THERMAL ANALYSIS OF MAGNETOHYDRODYNAMIC VISCOUS FLUID WITH INNOVATIVE FRACTIONAL DERIVATIVE

by

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In this study, an attempt is made to investigate a fractional model of unsteady and an incompressible MHD viscous fluid with heat transfer. The fluid is lying over a vertical and moving plate in its own plane. The problem is modeled by using the constant proportional Caputo fractional derivatives with suitable boundary conditions. The non-dimensional governing equations of problem have been solved analytically with the help of Laplace transform techniques and explicit expressions for respective field variable are obtained. The transformed solutions for energy and momentum balances are appeared in terms of series form. The analytical results regarding velocity and temperature are plotted graphically by MATHCAD software to see the influence of physical parameters. Some graphic comparisons are also mad among present results with hybrid fractional derivatives and the published results that have been obtained by Caputo. It is found that the velocity and temperature with constant proportional Caputo fractional derivative are portrait better decay than velocities and temperatures that obtained with Caputo and Caputo-Fabrizio derivative. Further, rate of heat transfer and skin friction can be enhanced with smaller values of fractional parameter.

Key words: *new analytical solutions, novel fractional derivative, thermal analysis, viscous fluid, MHD*

Introduction

Fractional calculus is a branch of mathematics deals with non-local differentiation and integration [1]. In fractional differentiation one can allow the order of differentiation beyond the natural number it may real or even a complex number. Fractional calculus has been used efficiently for the physical description of mathematical modelling of non-local behavior. Wang *et al*. [2] consider a fractional rate type fluid-flow in cylindrical domain and obtained analytical solutions for velocity and tangential stress. Imran *et al*. [3] generalized an unsteady free convective flow of Walter's-B fluid by considering the mass transport subject to Lorentz force with permeable medium. Some other studied regarding respective subject area of fractional differentiations are also presented in [4-7]. Kashif *et al*. [8] investigate the non-local behavior of a nanofluid by utilizing

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the ABC and CF approaches for fractional derivatives and outlined the effect of nanotubes. Yang *et al*. [9] presented an experimental study about parameter identification for fractional fractal diffusion model. Ahmad *et al.* [10] compared the two generalized solutions of Jeffrey fluid-flow obtained by considering fractional derivative of singular kernel (Caputo) and non-singular kernel (Caputo-Fabrizio). Nazar *et al.* [11] illustrated a generalized model of heat and mass transfer. Some recent applications of fractional calculus in heat transfer, epidemic model, HIV, non-linear fluid-flow, nanofluids can be seen in following references [12-26].

Baleanu *et al.* [27] gave the new direction in the field of fractional calculus and defined some new definitions and successfully applied in fractional partial differential equations and expressed the obtained results in terms of Mittage-Leffler function. The advantage of this new operator is that it consists of two fractional operators' namely constant proportional derivative and Caputo type with power law kernel. It interpolates between derivative and integral of the function in the limiting cases. This is the most recent fractional operator and there is no single application in the field of heat transfer in the existing literature with this operator.

Therefore, our pivotal intention for this investigation is to imposing the most recent fractional operator namely hybrid fractional operator for flow of Newtonian fluid near an infinite upright surface. Respective solutions for temperature and velocity are obtained in series form, via Laplace transform technique. Also the effects of potent parameters are shown by graphic outlines of velocity with the help of MATHCAD software. Furthermore, a comparative analysis of our results with the already published work is presented graphically to see that which fractional operator is better to control the fluid velocity.

Mathematical model of the problem

Consider an unsteady, laminar, and free convection flow of an incompressible fluid through porous medium in the presence of constant magnetic force. The Cartesian components are taken vertically and horizontally along the plate. The velocity of the fluid is described as $\vec{V} = [u(y, t), 0, 0]$ and the movement of fluid is affected due to applied magnetic field:

$$\vec{B} = B_0\left(-\cos\theta\hat{i} + \sin\theta\hat{j}\right),\ \theta \in \left(0,\ \frac{\pi}{2}\right)$$

The induced magnetic field and its electrical effects are assumed negligible. The contribution of magnetic field as a body force generally characterized as $\vec{F} = \vec{J} \times B$, where $\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B})$, is current density, $\vec{E}$ is electric field, and σ defined as electrical conductivity of the fluid. Under the imposed specifications the x-component of magnetic force takes the form $F_x = -\sigma B_0^2\sin^2\theta u(y, t)$. Initially, both the plate and the fluid are maintained at dynamic and thermal equilibrium, with the uniform temperature T_1. At the moment $t = 0+$, the plate start to move in its own plane with a constant velocity U_0 in the x-direction and consequently also the adjacent fluid. The mechanical and thermal balances which governing the hydro-magnetic-flow with the dimensionless form [28]:

$$u_t(y,t) - u_{yy}(y,t) - \mathrm{Gr}T(y,t) + \left[M\sin^2\theta\right]u(y,t) = 0 \tag{1}$$

$$u_t(y,t) - \frac{1}{\mathrm{Pr}}u_{yy}(y,t) = 0 \tag{2}$$

with the following dimensionless conditions:

$$u(y,0) = 0,\ T(y,0) = 0,\ y > 0 \tag{3}$$

$$u(0,t) = H(t),\ T(0,t) = 1,\ t > 0 \tag{4}$$

$$u(y,t) \to 0,\ T(y,t) \to 0 \quad \text{as} \quad y \to \infty \tag{5}$$

where ρ is the density of fluid, T – the fluid temperature, β_T – the thermal expansion coefficient ν – the kinematics viscosity, g – the gravity, σ – the current density, C_p – the specific heat at constant pressure, and k – the thermal conductivity, $\mathrm{Gr} = [\mathrm{g}\beta_T(T_w - T_\infty)\nu]/U_0^3$ – the thermal Grashof number, $\mathrm{Pr} = (\mu C_p)/k$ – the Prandtl number, and $M = (\sigma B_0 \nu)/U_0^2$ – the magnetic parameter.

Fractional model of eqs. (1) and (2) can be obtained by using the definition, eq. (7), [27]:

$${}^{\mathrm{CPC}}D_t^{\alpha} u(y,t) - u_{yy}(y,t) - \mathrm{Gr}T(y,t) + \left[M \sin^2\theta\right] u(y,t) = 0 \tag{6}$$

$${}^{\mathrm{CPC}}D_t^{\alpha} T(y,t) - \frac{1}{\mathrm{Pr}} u_{yy}(y,t) = 0 \tag{7}$$

where ${}^{\mathrm{CPC}}D_t^{\alpha} u(y,t)$ is the constant proportional-Caputo fractional derivative operator [27].

Solution of the problem

The respective initial-boundary value fractional model is solved by means of Laplace transform method. Since eq. (7) is not coupled with eq. (6), so first we solve eq. (7) by Laplace transform method and then eq. (12).

Temperature field

By taking Laplace transform of eq. (7) and eqs. $(3)_2$-$(5)_2$:

$$\left[\frac{k_1(\alpha)}{s} + k_0(\alpha)\right] s^{\alpha} \overline{T}(y,s) = \frac{1}{\mathrm{Pr}} \overline{T}_{yy}(y,s) \tag{8}$$

where $\overline{T}(y,s)$ is the transformed image of $T(y,t)$ under Laplace transform and have suppose that satisfies the transformed boundary conditions:

$$\overline{T}(0,s) = \frac{1}{s} \ \text{and} \ \overline{T}(y,s) = 0, \ \text{as} \ y \to \infty \tag{9}$$

The transformed temperature field is obtained by solving the eq. (8) subject to conditions eq. (9):

$$\overline{T}(y,s) = \frac{1}{s} \mathrm{e}^{-y\sqrt{\mathrm{Pr}\left[\frac{k_1(\alpha)}{s} + k_0(\alpha)\right] s^{\alpha}}} \tag{10}$$

Equation (10) is in exponential form and the root expression is very complicated so it is hard to find the inverse formula with this form. Therefore, we used the series form of exponential function and expressed in more suitable form to obtain inverse analytically:

$$\overline{T}(y,s) = \frac{1}{s} + \sum_{k=1}^{\infty}\sum_{p=0}^{\infty} \frac{\left(-y\sqrt{\mathrm{Pr}}\right)^k \left[k_1(\alpha)\right]^p}{k!\,p!\left[k_0(\alpha)\right]^{p-\frac{k}{2}} s^{1+p-\frac{\alpha k}{2}}} \frac{\Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(\frac{k}{2}+1-p\right)} \tag{11}$$

Now by applying the inverse Laplace to eq. (11), we have:

$$T(y,t) = 1 + \sum_{k=1}^{\infty}\sum_{p=0}^{\infty} \frac{\left(-y\sqrt{\mathrm{Pr}}\right)^k \left[k_1(\alpha)\right]^p}{k!\,p!\left[k_0(\alpha)\right]^{p-\frac{k}{2}} s} \frac{t^{p-\frac{\alpha k}{2}} \Gamma\left(\frac{k}{2}+1\right)}{\Gamma\left(1+p-\frac{\alpha k}{2}\right)\Gamma\left(\frac{k}{2}+1-p\right)} \tag{12}$$

Rate of heat transfer in terms of Nusselt number can be computed through the following relation and presented in tab. 1:

$$\mathrm{Nu} = -T_y(0,t) \tag{13}$$

Table 1. Statistically analysis of Nusselt number for the effect of fractional parameter

α	Nu, t = 1,5	Nu, t = 3	Nu, t = 5
0.1	2..402	3.029	3.693
0.2	2.318	2.840	3.385
0.3	2.231	2.655	3.095
0.4	2.141	2.476	2.822
0.5	2.047	2.302	2.565
0.6	1.950	2.133	2.325
0.7	1.851	1.970	2.100
0.8	1.750	1.813	1.891
0.9	1.648	1.662	1.696
1.0	1.545	1.518	1.515

Velocity field

Taking Laplace transform of eq. (6) and using eqs. $(3)_1$-$(5)_1$ and using the expression from eq. (10):

$$\left[\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha \bar{u}(y,s)-\bar{u}_{yy}(y,s)-\mathrm{Gr}\bar{T}(y,s)+\left[M\sin^2\theta\right]\bar{u}(y,s)=0 \tag{14}$$

where $\bar{u}(y, s)$ is the Laplace transform of the function $u(y, t)$ that satisfies:

$$\bar{u}(0,s)=\frac{1}{s} \text{ and } \bar{u}(y,s), \text{ as } y\to\infty \tag{15}$$

The solution of eq. (14) subject to eq. (15):

$$\bar{u}(y,s)=\frac{1}{s}\mathrm{e}^{-y\sqrt{\left[\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha+M\sin^2\theta}}+\frac{\mathrm{Gre}^{-y\sqrt{\left[\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha+M\sin^2\theta}}}{s\left\{(\mathrm{Pr}-1)\left[\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha-M\sin^2\theta\right\}}-$$
$$-\frac{\mathrm{Gre}^{-y\sqrt{\left[\mathrm{Pr}\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha}}}{s\left\{(\mathrm{Pr}-1)\left[\frac{k_1(\alpha)}{s}+k_0(\alpha)\right]s^\alpha-M\sin^2\theta\right\}} \tag{16}$$

Again it is very hard to find inverse Laplace with exponential form. So, it is better to express in series form to obtain inverse analytically:

$$\bar{u}(y,s)=\frac{1}{s}+\sum_{l=1}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-y)^l\left(M\sin^2\theta\right)^{\frac{l}{2}-m}\left[k_1(\alpha)\right]^n}{l!m!n!\left[k_0(\alpha)\right]^{n-m}s^{1+n-\frac{\alpha m}{2}}}\frac{\Gamma(1+\frac{l}{2})\Gamma(1+m)}{\Gamma(1+\frac{l}{2}-m)\Gamma(1+m-n)}-$$

$$-\frac{\mathrm{Gr}}{\mathrm{Pr}-1}\sum_{l=1}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-y)^l(-1)^r\left(M\sin^2\theta\right)^{\frac{l}{2}-m-r-1}\left[k_1(\alpha)\right]^{n+q}}{l!m!n!q!\left[k_0(\alpha)\right]^{n-m+q-r}s^{1+n-\alpha m+q-\alpha r}}\cdot$$

$$\cdot\frac{\Gamma(1+\frac{l}{2})\Gamma(1+m)\Gamma(1+r)}{\Gamma(1+\frac{l}{2}-m)\Gamma(1+m-n)\Gamma(1+r-q)}+$$

$$+\frac{\mathrm{Gr}}{\mathrm{Pr}-1}\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\sum_{r=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-y\sqrt{\mathrm{Pr}})^k(-1)^r\left[k_1(\alpha)\right]^{p+q}}{k!p!q!\left(M\sin^2\theta\right)^{1+r}\left[k_0(\alpha)\right]^{p-\frac{k}{2}+q-r}s^{1+p-\frac{\alpha k}{2}+q-\alpha r}}\frac{\Gamma(1+\frac{k}{2})\Gamma(1+r)}{\Gamma(1+\frac{k}{2}-p)\Gamma(1+r-q)}\tag{17}$$

Taking inverse Laplace of eq. (17), we have final expression for velocity:

$$u(y,t)=1+\sum_{l=1}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-y)^l\left(M\sin^2\theta\right)^{\frac{l}{2}-m}\left[k_1(\alpha)\right]^n}{l!m!n!\left[k_0(\alpha)\right]^{n-m}}\frac{t^{n-\alpha m}\Gamma(1+\frac{l}{2})\Gamma(1+m)}{\Gamma(1+n-\alpha m)\Gamma(1+\frac{l}{2}-m)\Gamma(1+m-n)}-$$

$$-\frac{\mathrm{Gr}}{\mathrm{Pr}-1}\sum_{l=1}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-y)^l(-1)^r\left(M\sin^2\theta\right)^{\frac{l}{2}-m-r-1}\left[k_1(\alpha)\right]^{n+q}}{l!m!n!q!\left(k_0(\alpha)\right)^{n-m+q-r}}\cdot$$

$$\cdot\frac{t^{n-\alpha m+q-\alpha r}\Gamma(1+\frac{l}{2})\Gamma(1+m)\Gamma(1+r)}{\Gamma(1+n-\alpha m+q-\alpha r)\Gamma(1+\frac{l}{2}-m)\Gamma(1+m-n)\Gamma(1+r-q)}+$$

$$+\frac{\mathrm{Gr}}{\mathrm{Pr}-1}\sum_{k=0}^{\infty}\sum_{p=0}^{\infty}\sum_{r=0}^{\infty}\sum_{q=0}^{\infty}\frac{(-y\sqrt{\mathrm{Pr}})^k(-1)^r\left[k_1(\alpha)\right]^{p+q}}{k!p!q!\left(M\sin^2\theta\right)^{1+r}\left[k_0(\alpha)\right]^{p-\frac{k}{2}+q-r}}\cdot$$

$$\cdot\frac{t^{p-\frac{\alpha k}{2}+q-\alpha r}\Gamma(1+\frac{k}{2})\Gamma(1+r)}{\Gamma(1+p-\frac{\alpha k}{2}+q-\alpha r)\Gamma(1+\frac{k}{2}-p)\Gamma(1+r-q)}\tag{18}$$

This solution is valid for $\mathrm{Pr}\neq 1$.

Skin friction can be computed through the following relation and presented in tab. 2:

$$C_f=-u_y(0,t)\tag{19}$$

Results and discussion

In this section evaluations are made about the factual deportment of the physical parameters overthe field variables to check the validity of our results. To achieve this purpose some comparisons of present temperature and velocity fields to existing temperature and velocity fields are made graphically. In fig. 1 temperature field obtained by hybrid fractional derivative is compare to that obtained by Caputo fractional derivative, Shah *et al.* [28], for fractional parameter α variation. The pattern of profiles depict that hybrid temperature profile remain detached and lagging behind from the Caputo temperature moreover this detachment become more significant for increasing value of α. This behavior is justifying the stableness of the hybrid fractional temperature. It clear from figures that hybrid solution is decay better than solution obtained with Caputo. Further, noticed that for greater α thermal boundary-layer thickness also increases. The velocity profile solved by using the constant proportional Caputo fractional derivative and that velocity obtained by Caputo [28] are plotted for different values of fractional parameter α in fig. 2, It is clear from the respective figures hybrid fractional velocity shows more decay behavior than that of the velocity obtained with Caputo fractional derivative [28]. Also, it is observed that for larger α momentum boundary-layer thickness reduces.

Also some graphs of velocity field are plotted against special co-ordinate y for respective parameter's variation. In fig. 3 the effect of α *is* presented and it is cleared from outlines of the velocity profile the parameter α confesses the dual behavior for large and small times. For the large time velocity decreases for an increasing value of α while it increases for the small times. So, solution exhibit dual behavior for smaller and larger values of time. The influence of fractional parameter α on Nusselt number and skin friction is studied numerically in tabs. 1 and 2, respectively. It is found that Nusselt number and skin friction are decreasing function of fractional parameter α and increase by increasing value of time.

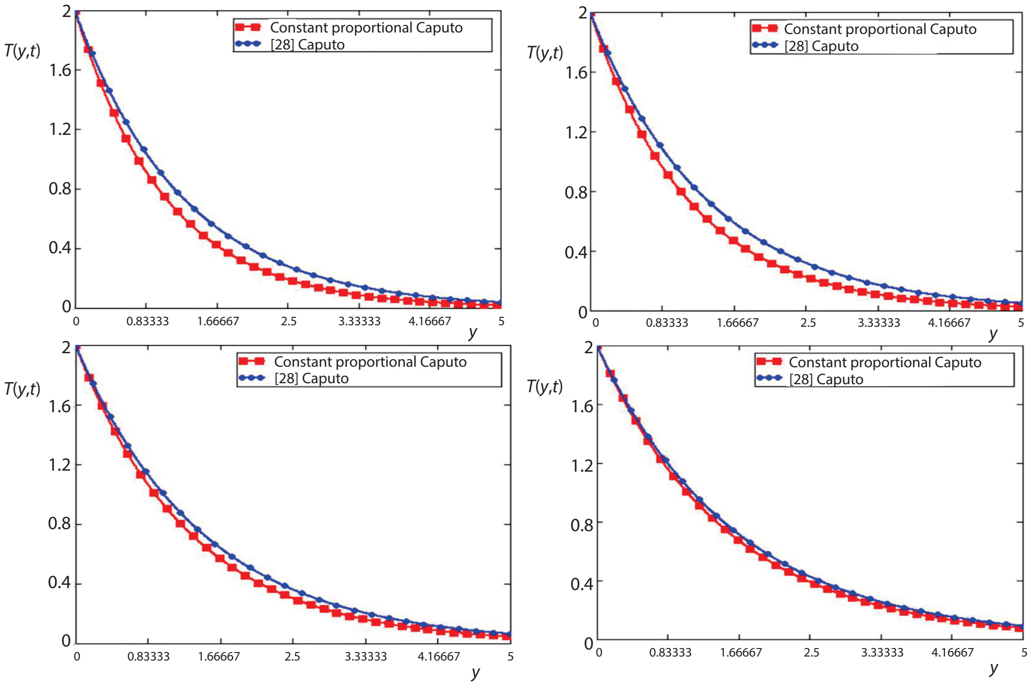

Figure 1. Temperature profile comparison for different values of fractional parameter $t = 0.5$, $\alpha = 0.2, 0.4, 0.6, 0.99$

Table 2. Statistically analysis of C_f for the effect of fractional parameter

α	C_f, $t = 1.5$	C_f, $t = 3$	C_f, $t = 5$
0.1	0.746	0.816	0.911
0.2	0.707	0.749	0.818
0.3	0.667	0.684	0.731
0.4	0.626	0.621	0.649
0.5	0.585	0.560	0.572
0.6	0.542	0.502	0.500
0.7	0.500	0.446	0.433
0.8	0.457	0.392	0.371
0.9	0.415	0.342	0.312
1.0	0.372	0.293	0.258

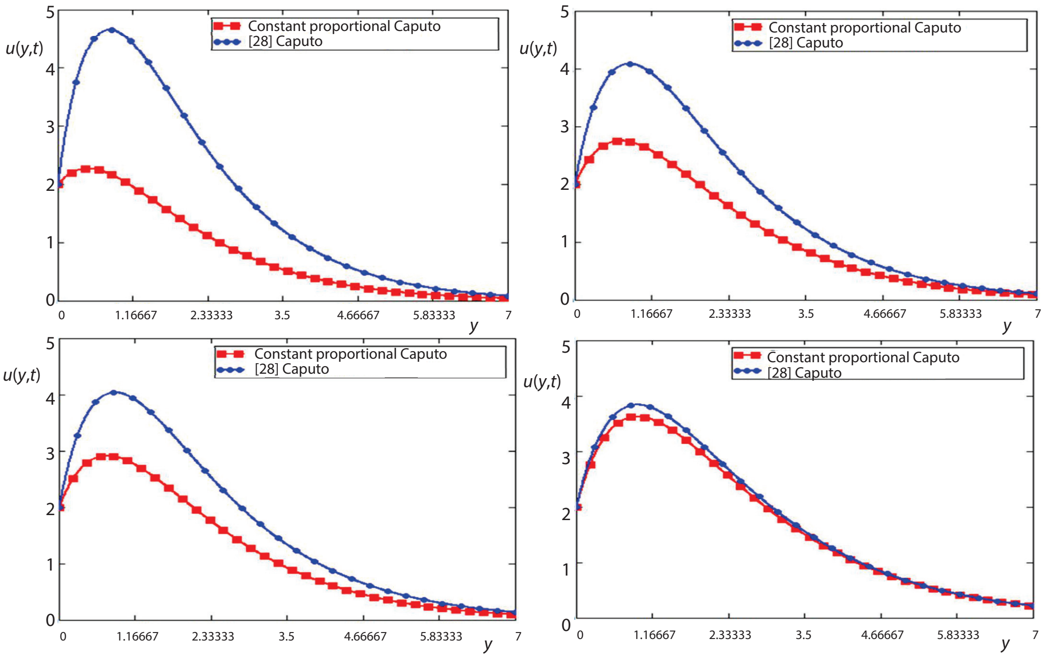

Figure 2. Velocity profile comparison for different values of fractional parameter $t = 0.5$, $\alpha = 0.2, 0.4, 0.6, 0.99$

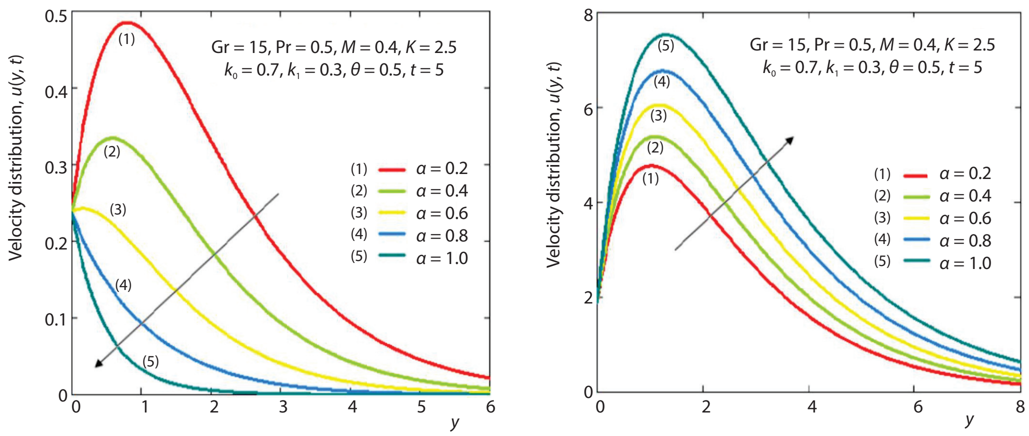

Figure 3. Velocity profile for small and large times for different values of fractional parameter

Conclusions

The concluded remarks are listed as follows.

- Rate of heat transfer and skin friction are maximum for smaller values of α and minimum for larger α.
- Constant proportional Caputo fractional temperature and velocity has shown more decay behavior than that of temperature and velocity obtained with *C* [28].
- Constant proportional Caputo fractional is the best choice to get the controlled velocity and temperatureprofiles for different values of fractional parameter.

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