FRACTAL APPROACH TO EXPLANATION OF SILKWORM COCOON'S BIOMECHANISM

by

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Fractal calculus is an excellent tool to explaining natural phenomena in porous media. In this paper, we first give a simple introduction on He's fractal derivative, and then it is used to establish a model for thermal conduction of silkworm cocoon reveal its biomechanism. The theoretical results obtained in this paper are helpful for the biomimetic design.

Key words: *fractal calculus, porous media, fractal space, silkworm cocoon*

Introduction

China is the first country in the world to raise silk from the silkworm cocoon, dating back to more than 5000 years [1]. He [1] claimed that silk is of China, and China is of silk. Both the words of *China* and *silk* came from a Chinese character si (丝) for silk. Silkworm fibres are extracted from silkworm cocoon and become a raw material for weaving silk. Silkworm fibres have long been used for textiles as an excellent natural materials, and silk fibroin [2-4] is widely used for fabrication of nanofibers for various advanced applications by either the electrospinning or the bubble spinning [5-11]. Silkworm cocoons have very good protective effect on silkworm pupa. Firstly, cocoon can protect silkworm pupa from other animals. Secondly, no matter how low/high the temperature outside, silkworm cocoons always can maintain a certain temperature suitable for silkworm life. Thirdly, when it is raining or wet outside, the silkworm cocoon always keeps its inside dry. Silkworm cocoons have such excellent properties that it would be very meaningful if we research them and applied them to textiles. Chen *et al*. [12], gave a theoretical study of silk's waterproof and dustproof properties [12], Chen *et al.* [13] suggested a fractal nanohydrodynamics for explanation of cocoon's excellent air permeability, and they claimed that the cocoon is a real emperor's new clothes. All the excellent properties of the silkworm cocoon come from its hierarchical structure. Many experimental studies revealed that air permeability can remarkably be improved by a hierarchical structure [14, 15].

In this paper, He's fractal calculus [16, 17] and two scale transform method are adopted to elucidate the heat conduction mechanism of silkworm cocoon. In addition, experiment will be done to explain the waterproof mechanism of silkworm cocoon.

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A brief introduction fractal calculus

The fractal calculus studies phenomena arising in porous media [16], so it is extremely suitable to reveal the hidden mechanism of the cocoon. In the past decades, both engineering and mathematicians have devoted considerable effort to the study of fractal geometry, fractional calculus and fractal calculus. The fractal calculus is a very excellent tool to elucidate a lot of natural phenomena [18-30]. The most used fractal derivative is (Chen's fractal derivative) [17]:

$$\frac{\mathrm{D}\Theta}{\mathrm{D}x^{\alpha}} = \lim_{\zeta \to x} \frac{\Theta(x) - \Theta(\zeta)}{x^{\alpha} - \zeta^{\alpha}}$$

where α is an index different from an integer number, generally $\alpha < 1$. Though it is simple in form, it has no physical meaning.

In this paper, we use the He's fractal derivative to elucidate the thermal transfer of silkworm cocoon. The He's fractal derivative is defined [17]:

$$\frac{\mathrm{D}\Xi}{\mathrm{D}x^{\alpha}}(x_0) = \Gamma(1+\alpha) \lim_{\substack{x - x_0 \to \Delta x \\ \Delta x \neq 0}} \frac{\Xi(x) - \Xi(x_0)}{(x - x_0)^{\alpha}}$$

where α is the fractal dimensions, Δx is a very small value different from zero, it is a porous size of the cocoon. He's fractal derivative has attracted the interest for many researchers. It can well explain many physical phenomena, for example, wool fibers, water permeation and heat transfer in fractal media. The He's fractal derivative is very similar to the local fractional derivative defined [17]:

$$\mathrm{D}_x^{(\alpha)} f(x_0) = f^{(\alpha)}(x_0) = \left. \frac{\mathrm{d}^{\alpha} f(x)}{\mathrm{d}x^{\alpha}} \right|_{x = x_0} = \lim_{x \to x_0} \frac{\Delta^{\alpha}[f(x) - f(x_0)]}{(x - x_0)^{\alpha}}$$

where

$$\Delta^{\alpha}[f(x) - f(x_0)] \cong \Gamma(1+\alpha)\Delta[f(x) - f(x_0)]$$

Fractal model for heat transfer of silkworm cocoon

The silkworm cocoon is a fractal porous hierarchy [12, 13]. In order to clearly explain the fractal model for heat transfer of silkworm cocoon, we use the Fourier's law of thermal conduction:

$$\sigma = -\lambda \frac{\Delta T}{\Delta x} \tag{1}$$

and

$$\frac{\Delta T}{\Delta t} = -\frac{\Delta \sigma}{\Delta x} \tag{2}$$

where λ is the material conductivity, σ – the heat flux and the temperature gradient is $\Delta T/\Delta x$.

In the continuous medium, the eq. (1) can be written:

$$\sigma = -\lambda \frac{\mathrm{DT}}{\mathrm{D}x} \tag{3}$$

and

$$\frac{\mathrm{D}T}{\mathrm{D}t} = -\frac{\mathrm{D}\sigma}{\mathrm{D}x} \tag{4}$$

Substitute eq. (4) into eq. (3), we obtain:

$$\frac{\mathrm{D}T}{\mathrm{D}t}+\frac{\mathrm{D}}{\mathrm{D}x}\left(\lambda\frac{\mathrm{D}T}{\mathrm{D}x}\right)=Q_0 \tag{5}$$

In this paper, we will use He's fractal derivative [16] to explain the fractal model of heat transfer of silkworm cocoon, so eq. (5) have to be modified for fractal media. We have:

$$\frac{\mathrm{D}T}{\mathrm{D}t}+\frac{\mathrm{D}}{\mathrm{D}x^\alpha}\left(\lambda\frac{\mathrm{D}T}{\mathrm{D}x^\alpha}\right)=Q_0 \tag{6}$$

with boundary conditions

$$\begin{aligned} T(0,t)&=T_0\\ T(L,t)&=T_\infty \end{aligned} \tag{7}$$

where T is the temperature, α – the fractal dimensions of fractal medium, Q_0 – the heat source and $\mathrm{D}/\mathrm{D}x^\alpha$ – the He's fractal derivative defined:

$$\frac{\mathrm{D}T}{\mathrm{D}x^\alpha}(x_0)=\Gamma(1+\alpha)\lim_{\substack{x_1-x_2\to\Delta x\\ \Delta x\neq 0}}\frac{T(x)-T(x_0)}{(x-x_0)^\alpha}$$

where α the fractal dimensions of silkworm cocoon, Δx – the very small value different from zero, it is the porous size of the cocoon. The value of fractal dimensions is equal to the order of fractal derivative. There are many analytical methods for solving eq. (6) [31-33], in this paper we adopt the two scale transform method [34, 35], which is to convert a fractal space to its continuous partner as explained in [36]. Let:

$$X=x^\alpha \tag{8}$$

Equation (6) can be written into its partner:

$$\frac{\mathrm{D}T}{\mathrm{D}t}+\frac{\mathrm{D}}{\mathrm{D}X}\left(\lambda\frac{\mathrm{D}T}{\mathrm{D}X}\right)=Q_0 \tag{9}$$

A Taylor series solution [37] can be easily found for eq. (9), in this paper, we mainly consider the steady case of the model of heat transfer of silkworm cocoon, eq. (9) can be written into the form:

$$\frac{\mathrm{D}}{\mathrm{D}X}\left(\lambda\frac{\mathrm{D}T}{\mathrm{D}X}\right)=Q_0 \tag{10}$$

Its exact solution:

$$T=\frac{Q_0}{2\lambda}X^2+\kappa X+\mu \tag{11}$$

Substituting eq. (8) into eq. (11), we have:

$$T=\frac{Q_0}{2\lambda}x^{2\alpha}+\kappa x^\alpha+\mu \tag{12}$$

Using the boundary condition, we can obtain the result:

$$T=\frac{Q_0}{2\lambda}x^{2\alpha}+\frac{\left(T_\infty-T_0-\dfrac{Q_0}{2\lambda}\right)x^\alpha}{L^\alpha}+T_0 \tag{13}$$

The temperature of silkworm cocoon change is very slowly on its inner-surface. So we can obtain:

$$\frac{\mathrm{D}T}{\mathrm{D}x}=\frac{\alpha Q_0}{\lambda}x^{2\alpha-1}+\frac{\left(T_\infty-T_0-\dfrac{Q_0}{2\lambda}L^{2\alpha}\right)\alpha x^{\alpha-1}}{L^\alpha} \tag{14}$$

$$\frac{\mathrm{D}^2T}{\mathrm{D}x^2}=\frac{\alpha Q_0(2\alpha-1)}{\lambda}x^{2\alpha-2}+\frac{\left(T_\infty-T_0-\dfrac{Q_0}{2\lambda}L^{2\alpha}\right)\alpha(\alpha-1)x^{\alpha-2}}{L^\alpha} \tag{15}$$

If the fractal dimensions [38-45] of silkworm cocoon are $\alpha > 1.5$, we have:

$$\lim_{x\to 0}x^{\alpha-2}=\infty \tag{16}$$

It contradicts to the fact of the very slow change of body temperature for silkworm cocoon. In order to eliminate the contradictory fact, we have:

$$\frac{\left(T_\infty-T_0-\dfrac{Q_0}{2\lambda}L^{2\alpha}\right)\alpha(\alpha-1)}{L^\alpha}=0 \tag{17}$$

namely

$$T_\infty-T_0-\frac{Q_0}{2\lambda}L^{2\alpha}=0 \tag{18}$$

From eq. (18) we obtain a critical thickness for the cocoon, less than which the bio-mechanism becomes invalid.

Thus, eq.(13) can be written into:

$$T=\frac{Q_0}{2\lambda}x^{2\alpha}+T_0 \tag{19}$$

The fractal dimensions of silkworm cocoon are $\alpha > 1.5$, we can easily find the results:

$$\left.\frac{\mathrm{D}T(x)}{\mathrm{D}x}\right|_{x=0}=0 \tag{20}$$

$$\left.\frac{\mathrm{D}^2T(x)}{\mathrm{D}x^2}\right|_{x=0}=0 \tag{21}$$

$$\left.\frac{D^3T(x)}{Dx^3}\right|_{x=0}=0 \tag{22}$$

Equations (20)-(22) show that the temperature change on the inner surface of silkworm cocoon surface is quite slow regardless of the temperature of external environment. Equation (19) shows the properties of the solution for different fractal dimensions α. It is clearly when the fractal dimensions $\alpha > 1$, the inner temperature of the cocoon changes very small regardless of the temperature of external environment of the silkworm cocoon.

When $Q_0 = 0$, the solution of eq. (10):

$$T=\kappa X+\mu \tag{23}$$

Substitute eq. (8) into eq. (23), we have:

$$T=\kappa x^\alpha+\mu \tag{24}$$

Using the boundary condition, we can obtain:

$$T = \frac{(T_\infty - T_0)x^\alpha}{L^\alpha} + T_0 \tag{25}$$

The eq. (25) has the following property:

– when $\alpha > 1$, we have

$$\left.\frac{\mathrm{D}T(x)}{\mathrm{D}x}\right|_{x=0} = 0 \tag{26}$$

– when $\alpha = 1$, we have

$$\left.\frac{\mathrm{D}T(x)}{\mathrm{D}x}\right|_{x=0} = \frac{L_\infty - L_0}{L} \tag{27}$$

– when $\alpha < 1$, we have

$$\left.\frac{\mathrm{D}T(x)}{\mathrm{D}x}\right|_{x=0} = \infty \tag{28}$$

According to eq. (25), the slope at $x = 0$ depends on the value of α.

Conclusion

In this paper, we give a simple introduction He's fractal calculus [16]. The model of thermal conduction of silkworm cocoon is described by He's fractal derivative. Our results show that the fractal calculus is a powerful and effective tool to dealing with natural phenomena in porous media. The establishment of thermal conduction mechanisms for the silkworm cocoon will be helpful for the biomimetic design.

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References

[1] He, J.-H., Silk is of China, and China is of Silk: A Response to Good *et al*. (2009), *Archaeometry*, *53* (2011), 2, pp. 411-412
[2] Liu, F. J., He, J. H., Bubble Electrospun Silk Fibroin for Tissue Engineering Scaffolds, *Journal of Controlled Release*, *172* (2013), 1, pp. E127-E127
[3] Rockwood, D. N., *et al*., Materials Fabrication from Bombyx Mori Silk Fibroin, *Nature Protocols*, 6 (2011), 10, pp. 1612-1631
[4] Min, B. M., *et al*., Electrospinning of Silk Fibroin Nanofibers and Its Effect on the Adhesion and Spreading of Normal Human Keratinocytes and Fibroblasts in Vitro, *Biomaterials*, *25* (2004), 7-8, pp. 1289-1297
[5] Li, X. X., *et al*., The Effect of Sonic Vibration on Electrospun Fiber Mats, *Journal of Low Frequency Noise Vibration and Active Control*, *38* (2019), 3-4, pp. 1246-1251
[6] Zhao, J. H., *et al*., Needle's Vibration in Needle-Disk Electrospinning Process: Theoretical Model and Experimental Verification, *Journal of Low Frequency Noise Vibration and Active Control*, *38* (2019), 3-4, pp. 1338-1344
[7] Wu, Y. K., Liu, Y., Fractal-Like Multiple Jets in Electrospinning Process, *Thermal Science*, *24* (2020), 4, pp. 2499-2505
[8] He, J. H., Advances in Bubble Electrospinning, *Recent Patents on Nanotechnology*, *13* (2019), 3, pp. 162 -163
[9] He, J. H., Liu, Y. P., Bubble Electrospinning: Patents, Promises and Challenges. Recent Patents on Nanotechnology, *14* (2020), 1, pp. 3-4
[10] He, C. H., *et al*., Taylor Series Solution for Fractal Bratu-Type Equation Arising in Electrospinning Process, *Fractals*, *28* (2020), 1, 2050011

[11] He, J. H., On the Height of Taylor Cone in Electrospinning, *Results in Physics*, *17* (2020), June, 103096
[12] Chen, R. X., *et al*., Waterproof and Dustproof of Wild Silk: A Theoretical Explanation, Journal of NanoResearch, *22* (2013), May, pp. 61-63
[13] Chen, R. X., *et al.*, Silk Cocoon: Emperor's New Clothes for Pupa: Fractal Nanohydrodynamical Approach, *Journal of NanoResearch, 22* (2013), May, pp. 65-70
[14] Yang, Z. P., et al., A Fractal Model for Pressure Drop through a Cigarette Filter, *Thermal Science*, *24* (2020), 4, pp. 2653-2659
[15] Yang, Z. P., *et al*., On the Cross-Section of Shaped Fibers in the Dry Spinning Process: Physical Explanation by the Geometric Potential Theory, *Results in Physics*, *14* (2019), Sept., 102347
[16] He, J. H., Fractal Calculus and Its Geometrical Explanation, *Result in Physics*, *10* (2018), Sept., pp. 272-276
[17] He, J. H., Ain, Q. T., New Promises and Future Challenges of Fractal Calculus: from Two-Scale Thermodynamics to Fractal Variational Principle, *Thermal Science*, *24* (2020), 2A, pp. 659-681
[18] He, J. H., A Tutorial Review on Fractal Spacetime and Fractional Calculus, *Int. J. Theor. Phys*., *53* (2014), June, pp. 3698-718
[19] Wang, K. L., He, C. H., A Remark on Wang's Fractal Variational Principle, *Fractals*, *27* (2019), 08, 1950134
[20] Wang, Q. L., *et al*., Fractal Calculus and Its Application Explanation of Biomechanism of Polar Bear Hairs, *Fractals*, *26* (2018), 6, 1850086
[21] Wang, Q. L., *et al*., Fractal Calculus and Its Application Explanation of Biomechanism of Polar Hairs (Vol. 26, 1850086, 2018), *Fractals*, *27* (2019), 5, 1992001
[22] Wang, Y., Deng, Q. G., Fractal Derivative Model for Tsunami Travelling, *Fractals*, *27* (2019), 1, 1950017
[23] Hu, Y., He, J. H., On Fractal Space Time and Fractional Calculus, *Thermal Science*, *20* (2016), 3, pp. 773-777
[24] Wang, K. L., Wang, K. J., A Modification of the Reduced Differential Transform Method for Fractional Calculus, *Thermal Science*, *22* (2018), 4, pp. 1871-1875
[25] Fei, D. D., *et.al*., Fractal Approach to Heat Transfer in Silkworm Cocoon Hierarchy, *Thermal Science*, *17* (2013), 5, pp. 1546-1548
[26] Liu, F. J., *et al.*, A Delayed Fractional Model for Cocoon Heat-Proof Property, *Thermal Science*, *21* (2017), 4, pp. 1867-1871
[27] Liu, F .J., *et al*., A Fractional Model for Insulation Clothings with cocoon-like porous structure, *Thermal Science, 20* (2016), 3, pp. 779-784
[28] Wang, Q. L., *et al*., Fractional Model for heat conduction in polar bear hairs, *Thermal Science*, *16* (2012), 2, pp. 339-342
[29] Fan, J., *et al*., Fractal Calculus for Analysis of Wool Fiber: Mathematical Insight of Its Biomechanism, *Journal of Engineered Fibers and Fabrics*, On-line first, https://doi.org/10.1177/1558925019872200, 2019
[30] Liu, F. J., *et al*., Silkworm (Bombyx Mori) Cocoon *vs*. Wild Cocoon: Multi-Layer Structure and Performance Characterization, *Thermal Science*, *23* (2019), 4, pp. 2135-2142
[31] He, J H., A Fractal Variational Theory for 1-D Compressible Flow in a Microgravity Space, *Fractals*, *28* (2020), 2, 2050024
[32] He, J. H., A Simple Approach to 1-D Convection-Diffusion Equation and Its Fractional Modification for E Reaction Arising in Rotating Disk Electrodes, *Journal of Electroanalytical Chemistry*, *854* (2019), 113565
[33] He, J. H., A Short Review on Analytical Methods for to a Fully Fourth Order Non-Linear Integral Boundary Value Problem with Fractal Derivatives, *International Journal of Numerical Methods for Heat and Fluid-Flow*, *30* (2020), 11, pp. 4933-4943
[34] Ain, Q. T., He, J. H., On Two-Scale Dimension and Its Applications, *Thermal Science*, *23* (2019), 3A, pp. 1313-1318
[35] He, J. H., Ji, F. Y., Two-Scale Mathematics and Fractional Calculus for Thermodynamics, *Thermal Science*, *23* (2019), 4, pp. 2131-2133
[36] Wang, K. L, *et al*., Physical Insight of Local Fractional Calculus and Its Application Fractional Kdv-Burgers-Kuramoto Equation, *Fractals*, *27* (2019), 07, 1950122
[37] He, J. H., Taylor Series Solution for a Third Order Boundary Value Problem Arising in Architectural Engineering, *Ain Shams Engineering Journal*, On-line first, https://doi.org/10.1016/j.asej.2020.0106, 2020
[38] Wang, K. L., *et.al.*, A Fractal Variational Principle for the Telegraph Equation with Fractal Derivatives, *Fractals*, *28* (2020), 4, 2050058
[39] Wang, K. L., He's Frequency Formulation for Fractal Nonlinear Oscillator Arising in a Microgravity space, *Numerical Methods for Partial Differential Equations*, On-line first, https://doi.org/10.1002/num.22584, 2020

[40] Wang, K. L., A Novel Approach for Fractal Burgers-BBM Equation and its Variational Principle, 2020, *Fractals*, On-line first, https://doi.org/10.1142/S0218348X2150059, 2020
[41] Wang, K. L., Effect of Fangzhu's Nanoscale Surface Morphology on Water Collection, *Mathematical Method in the Applied Sciences*, On-line first, https://doi.org/10.1002/mma.6569, 2020
[42] Wang, K. J., Wang, K. L., Variational Principles for Fractal Whitham-Broer-Kaup Equations in Shallow Water, *Fractals*, On-line first, https://doi.org/10.1142/S0218348X21500286, 2020
[43] Wang, K. J., A New Fractional Nonlinear Singular Heat Conduction Model for the Human Head Considering the Effect of Febrifuge, *Eur. Phys. J. Plus*, *135* (2020), Nov., 871
[44] Wang, K. J., Variational Principle and Approximate Solution for the Generalized Burgers-Huxley Equation with Fractal Derivative, *Fractals*, On-line first, https://doi.org/10.1142/S0218348X21500444, 2020
[45] Wang, K. J., Variational Principle and Approximate Solution for the Fractal Vibration Equation in a Microgravity Space, *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, On-line first, https://doi.org/10.1007/s40997-020-00414-0, 2020

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