O$_3$</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>997.1</td>
<td>3970</td>
<td>8933</td>
</tr>
<tr>
<td>$C_p$</td>
<td>4179</td>
<td>765</td>
<td>385</td>
</tr>
<tr>
<td>$k$</td>
<td>0.613</td>
<td>40</td>
<td>401</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$21 \times 10^{-5}$</td>
<td>$0.85 \times 10^{-5}$</td>
<td>$1.67 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.05</td>
<td>$1 \times 10^{-10}$</td>
<td>$5.96 \times 10^7$</td>
</tr>
</tbody>
</table>

Under all the aforementioned assumptions, the governing equations of the problem are introduced as, see [14, 26-27].

**Hybrid Nanofluid phase**

$$\frac{\partial v}{\partial R} + \frac{R_1}{R + R_1} \frac{\partial u}{\partial x} + \frac{v}{R + R_1} = 0$$ (1)
\[
\rho_{hfnf} \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + \frac{U R_1}{R + R_1} \frac{\partial V}{\partial x} + \frac{U^2}{R + R_1} = - \frac{\partial P}{\partial R} + \frac{\mu_{hfnf}}{R + R_1} \left[ \frac{\partial}{\partial R} \left( \left( R + R_1 \right) \tau_{RR} \right) + \frac{R_1}{R + R_1} \frac{\partial \tau_{XR}}{\partial x} - \frac{\tau_{XX}}{R + R_1} \right] + \frac{\rho_s}{\tau_m} (V_s - V)
\] (2)

\[
\rho_{hfnf} \frac{\partial U}{\partial t} + V \frac{\partial U}{\partial x} + \frac{U R_1}{R + R_1} \frac{\partial U}{\partial x} + \frac{U V}{R + R_1} = - \frac{R_1}{R + R_1} \frac{\partial P}{\partial x} + \frac{\mu_{hfnf}}{(R + R_1)^2} \frac{\partial}{\partial R} \left( \left( R + R_1 \right)^2 \tau_{RX} \right) + \left( \rho \beta \right)_{hfnf} g (T - T_0) + \frac{\mu_{hfnf}}{R + R_1} \frac{\partial \tau_{RX}}{\partial x} + \frac{\rho_s c_s}{\tau_m} \left( U_s - U \right) - \frac{\sigma_{hfnf} B_0^2}{(\rho c_p)_{hfnf}} U
\] (3)

\[
\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} + \frac{U R_1}{R + R_1} \frac{\partial T}{\partial x} = \frac{k_{hfnf}}{(\rho c_p)_{hfnf}} \nabla^2 T + \frac{\mu_{hfnf}}{(\rho c_p)_{hfnf}} \left[ 4 \left( \frac{\partial V}{\partial R} \right)^2 + \left( \frac{\partial U}{\partial R} + \frac{U V}{R + R_1} + \frac{\partial V}{\partial x} \right)^2 \right] + \frac{\rho_s c_s}{\tau_t (\rho c_p)_{hfnf}} \left( T_s - T \right) + \frac{\mu_{hfnf}}{R + R_1} \frac{\partial \tau_{RX}}{\partial x} + \frac{\sigma_{hfnf} B_0^2}{(\rho c_p)_{hfnf}(R + R_1)^2} U^2
\] (4)

**Dusty particles**

\[
\frac{\partial V_s}{\partial R} + \frac{R_1}{R + R_1} \frac{\partial U_s}{\partial x} + \frac{V_s}{R + R_1} = 0
\] (5)

\[
\rho_s \left[ \frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial x} + \frac{U_s R_1}{R + R_1} \frac{\partial V_s}{\partial x} - \frac{U_s^2}{R + R_1} = - \frac{\partial P_s}{\partial R} - \frac{\rho_s}{\tau_m} (V_s - V) \right]
\] (6)

\[
\rho_s \left[ \frac{\partial U_s}{\partial t} + V_s \frac{\partial U_s}{\partial x} + \frac{U_s R_1}{R + R_1} \frac{\partial U_s}{\partial x} + \frac{U_s V_s}{R + R_1} \right] = - \frac{R_1}{R + R_1} \frac{\partial P_s}{\partial x} - \frac{\rho_s}{\tau_m} (U_s - U)
\] (7)

\[
\rho_s c_s \left[ \frac{\partial T_s}{\partial t} + V_s \frac{\partial T_s}{\partial x} + \frac{U_s R_1}{R + R_1} \frac{\partial T_s}{\partial x} \right] = - \frac{\rho_s c_s}{\tau_t} (T_s - T)
\] (8)

where

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R + R_1} \frac{\partial}{\partial x} \left( \frac{R_1}{R + R_1} \frac{\partial}{\partial x} \right)
\]

\[
\tau_{XX} = \mu_{hfnf} \left( \frac{2 R_1}{R + R_1} \frac{\partial U}{\partial x} + \frac{2 V}{R + R_1} \right)
\]

\[
\tau_{XR} = \mu_{hfnf} \left( \frac{R_1}{R + R_1} \frac{\partial V}{\partial x} - \frac{U}{R + R_1} + \frac{\partial U}{\partial x} \right), \quad \tau_{RR} = 2 \mu_{hfnf} \frac{\partial V}{\partial R}
\]

In Eqs. (1)-(8), \( U, V, U_s, V_s \) are the velocity components of the fluid and dust phase, in the laboratory frame \( (R, X) \); \( P, P_s \) is the pressure, of the fluid and dust phase, \( \sigma_{hfnf} \) are the electrical conductivity, \( \rho_f, \rho_s \) is density of the fluid and dust phase, \( \mu \) is the dynamic viscosity, \( \eta \) is the kinematic viscosity, \( \kappa \) is the thermal conductivity, \( \kappa_p \) is the specific heat at constant pressure, \( \kappa \) is the concentration and \( T \) is the temperature of the fluid, \( \alpha \) is the coefficient of linear thermal expansion of the fluid, \( \beta \) is the coefficient of expansion with concentration, and \( g \) is acceleration due to gravity. If \( (r, x) \) and \( (u, v), (u_s, v_s) \) are the coordinates and velocity components in the wave frame then:

\[
\bar{x} = X - ct, \quad \bar{r} = R, \quad \bar{u} = U - c, \quad \bar{v} = V, \quad \bar{P}(x) = P(X, t) \bar{u}_s = \bar{U}_s - c, \quad \bar{v}_s = V_s, \quad \bar{P}_s(x) = P_s(X, t)
\] (9)

Also, the following non-dimensional quantities are introduced:

\[
x = \frac{\bar{x}}{c}, \quad r = \frac{\bar{r}}{d_1}, \quad u = \frac{\bar{u}}{c}, \quad v = \frac{\bar{v}}{c}, \quad \delta = \frac{d_1}{\lambda}, \quad \tau = \frac{P}{\rho c_0}, \quad \eta = \frac{\bar{\eta}}{c_0}, \quad e = \frac{\alpha}{d_1}, \quad k = \frac{R_1}{d_1}, \quad \phi = \frac{C - C_0}{C_0}, \quad \theta = \frac{T - T_0}{T_0}
\]

\[
u_s = \frac{u_s}{c}, \quad \nu_s = \frac{v_s}{c}, \quad P_s = \frac{\bar{P}_s}{c_0 \lambda}
\] (10)
In Eq. (10), the subscript s refers to the dusty phase and 0 refers to the conditions at the channels walls. Moreover, definitions of the stream function for the nanofluid phase \( \psi \) and the dusty particles phase \( \bar{\psi} \) are expressed as:

\[
\frac{\partial \bar{\psi}}{\partial z}, \quad u = -\frac{\partial \psi}{\partial r}, \quad \nu_s = \frac{\delta k \frac{\partial \psi}{\partial k}}{r+k} \quad \text{and} \quad \nu = \frac{\delta k \frac{\partial \psi}{\partial k}}{r+k} \quad \text{(11)}
\]

When the previous equations (Eqs. (9)-(10)) are inserted in the systems (1)-(8) and taking into account estimations of the low Reynolds number and long of the wave length, the governing systems are given as:

**Hybrid nanofluid phase:**

\[
\frac{\partial \rho}{\partial r} = 0 \quad \text{(12)}
\]

\[
-\frac{k}{r+k} \frac{\partial \rho}{\partial x} + \frac{\mu_{\text{hnf}}}{\mu_f} \frac{\rho_f}{\rho_{\text{hnf}}} \frac{1}{r+k} \frac{\partial}{\partial r} [(r+k)^2 (\psi_{rrr} - \frac{\psi_{r}}{r+k})] + \frac{(\rho \beta)_{\text{hnf}}}{(\rho \beta)_f} Gr \theta + D_s \alpha_d (\bar{\psi}_r - \psi_r) - \\
\frac{\sigma_{\text{hnf}}}{\sigma_f (r+k)} Ha^2 (1 - \psi_r) = 0 \quad \text{(13)}
\]

\[
\frac{1}{Pr} \frac{(\rho c_p)_f}{\rho_{\text{hnf}}} \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r+k} \frac{\partial \theta}{\partial r} + \frac{Ec}{Pr} \frac{(\rho c_p)_f}{\rho_{\text{hnf}}} \frac{\mu_{\text{hnf}}}{\mu_f} \left( \psi_{rrr} + \psi_{r} - \frac{\psi_{r}}{r+k} \right) + \frac{2}{3 Pr} \frac{(\rho c_p)_f}{\rho_{\text{hnf}}} D_s \alpha_d (\theta_s - \theta) + \\
\frac{\sigma_{\text{hnf}}}{\sigma_f (r+k)} D_s \alpha_d Ec (\bar{\psi}_r - \psi_r)^2 + \frac{\sigma_{\text{hnf}}}{\sigma_f (r+k)} Ec Ha^2 \frac{1}{(r+k)^2} (1 - \psi_r)^2 = 0 \quad \text{(14)}
\]

**Dusty particles:**

\[
\frac{\partial P_s}{\partial r} = 0 \quad \text{(15)}
\]

\[
0 = -\frac{k}{r+k} \frac{\partial P_s}{\partial x} - \alpha_d (\bar{\psi}_r - \psi_r) \quad \text{(16)}
\]

\[
0 = (\theta_s - \theta) \quad \text{(17)}
\]

where

\[
Pr = \frac{v_f (\rho c_p)_f}{\kappa_f}, \quad Re = \frac{\rho_f c_{d1}}{\mu_f}, \quad Ec = \frac{\rho f c^2}{(\rho c_p)_f r_0}, \quad Gr = \frac{\beta^2 (\rho \beta)_f r_0}{\mu_f c^2}, \quad Ha^2 = \frac{\beta^2 \sigma_f}{\mu_f},
\]

\[
\gamma = \frac{c_s}{c_p}, \quad \alpha_d = \frac{d^2}{\tau_m}, \quad \tau_t = \frac{3}{2} \tau_m \gamma Pr, \quad D_s = \frac{\rho_s}{\rho_f}
\]

\[
\text{In Eq.}(18), \text{ Pr is the Prandtl number, Re is the Reynolds number, Ec is the Eckert number, } Gr \text{ is the thermal buoyancy parameter, Ha Hartman number, } \gamma \text{ is the specific heat ratio of the mixture, } D_s \text{ is the mass concentration of particle phase and } \alpha_d \text{ is the dust parameter. Also, Eqs.}(12)-(13) \text{ and } (15)-(16) \text{ after eliminating the pressure terms are written as:}
\]

\[
\frac{\mu_{\text{hnf}}}{\mu_f} \left[ (r+k) \psi_{rrr} + 2 \psi_{rr} - \frac{\psi_{r}}{r+k} \right] - \left[ (r+k) \left( Gr \frac{(\rho \beta)_{\text{hnf}}}{(\rho \beta)_f} \frac{\partial \theta}{\partial r} + D_s \alpha_d \frac{\partial \bar{\psi}_r}{\partial r} - \psi_r \right) + \\
\frac{\sigma_{\text{hnf}}}{\sigma_f (r+k)} \left( Ha^2 (1 - \psi_r) + (r+k) \psi_{rr} \right) \right] - \left[ Gr \frac{(\rho \beta)_{\text{hnf}}}{(\rho \beta)_f} \theta + D_s \alpha_d (\bar{\psi}_r - \psi_r) - \frac{\sigma_{\text{hnf}}}{\sigma_f (r+k)} Ha^2 (1 - \psi_r) \right] = \\
\frac{\mu_{\text{hnf}}}{\mu_f} \frac{1}{(r+k)^2} \quad \text{(19)}
\]
\[ \bar{\psi}_{rr} - \psi_{rr} + \frac{1}{r+k} (\bar{\psi}_{r} - \psi_{r}) = 0 \]  
(20)

\[ \Delta P^* = k \int_0^1 \left[ \mu_{hf} \frac{1}{\mu_f} \left( -\psi_{rr} - \frac{(1-\psi_r)}{r+k} \right) + (r + k) \left( -\frac{\mu_{hf}}{\mu_f} \psi_{rrr} + G_T \frac{(\rho\beta)_{hf}}{(\rho\beta)_{bf}} \theta + D_s \alpha_d (\bar{\psi}_r - \psi_r) - \frac{\sigma_{hf}}{\sigma_f (r+k)} \right) \right] dx \]  
(21)

The corresponding boundary conditions are given by:

\[ \psi = \bar{\psi} \frac{F}{r}, \frac{\partial \psi}{\partial r} = 1, \theta = 0, \phi = 0, \bar{\psi} = \bar{\psi} \frac{F}{r}, \frac{\partial \bar{\psi}}{\partial r} = 1, \beta_s = 0 \text{ at } r = \pm(1 + \epsilon \sin 2\pi X) \]  
(22)

On the other hand, rate of the volume flows in the laboratory frame and in the wave frame are, respectively given by:

\[ Q = \int_{-\bar{\eta}}^{\bar{\eta}} U(R,X,t) \, dR \]  
(23)

\[ q = \int_{-\bar{\eta}}^{\bar{\eta}} u(r,x) \, dr \]  
(24)

The dimensionless mean flows in the laboratory \( \Theta \) and in the wave frame \( F \) are defined as:

\[ \Theta = \frac{\bar{Q}}{c d_s}, F = \frac{F}{c d_s} \]  
(25)

where, \( \bar{Q} \) is the time averaged flow over a period \( \bar{T} \) and it is given by:

\[ \bar{Q} = \frac{1}{\bar{T}} \int_0^{\bar{T}} Q \, dt \]  
(26)

From the previous equations, the following relations are obtained:

\[ \Theta = F + 2 \]  
(27)

\[ F = \int_{-\bar{\eta}}^{\bar{\eta}} \psi_r \, dr \]  
(28)

Additionally, the hybrid nanofluid properties are evaluated according to the following correlations:

\[ \rho_{huf} = \phi_{Al,O_i} \rho_{Al,O_i} + \phi_{\text{Cu}} \rho_{\text{Cu}} + (1-\phi) \rho_f \]  
(29a)

\[ (\rho C_p)_{huf} = \phi_{Al,O_i} (\rho C_p)_{Al,O_i} + \phi_{\text{Cu}} (\rho C_p)_{\text{Cu}} + (1-\phi)(\rho C_p)_f \]  
(29b)

\[ (\rho \beta)_{huf} = \phi_{Al,O_i} (\rho \beta)_{Al,O_i} + \phi_{\text{Cu}} (\rho \beta)_{\text{Cu}} + (1-\phi)(\rho \beta)_f \]  
(29c)

\[ \alpha_{huf} = \frac{k_{huf}}{(\rho C_p)_{huf}} \]  
(29d)

\[ \frac{k_{huf}}{k_{bf}} = \left( \frac{(\phi_{Al,O_i}, k_{Al,O_i} + \phi_{\text{Cu}} k_{\text{Cu}})}{\phi} + 2k_{bf} + 2(\phi_{Al,O_i}, k_{Al,O_i} + \phi_{\text{Cu}} k_{\text{Cu}}) - 2\phi k_{bf} \right) \times \left( \frac{(\phi_{Al,O_i}, k_{Al,O_i} + \phi_{\text{Cu}} k_{\text{Cu}})}{\phi} + 2k_{bf} - (\phi_{Al,O_i}, k_{Al,O_i} + \phi_{\text{Cu}} k_{\text{Cu}}) + \phi k_{bf} \right)^{-1} \]  
(29e)
\[
\mu_{nf} = \frac{\mu_{nf}}{(1-(\phi_{Al_{2}O_{3}}+\phi_{Cu}))^{2.5}}
\]
(29f)

\[
\sigma_{nf} = 1. + \frac{3\left(\frac{\phi_{Al_{2}O_{3}}\sigma_{Al_{2}O_{3}}+\phi_{Cu}\sigma_{Cu}}{\sigma_f}\right) - \left(\frac{\phi_{Al_{2}O_{3}}\sigma_{Al_{2}O_{3}}+\phi_{Cu}\sigma_{Cu}}{\sigma_f}\right)}{\sigma_f}
\]
(29g)

3. Method of solution

To solve the governing equations (14), (15), (18), (20) and (21), it is needed to map the wavy boundaries into a rectangular computational domain. Therefore, the following new independent variables are introduced:

\[
\xi = X, \quad \eta' = \frac{r}{1+\epsilon \sin 2\pi x}
\]
(30)

The partial derivatives for the dependent variables are obtained as follows:

\[
\begin{bmatrix}
\frac{\partial \Phi}{\partial \chi} \\
\frac{\partial \Phi}{\partial r}
\end{bmatrix} = \frac{1}{|J|} \begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix} \begin{bmatrix}
\frac{\partial \Phi}{\partial \xi} \\
\frac{\partial \Phi}{\partial \eta'}
\end{bmatrix}
\]
(31)

Where \(\beta_{11} = \frac{\partial r}{\partial \eta'}, \beta_{12} = -\frac{\partial r}{\partial \xi'}, \beta_{21} = -\frac{\partial \chi}{\partial \eta'}, \beta_{22} = \frac{\partial \chi}{\partial \xi'}\) and \(|J| = \beta_{11}\beta_{22} - \beta_{21}\beta_{12}\)
Using Eqs. (30) and (31), the computational domain is transformed to $-1 \leq \eta' \leq 1$ which makes the applying of the numerical method is available. Here the Runge-Kutta method with shooting technique is used to solve the resulting system of the equations. The number of the grid points are taken to be equal 401 and the convergence criteria is $10^{-6}$. In addition, a validation test consisting of comparisons with previously published results is performed and presented in Fig. 2. It is found very good agreements are observed between the presented study (in special cases) and those obtained by Hina et al. [14].

4. Results and discussion

Discussion of the obtained results is notified in this section. A set of graphical results in terms of the velocity profiles for the dusty particles, temperature distributions and nanoparticle volume fraction are presented in Figs. 3-16. During these computations, the governing parameters are considered in wide ranges i.e range of the thermal buoyancy parameter $1 \leq Gr \leq 4$, range of the Hartmann parameter $Ha$ is $0 \leq Ha \leq 4$, range of amplitude ratio $\epsilon$ is $1 \leq \epsilon \leq 5$, range the Eckert number is $0.1 \leq Ec \leq 0.5$ and range of the copper nanoparticles is $0 \leq \phi_{Cu} \leq 0.05$. Here it should be mentioned that from Eq. (17), the profiles of the temperature of the dusty particles are the same of nanofluid temperature. Also, values of the mass concentration of the dusty particles $Ds$ and the dusty parameters $\alpha_d$ are assumed 0.1 and 10, respectively.

Figures 3-5 show the profiles of the fluid velocity, dusty velocity and fluid temperature for various values of $Ha$ at $Gr = 1, \epsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, \phi_{Cu} = 0.01$. The results revealed that the growing in the magnetic parameter $Ha$ causes a diminishing in the convective process and hence both of the fluid and dusty velocities are reduced. Physically, these behaviors are due to the Lorentz force resulting from the magnetic field that works against to the hybrid nanofluid flow. On the contrary, the temperature distributions are enhanced as $Ha$ is increased due to the increase in the thermal boundary layer as the magnetic force is enhanced.
Fig. 3. Fluid velocity for different values of $\text{Ha}$ at $\text{Gr} = 1, \varepsilon = 0.2, \text{Ec} = 1, \text{Pr} = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, \phi_{cu} = 0.01$

Fig. 4. Dusty velocity for different values of $\text{Ha}$ at $\text{Gr} = 1, \varepsilon = 0.2, \text{Ec} = 1, \text{Pr} = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, \phi_{cu} = 0.01$

Fig. 5. Profiles of the fluid temperature for different values of $\text{Ha}$ at $\text{Gr} = 1, \varepsilon = 0.2, \text{Ec} = 1, \text{Pr} = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, \phi_{cu} = 0.01$

Figures 6-8 display the hybrid nanofluid velocity, dusty velocity and mixture temperature for various values of $\text{Gr}$ at $\phi_{cu} = 0.01, \varepsilon = 0.2, \text{Ec} = 1, \text{Pr} = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, \text{Ha} = 1$.

It is noted that in the range $0.25 \leq \eta \leq 0.75$, the alteration of $\text{Gr}$ causes that the hybrid nanofluid and dusty velocities tend to decrease while the inverse behaviors are noted for the other values of $\eta$. Additionally, the hybrid nanofluid temperature is increasing as $\text{Gr}$ is growing, for all values of $\eta$. 

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Profiles of the hybrid nanofluid velocity, dusty velocity and the suspension temperature for various values of amplitude ratio $\varepsilon$ at $\Phi_{cu} = 0.01, \varepsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$ are displayed in Figures 9-11. Like impacts of $Gr$ on the hybrid nanofluid and dusty velocities, the variations of $\varepsilon$ reduces the velocity components in the range $0.25 \leq \eta \leq 0.75$ while these components are enhanced in the remaining values of $\eta$. Furthermore, a clear enhancement in the temperature distributions is given as the amplitude ratio $\varepsilon$ is growing. The increasing behaviors of the temperature are noted for all values of $\eta$ with higher variations of the temperature when $\varepsilon$ is altered from 0.8 to 1 comparing to the other values of $\varepsilon$. 

Fig. 6. Fluid velocity for different values of $Gr$ at $\Phi_{cu} = 0.01, \varepsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

Fig. 7. Dusty velocity for different values of $Gr$ at $\Phi_{cu} = 0.01, \varepsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

Fig. 8. Profiles of the fluid temperature for different values of $Gr$ at $\Phi_{cu} = 0.01, \varepsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$
Fig. 9. Fluid velocity for different values of $\varepsilon$ at $\phi_{\text{cu}} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

Fig. 10. Dusty fluid velocity for different values of $\varepsilon$ at $\phi_{\text{cu}} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

Fig. 11. Profiles of the fluid temperature for different values of $\varepsilon$ at $\phi_{\text{cu}} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

In Figures 12-14, impacts of the curvature parameter $k$ on the velocities of the hybrid nanofluids and dusty phases together with the temperature distributions at $\phi_{\text{cu}} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, \alpha_d = 10, D_s = 0.1, Ha = 1, \varepsilon = 0.2$ are illustrated. The findings disclosed that there are two opposite behaviors as $k$ is increased. Those are enhancements of the velocities as $k$ is growing in the lower part of the channel and a weakness in the flows as $k$ increases in the upper parts of the channel. However, the temperature distributions are diminishing as $k$ is altered in the whole domain. All these behaviors show that the curvature parameter is a good factor in controlling the flow and thermal fields.
Fig. 12. Fluid velocity for different values of \( k \) at \( \Phi_{Cu} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, \alpha_d = 10, D_a = 0.1, Ha = 1, \epsilon = 0.2 \)

Fig. 13. Dusty velocity for different values of \( k \) at \( \Phi_{Cu} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, \alpha_d = 10, D_a = 0.1, Ha = 1, \epsilon = 0.2 \)

Fig. 14. Profiles of the fluid temperature for different values of \( k \) at \( \Phi_{Cu} = 0.01, Gr = 1, Ec = 1, Pr = 6.82, \alpha_d = 10, D_a = 0.1, Ha = 1, \epsilon = 0.2 \)

The temperature features for the different values of the Eckert number \( Ec \) and the copper nanoparticles volume fraction \( \Phi_{Cu} \) at \( Gr = 1, k = 5, Pr = 6.82, \alpha_d = 10, D_a = 0.1, Ha = 1, \epsilon = 0.2 \) are displayed in Figures 15 and 16. The results revealed that the temperature distributions are augmented as \( Ec \) is increased. Physically, the growing in \( Ec \) means the increase in the viscous dissipation process which heated up the mixture and hence the temperature distributions are enhanced. On the contrary, the growing in \( \Phi_{Cu} \) makes the mixture more viscous and thus the convective process is reduced and as a result the temperature distributions are diminishing as \( \Phi_{Cu} \) is increasing.
Fig. 15. Profiles of the fluid temperature for different values of $\text{Ec}$ at $\Phi_{\text{cu}} = 0.01, Gr = 1, k = 5, Pr = 6.82, \alpha_d = 10, D_s = 0.1, Ha = 1, \varepsilon = 0.2$

Fig. 16. Profiles of the fluid temperature for different values of $\Phi_{\text{cu}}$ at $Gr = 1, \varepsilon = 0.2, Ec = 1, Pr = 6.82, k = 5, \alpha_d = 10, D_s = 0.1, Ha = 1$

5. Conclusions

The current study carried out numerical simulations for the magnetic convective peristaltic flow of a dusty hybrid nanofluid in curved channels. The low Reynolds number and the long wave length approach are assumed. Two systems of the equations are presented for the hybrid nanofluid and the dusty particles phases. A mathematical form for the pressure distributions is introduced and validation tests with previously published results are performed. The main outcomes of this study can be summarized as:

- The temperature lineaments are supported as either the Hartmann number or the Grashof number are growing.
- An increase in the Eckert number enhances the viscous dissipation process and hence the temperature distributions tend to increase.
- A weakness hybrid nanofluid and dusty flows are given as the magnetic force is enhanced.
- Values of the dusty temperature are equal values of the nanofluid temperature regardless variations of the governing parameter.
- Velocity of the dusty particles is reduced as the Grashof number and the amplitude ratio increase (particularly at the mid-section of the channel).
- The current study can be extended in the future to include the radiation impacts, variable magnetic field and non-linear Boussinesq approximation.
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References


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