MAGNETIC FIELD ON SURFACE WAVES PROPAGATION IN GRAVITATIONAL THERMOELASTIC MEDIA WITH TWO TEMPERATURE AND INITIAL STRESS IN THE CONTEXT OF THREE THEORIES

by

Jamel BOUSLIMI[a,b*], Sayed Mohamed ABO-DAHAB[c], Khaled LOTFY[d], Sayed ABDEL-KHALEK[e,f], Eied M. KHALIL[e,g], and Mohamed OMRI[h]

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[a] Physics Department, Faculty of Science, Taif University, Taif, Saudi Arabia
[b] Department of Engineering Physics and Instrumentation,
National Institute of Applied Sciences and Technology, Carthage University, Tunisia
[c] Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt
[d] Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
[e] Department of Mathematics and Statistics, Faculty of Science, Taif University, Taif, Saudi Arabia
[f] Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt
[g] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasser City, Egypt
[h] Deanship of Scientific Research, King Abdulaziz University, Jeddah, Saudi arabia

In this paper is investigating the theory of generalized thermoelasticity under two temperature is used to solve boundary value problems of 2-D half-space its boundary with different types of heating under gravity effect. The governing equations are solved using new mathematical methods under the context of Lord-Shulman, Green-Naghdi theory of type III (G-N III) and the three-phase-lag model to investigate the surface waves in an isotropic elastic medium subjected to gravity field, magnetic field, and initial stress. The general solution obtained is applied to a specific problem of a half-space and the interaction with each other under the influence of gravity. The physical domain by using the harmonic vibrations is used to obtain the exact expressions for the Waves velocity and attenuation coefficients for Stoneley waves, Love waves, and Rayleigh waves. Comparisons are made with the results between the three theories. Numerical work is also performed for a suitable material with the aim of illustrating the results. The results obtained are calculated numerically and presented graphically with some comparisons in the absence and the presence the influence of gravity, initial stress and magnetic field. It clears that the results obtained agree with the physical practical results and agree with the previous results if the gravity, two temperature, and initial stress neglect as special case from this study.

Key words: *generalized thermoelasticity, magnetic field, surface waves, two temperature, three-phase-lag model Lord-Shulman theory, gravity*

Introduction

Recently, the theory of classical coupled thermoelasticity has more attentions in diverse topics because its applicant and interesting in industry, science and technology. Biot [1] is the prior study the classical coupled thermoelastic theory for strain-rate term in the Fourier heat conduction equation in parabolic-type heat conduction equation form (diffusion equation). A generalized thermoelasticity theories have been developed by Lord and Shulman [2] as well as

* Corresponding author, e-mail: jamelabouaysem@yahoo.fr

Green and Lindsay [3] determining the finite speed for thermal wave in solids. Brief reviews of this topic have been reported by Chandrasekharaiah *et al*. [4, 5]. Amin *et al*. [6] discussed surface waves propagation in a generalized thermoelastic media with magnetic field and rotation and its applications in geophysics and engineering. Among the authors who considered the generallized magnetothermoelastic equations are Nayfeh and Nemat-Nasser [7] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They have obtained the governing equations in the general case and the solution for some particular cases. Ezzat *et al.* [8] have established the model of 2-D equations of generalized magneto-thermoelasticity.

Youssef and El-Bary [9] analysis the generalized thermoelastic infinite layer problem under three theories using state space approach. The theory of heat conduction in a deformable body, formulated by Chen and Gurtin [10] and Youssef [11] presented a new model of thermoelasticity depends on two temperatures T and φ in generalized form. Dual phase lag model problem on an infinite non-homogeneous spherical cavity solid under magnetothermoelasticity theory is investigated also [12]. Love [13] discussed the gravity field effect on the velocity of Rayleigh waves that show that it increases to a significant extent by influence of gravity. De and Sengupta [14, 15] studied surface waves propagation in an elastic layer and Lamb's phenomenon. The propagation of waves in a thermoelastic layer with gravity discussed by Sengupta and Acharya [16]. Das *et al.* [17] investigated surface waves under the gravity and rotation in a non-homogeneous in generalized thermoelastic solid. Abd-Alla and Ahmed [18] and Abd-Alla [19] presented the influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space. Lotfy *et al.* [20-24] studied normal mode method for two-temperature generalized thermoelasticity under thermal shock problem.

Recently, Abo-Dahab [25] investigated surface waves in coupled and generalized thermoelasticity. Abo-Dahab [26] discussed GL model on propagation of surface waves in magneto-thermoelastic materials with voids and initial stress. Abo-Dahab *et al.* [27] pointed out problem of rotation, magnetic field and stiffness effect on propagation of surface waves in an elastic layer lying over a generalized thermo-elastic diffusive half-space with imperfect boundary. Abd-Alla *et al.* [28] investigated the effect of several fields on a generalized thermoelastic medium with voids in the context of Lord-Shulman or Dual-Phase-Lag Models.

In this paper, the theory of two temperature generalized thermoelasticity is used to solve boundary value problems of 2-D half-space its boundary with different types of heating under gravity effect. The governing equations are solved using new mathematical methods under the context of Lord-Shulman (L-S), Green-Naghdi theory of type III (G-N III), and the three-phase-lag model (3PHL) to investigate the surface waves in an isotropic elastic medium subjected to gravity field, magnetic field and initial stress. The general solution obtained is applied to a specific problem of a half-space and the interaction with each other under the influence of gravity. The physical domain by using the harmonic vibrations is used to obtain the exact expressions for the Waves velocity and attenuation coefficients for:
– Stoneley waves
– Love waves
– Rayleigh waves

Comparisons are made with the results between the three theories. Numerical work is also performed for a suitable material with the aim of illustrating the results. The results obtained are calculated numerically and presented graphically with some comparisons in the absence and the presence the influence of gravity, initial stress and magnetic field. If the gravity, two temperature, and initial stress neglect, the results obtained agree with the physical practical results and agree with the previous results of [25] as special case from this study.

Formulation of the problem

Considering two semi-infinite elastic anisotropic solid media denoted $\coprod_1$ and $\coprod_2$ are perfectly welded in contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress, *etc.* Two media assumed have mechanical properties. These media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and $\coprod_2$ is to be taken above $\coprod_1$. Let *Oxyz* be a set of orthogonal Cartesian co-ordinates, we consider the possibility of a type of wave travelling in the direction *Ox* as shown in fig. 1.

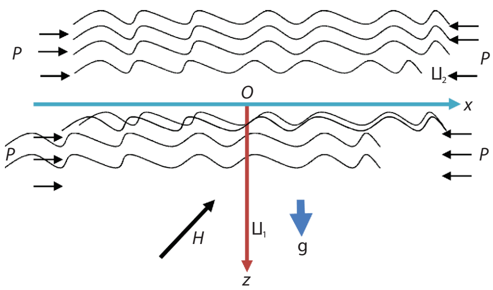


Figure 1. Schematic of the problem

Formulation of the problem

– The constitutive equation:

$$\sigma_{ij} = (\lambda\theta - P)\delta_{ij} + 2\mu e_{ij} - \gamma T\delta_{ij} - Pw_{ij}, \; w_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right) \tag{1}$$

The heat conduction equation:

$$\left(K^* + \tau_v^* \frac{\partial}{\partial t} + K\tau_T \frac{\partial^2}{\partial t^2}\right)\nabla 2\varphi = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[\rho C_e \ddot{T} + \gamma T_0 \ddot{e}\right], \; \tau_v^* = K + K^*\tau_\upsilon \tag{2}$$

– The equation of motion with body force takes the form:

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i, \; (i, j = 1,2,3) \tag{3}$$

– The relation between the heat conduction and dynamical heat:

$$\varphi - T = a\nabla^2\varphi \tag{4}$$

Taking into account the absence of displacement current, the linearized Maxwell's equations governing the magnetic field for a slowly moving solid medium having a perfect electrical conductivity:

$$\begin{gathered} \text{curl}\vec{h} = \vec{J}, \;\; \text{curl}\vec{E} = -\mu_e \vec{h}, \;\; \vec{E} = -\mu_e\left(\vec{u}\times\vec{H}_0\right) \\ \vec{h} = \text{curl}\left(\vec{u}\times\vec{H}_0\right), \;\; \text{div}\vec{h} = 0, \;\; \text{div}\vec{E} = 0, \;\; \vec{H}_0 = \vec{H}_0 + \vec{h}(x,y,t), \;\; \vec{H}_0 = (0,0,H) \end{gathered} \tag{5}$$

Using eq. (5) we obtain:

$$F_x = \mu_e H_0^2 \frac{\partial e}{\partial x}, \;\; F_z = \mu_e H_0^2 \frac{\partial e}{\partial z}, \;\; F_y = 0 \tag{6}$$

Maxwell's stress equation can be given:

$$\tau_{ij} = \mu_e\left[H_i h_j + H_j h_i - \left(H_k \times h_k\right)\delta_{ij}\right], \; i,j = 1,2,3 \tag{7a}$$

where τ_{ij} is Maxwell's stress tensor

$$\tau_{xx} = \tau_{zz} = \mu_e H_0^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z}\right), \;\; \tau_{xz} = 0 \tag{7b}$$

Equations (1) and (2) are the field equations of the generalized thermoelastic solid:

- The (L-S) theory: $K^* = \tau_v = \tau_T = \tau_q^2 = 0$, $\tau_q > 0$
- The (G-N II) theory: $\tau_v = \tau_T = \tau_q = 0$
- The (3PHL) theory: $\tau_v < \tau_T < \tau_q > 0$

Then on dimensional variables take the form:

$$\left(\sigma'_{ij},\tau_{ij}\right)=\frac{\left(\sigma'_{ij},\tau_{ij}\right)}{\rho C_0^2},\quad g'=\frac{g}{C_0^3\eta}\left(x',z',u',v'\right)=C_0\eta\left(x,z,u,v\right),$$
$$\left(t',\tau'_T,\tau'_v,\tau'_q\right)=C_0^2\eta\left(t,\tau_T,\tau_v,\tau_q\right),\ \left(\theta',\varphi'\right)=\frac{\left(T,\varphi\right)-T_0}{T_0} \tag{8}$$

where $\eta = (\rho C_e)/K$, $C_2^2 = \mu/\rho$, and $= (\lambda + 2\mu)/\rho$.

Substitute from eq. (5) into eqs. (2)-(4), we get:

$$\left(C_k + C_v\frac{\partial}{\partial t}+C_T\frac{\partial_2}{\partial t_2}\right)\nabla^2\varphi-\left(1+T_q\frac{\partial}{\partial t}+\frac{T_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\ddot{\theta}+\frac{\gamma}{\rho C_e}\ddot{e}\right) \tag{9}$$

$$\varphi-\theta=\nabla^2\varphi \tag{10}$$

where

$$C_k=\frac{K^*}{\rho C_e C_0^2},\quad C_v=\frac{\tau_v^*}{\rho C_e C_0^2},\quad C_k=\frac{K^*\eta\tau_T}{\rho C_e}$$

Equations of motion are:

$$a_1^*\nabla^2 u+a_2\frac{\partial e}{\partial x}-a_0\frac{\partial\theta}{\partial x}+g\frac{\partial w}{\partial x}=\beta\ddot{u} \tag{11}$$

$$a_1^*\nabla^2 w+a_2\frac{\partial e}{\partial z}-a_0\frac{\partial\theta}{\partial z}+g\frac{\partial u}{\partial x}=\beta\ddot{w} \tag{12}$$

where

$$\varepsilon=\frac{\gamma}{\rho C_e},\quad a_1^*=\frac{2\mu-P}{2\rho C_0^2},\quad a_2=\frac{2\lambda+2\mu+P+2\mu_e H_0^2}{2\rho C_0^2},\quad a_0=\frac{\gamma T_0}{\rho C_0^2}$$

Assuming the scalar and vector potential functions $\prod$ and ψ:

$$u=\frac{\partial\prod}{\partial x}-\frac{\partial\psi}{\partial z}\qquad w=\frac{\partial\prod}{\partial z}+\frac{\partial\psi}{\partial x} \tag{13}$$

Using eq. (13) into eqs. (11) and (12) we get:

$$\left(\nabla^2-\beta^*\frac{\partial^2}{\partial t^2}\right)\prod-a_3^*\frac{\partial\psi}{\partial x}-a_0^*\theta=0 \tag{14}$$

$$\left(\nabla^2-\beta^{**}\frac{\partial^2}{\partial t^2}\right)\psi+a_4\partial\frac{\partial\prod}{\partial x}=0 \tag{15}$$

where

$$R_H=\frac{\mu_e H_0^2}{\rho C_0^2},\quad a_0^*=\frac{a_0}{1+R_H},\quad a_3^*=\frac{g}{1+R_H},\quad a_4^*=\frac{a_3}{a_1^*},\quad \beta^*=\frac{1}{1+R_H},\quad \beta^{**}=\frac{\beta^*}{a_1^*}$$

The temperature eq. (9) tends:

$$\left(C_k + C_v \frac{\partial}{\partial t} + C_T \frac{\partial^2}{\partial t^2}\right)\nabla^2 \varphi - \left(1 + T_q \frac{\partial}{\partial t} + \frac{T_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2}{\partial t2}\left(\theta + \varepsilon \nabla^2 \Pi\right) \tag{16}$$

and similar relations in M_2 with replaced ρ, λ, μ, α, γ, by ρ^*, λ^*, μ^*, α^*, γ^*.

Solution of the problem

To solve eqs. (7), (11)-(13), we assume the following normal mode method:

$$\left[\pi, \psi, \varphi, \theta, \sigma_{ij}\right](x, z, t) = \left[\pi^*, \psi^*, \varphi^*, \theta^*, \sigma_{ij}^*\right](z)\mathrm{e}^{i\omega(x-ct)} \tag{17}$$

Substituting from eq. (17), into eqs. (14) and (15) using $D = d/dz$:

$$\left[D^2 - A_1\right]\pi^* - a_3^{**}\psi^* - a_0^*\theta^* = 0 \tag{18}$$

$$\left[D^2 - A_2\right]\psi^* + a_4^*\pi^* = 0 \tag{19}$$

Equation (7) tends:

$$\left[D^2 - A_3\right]\varphi^* + \beta^{-1}\theta^* = 0 \tag{20}$$

Also eq. (16) tends:

$$\left(D^2 - \omega^2\right)\varphi^* + B\left(D^2 - \omega^2\right)\pi^* + A\theta^* = 0 \tag{21}$$

where

$$A_1 = \omega^2\left(1 - c^2\beta^*\right),\ a_3^{**} = i\omega a_3^*,\ A_2 = \omega^2\left(1 - c^2\beta^{**}\right),\ a_4^{**} = i\omega a_4^*,\ A_3 = \frac{\beta\omega^2 + 1}{\beta}$$

where

$$B = \varepsilon A,\ A = \omega^2 c^2 \left[\frac{1 - i\omega c\left(\tau_q - \frac{i\omega\tau_q^2}{2}\right)}{C_k - i\omega\left(C_v - i\omega c C_T\right)}\right]$$

Eliminating, π^*, ψ^*, φ^*, and θ^* from eqs. (18)-(21), we obtain:

$$\begin{vmatrix} D^2 - A_1 & -a_3^{**} & 0 & -a_0^* \\ a_4^* & D^2 - A_2 & 0 & 0 \\ 0 & 0 & D^2 - A_3 & \beta^{-1} \\ B(D^2 - \omega^2) & 0 & D^2 - \omega^2 & A \end{vmatrix} = 0$$

which tends:

$$\left[D^6 + ED^4 + FD^2 + G\right]\pi^*(x) = 0 \tag{22}$$

where

$$E=-\frac{\left(A-\beta^{-1}\right)\left(A_1+A_2\right)-\beta^{-1}\omega^2+AA_3+a_0^*B\left(\omega^2+A_2+A_2\right)}{\left(A-\beta^{-1}+a_0^*B\right)}$$

$$F=\frac{\left(A-\beta^{-1}\right)\left[A_1A_2+a_3^{**}a_4^*\right]-\left(A_1+A_2\right)\left(\beta^{-1}\omega^2-AA_3\right)+a_0^*B\left[\omega^2\left(A_2+A_3\right)\right]+A_2+A_2}{\left(A-\beta^{-1}+a_0^*B\right)} \tag{23}$$

$$G=\frac{\left(\beta^{-1}\omega^2-AA_3\right)\left[A_1A_2+a_3^{**}a_4^*\right]-a_0^*B\omega^2A_2A_3}{\left(A-\beta^{-1}+a_0^*B\right)}$$

Also, in a similar manner, we get:

$$\left[D^6+ED^4+FD^2+G\right]\left(\psi^*,\varphi^*,\theta^*,\sigma_{ij}^*\right)(x)=0 \tag{24}$$

which can be factorized:

$$\left(D^2-k_1^2\right)\left(D^2-k_2^2\right)\left(D^2-k_3^2\right)\left(\psi^*,\varphi^*,\theta^*,\sigma_{ij}^*\right)(x)=0 \tag{25}$$

where $k_n^2(n = 1, 2, 3)$ are the roots of the characteristic equation:

$$K^6+EK^4+FK^2+G=0 \tag{26}$$

The solution of eq. (25), as $z \to \infty$, take the form:

$$\pi^*(z)=\sum_{n=1}^{3}M_n\exp\left(-k_nz\right) \tag{27}$$

$$\theta^*(z)=\sum_{n=1}^{3}M'_n\exp\left(-k_nz\right) \tag{28}$$

$$\psi^*(z)=\sum_{n=1}^{3}M''_n\exp\left(-k_nz\right) \tag{29}$$

$$\varphi^*(z)=\sum_{n=1}^{3}M'''_n\exp\left(-k_nz\right) \tag{30}$$

since

$$u^*(z)=i\omega\pi^*-D\psi^* \tag{31}$$

$$v^*(z)=D\pi^*+i\omega\psi^* \tag{32}$$

$$e^*(z)=i\omega u^*+Dv^* \tag{33}$$

Using eqs. (31) and (32) in order to obtain the displacements amplitudes u and v taking into account that are bounded as $x \to \infty$, we get:

$$u^*(z)=i\omega\sum_{n=1}^{3}\left[M_n+k_nM''_n\exp\left(-k_nz\right)\right] \tag{34}$$

$$v^*(z)=-i\omega\sum_{n=1}^{3}\left(k_nM_n+i\omega M''_n\right)\exp\left(-k_nz\right) \tag{35}$$

where M_n, M'_n, M''_n, and M'''_n are parameters depend on β, c, and ω.

Substituting from eqs. (27)-(30) into eqs. (18)-(21):

$$M'=\frac{\left(k_n^2-A_1\right)\left(k_n^2-A_2\right)+a_3^{**}a_4^*}{a_0^*\left(k_n^2-A_2\right)}M_n=H_{1n}M_n \tag{36}$$

$$M_n'' = \frac{a_4^*}{K_n^2 - A_2} M_n = H_{2n} M_n \tag{37}$$

$$M_n''' = -\beta^{-1} \left[\frac{\left(k_n^2 - A_1\right)\left(k_n^2 - A_2\right) + a_3^{**} a_4^*}{a_0^* \left(k_n^2 - A_2\right)\left(k_n^2 - A_3\right)} \right] M_n = H_{3n} M_n \tag{38}$$

where

$$H_{1n} = \frac{\left(k_n^2 - A_1\right)\left(k_n^2 - A_2\right) + a_3^{**} a_4^*}{a_0^* \left(k_n^2 - A_2\right)}, \quad H_{2n} = -\frac{a_4^*}{K_n^2 - A_2}$$

$$H_{3n} = -\beta^{-1} \frac{\left(k_n^2 - A_1\right)\left(k_n^2 - A_2\right) + a_3^{**} a_4^*}{a_0^* \left(k_n^2 - A_2\right)\left(k_n^2 - A_3\right)}, \quad n = 1, 2, 3$$

thus

$$\theta^*(z) = \sum_{n=1}^{3} H_{1n} M_n \exp\left(-k_n z\right) \tag{39}$$

$$\psi^*(z) = \sum_{n=1}^{3} H_{2n} M_n \exp\left(-k_n z\right) \tag{40}$$

$$\varphi^*(z) = \sum_{n=1}^{3} H_{3n} M_n \exp\left(-k_n z\right) \tag{41}$$

From eqs. (5), (14), and (33)-(35), into eq. (1), we can obtain:

$$\sigma_{xx}^*(z) = \sum_{n=1}^{3} h_n M_n \exp\left(-k_n z\right) - \frac{P}{\lambda + 2\mu} \tag{42}$$

$$\sigma_{zz}^*(z) = \sum_{n=1}^{3} h_n' M_n \exp\left(-k_n z\right) - \frac{P}{\lambda + 2\mu} \tag{43}$$

$$\sigma_{xz}^*(z) = \sum_{n=1}^{3} h_n'' M_n \exp\left(-k_n z\right) \tag{44}$$

$$\tau_{xz}^* = \mu_e H_0^2 \left[k_n \left(k_n - i\omega H_{2n}\right) - i\omega\left(i\omega + k_n H_{2n}\right) \right] \tag{45}$$

$$h_n = i\omega\left(i\omega + k_n H_{2n}\right) + \frac{\lambda}{\lambda + 2\mu} k_n \left(k_n - i\omega H_{2n}\right) - \frac{\gamma T_0}{\lambda + 2\mu} H_{1n} \tag{46}$$

$$h_n' = k_n \left(k_n - i \quad H_{2n}\right) + \frac{i\omega\lambda\left(i\omega \quad k_n H \quad _n\right)}{\lambda + 2\mu} - \frac{\quad}{\lambda + 2\mu} H_{1n} \tag{47}$$

$$h_n'' = -\left(\frac{\mu + \frac{P}{2}}{\lambda + 2\mu} \right) k_n \left(i\omega + k_n H_{2n}\right) - \left(\frac{\mu - \frac{P}{2}}{\lambda + 2\mu} \right) i\omega\left(-k_n + i\omega H_{2n}\right) \tag{48}$$

Boundary conditions (application)

We will take the following application considering the thermal shock:

– $\theta(x, 0, t) = f(x, 0, t)$

which tends:

$$\sum_{n=1}^{3} H_{1n} M_n = f^* \tag{49}$$

– $\sigma_{zz} + \tau_{zz} = -P/\rho C_0^2$

which with helping eq. (7) trends:

$$\sum_{n=1}^{3} \overline{h}'_n M_n = 0 \tag{50}$$

where

$$\overline{h}'_n = \left[k_n \left(k_n - i\omega H_{2n}\right)\left(\frac{\lambda + 2\mu + \mu_e H_0^2}{\lambda + 2\mu}\right) - i\omega\left(i\omega + k_n H_{2n}\right)\left(\frac{\lambda + \mu_e H_0^2}{\lambda + 2\mu}\right) - \frac{\gamma T_0}{\lambda + 2\mu} H_{1n} \right]$$

finally

– $\sigma_{xz} + \tau_{xz} = 0$

winich tends:

$$\sum_{n=1}^{3} h''_n M_n = 0 \tag{51}$$

Equations (45)-(47) can be re-written in the matrices form:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ \overline{h}'_1 & \overline{h}'_2 & \overline{h}'_3 \\ h''_1 & h''_2 & h''_3 \end{pmatrix} \begin{pmatrix} f^* \\ 0 \\ 0 \end{pmatrix} \tag{52}$$

Special cases

Stoneley waves

In Stoneley waves we assume that the waves are propagated along the common boundary of two semi-infinite media ∐1 and ∐2.

– The components of displacement at the surface between the two media ∐1 and ∐2 must be continuous independent on position and time, this means:

$$[u,v]\amalg_1 = [u,v]\amalg_2, \text{ at } z = 0 \tag{53}$$

Substituting from eqs. (34) and (35) we get:

$$\left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})M_n\right]\amalg_1 = \left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})M_n\right]\amalg_2 \tag{54}$$

$$\left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\right]\amalg_1 = \left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\right]\amalg_2 \tag{55}$$

– The total stress components σ_{xx}, σ_{xz}, σ_{zz}, τ_{xx}, τ_{xz}, and τ_{zz}, must be continuous at the boundary z =0:

$$\left[\sigma_{xy} + \tau_{xy},\, \sigma_{xx} + \tau_{xx},\, \sigma_{zz} + \tau_{zz}\right]\amalg_1 = \left[\sigma_{xy} + \tau_{xy},\, \sigma_{xx} + \tau_{xx},\, \sigma_{zz} + \tau_{zz}\right]\amalg_2 \tag{56}$$

Substituting from eqs. (39)-(42) we obtain:

$$\left[\sum_{n=1}^{3} h''_n M_n\right]\amalg_1 = \left[\sum_{n=1}^{3} h''_n M_n\right]\amalg_2 \tag{57}$$

$$\left[\sum_{n=1}^{3} h_n M_n\right]Ш_1 = \left[\sum_{n=1}^{3} h_n M_n\right]Ш_2 \tag{58}$$

$$\left[\sum_{n=1}^{3} \bar{h}'_n M_n\right]Ш_1 = \left[\sum_{n=1}^{3} \bar{h}'_n M_n\right]Ш_2 \tag{59}$$

– Thermal insulated (*i. e.*):

$$\left[\frac{\partial\theta^*}{\partial z}\right]Ш_1 = \left[\frac{\partial\theta^*}{\partial z}\right]Ш_2$$

Substitute from eq. (39), we get:

$$\left[\sum_{n=1}^{3} K_n H_{1n} M_n\right]Ш_1 = \left[\sum_{n=1}^{3} K_n H_{1n} M_n\right]Ш_2 \tag{60}$$

Therefore, eqs. (54), (55), and (57)-(60) determine the velocity equation of Stoneley waves in anelastic solid media under with gravity.

It is shown from eqs. (54), (55), and (57)-(60) that the Stoneley waves velocity depends on the two temperature of the material medium, densities of both media and gravity. Since the wave velocity eqs. (54), (55), and (57)-(60) for Stoneley waves under the present circumstances depends on the particular value of ω and creates a dispersion of a general wave form.

Love waves

– The displacement components at the surface between the two media for medium $\coprod_1$ must be continuous at all times and positions, but in medium $\coprod_2$ must continuous at all time for position $z = L$ (thickness) (*i. e.*, for Love waves considering the first medium is half space $z > 0$, but the second medium with thickness $z = L$).

This means:

$$[u,v]Ш_1(z) = [u,v]Ш_2,\ (z = L) \tag{61}$$

Substituting from eqs. (34) and (35) we get:

$$\left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})Mn\exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3}(i\omega - k_n H_{2n})M_n\exp(-k_n L)\right]Ш_2 \tag{62}$$

$$\left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3}(k_n + i\omega H_{2n})M_n\exp(-k_n L)\right]Ш_2 \tag{63}$$

– The total stress components σ_{xx}, σ_{xz}, σ_{zz}, τ_{xx}, τ_{xz}, and τ_{zz}, must be continuous in medium at all times and positions but in medium $\coprod_2$ must continuous at all time for position $z = L$ (thickness):

$$[\sigma_{xy} + \tau_{xy}, \sigma_{xx} + \tau_{xx}, \sigma_{zz} + \tau_{zz}]Ш_1(z) = [\sigma_{xy} + \tau_{xy}, \sigma_{xx} + \tau_{xx}, \sigma_{zz} + \tau_{zz}]Ш_2,\ (z = L) \tag{64}$$

Substituting from eqs. (39)-(42) we obtain:

$$\left[\sum_{n=1}^{3} h''_n M_n\exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3} h''_n M_n\exp(-k_n L)\right]Ш_2 \tag{65}$$

$$\left[\sum_{n=1}^{3} h_n M_n \exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3} h_n M_n \exp(-k_n L)\right]Ш_2 \tag{66}$$

$$\left[\sum_{n=1}^{3} \bar{h}_n' M_n \exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3} \bar{h}_n' M_n \exp(-k_n L)\right]Ш_2 \tag{67}$$

– Thermal insulated, *i. e.*

$$\left[\frac{\partial \theta^*}{\partial z}\right]Ш_1(z) = \left[\frac{\partial \theta^*}{\partial z}\right]Ш_2, \ (z = L)$$

Substitute from eq. (39):

$$\left[\sum_{n=1}^{3} K_n H_{1n} M_n \exp(-k_n z)\right]Ш_1 = \left[\sum_{n=1}^{3} K_n H_{1n} M_n \exp(-k_n L)\right]Ш_2 \tag{68}$$

Rayleigh waves

Concerns the Rayleigh waves in anisotropic fibre-reinforced elastic media, the medium $Ш_2$ will replaced by a vacuum. So the stress boundary condition in this case may be expressed:

$$\sigma_{xy} + \tau_{xy} = \sigma_{xx} + \tau_{xx} = \sigma_{zz} + \tau_{zz} = 0$$

Numerical results

For illustrating the analytical procedure, we will consider a numerical example for which appropriate the media considered in the previous calculations. The results display the variation of displacement, temperature, and stress considering two theories for the copper material [26].

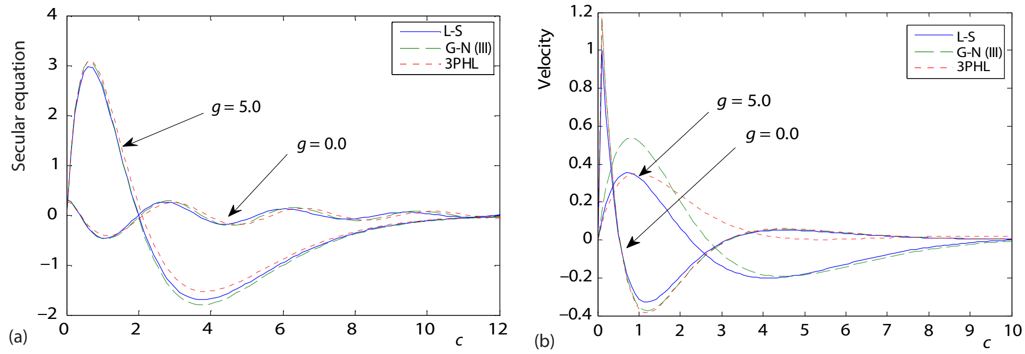
Table 1. Physical constants of the copper material [26]

Parameter	Value	Parameter	Value	Parameter	Value
λ	$759 \cdot 10^9$ N/m^2	K	386 N/Ks	ε	0.0168
μ	$386 \cdot 10^{10}$ kg/ms^2	C_E	383.1 J/kgk	$a = \xi$	1
ρ	7800 kg/m^2	α	–1.28N/m^2	ω_0	2
T_0	293 K	α_t	$1.78 \cdot 10^{-5}$ N/m^2	η	8886.73 m/s^2

The computations were carried out for a value of $\omega = \omega 0 + i\xi$ and time $t = 0.001$ seconds.

Figures 2 and 3 display variation of secular equation, velocity and attenuation coefficient for Rayleigh wave with respect to phase velocity c with and without gravity and, magnetic field, respectively, in the context of three theories (L-S), (G-N III), and (3PHL).

Generally, we conclude that all dependent variables (secular equation, velocity and attenuation coefficient) start from zero at $c = 0$ and tend to zero as phase velocity c tends to infinity. Also, it is obvious that there are clear changes between the three thermoelastic models. From fig. 2, we show that frequency equation and attenuation coefficient take greater value in presence of gravity comparing with the corresponding values if the gravity is neglected $0 < c < 2$, but, takes the inverse behavior for large values of phase velocity (*i. e.*, $c > 2$). It appear that the attenuation coefficient increase with absence of gravity nearly at $0 < c < 5$, but, take inverse behavior with presence of gravity when $c > 5$. Physically, it clears that smaller values of phase velocity in presence of gravity act positively in secular equation and attenuation co-



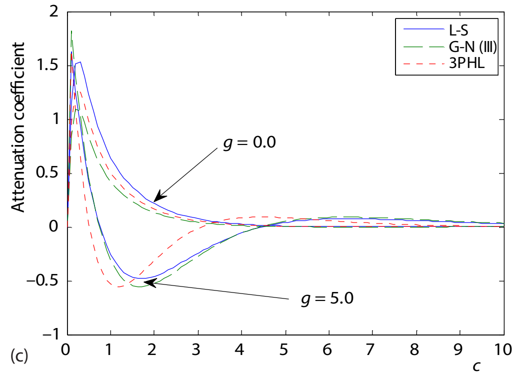
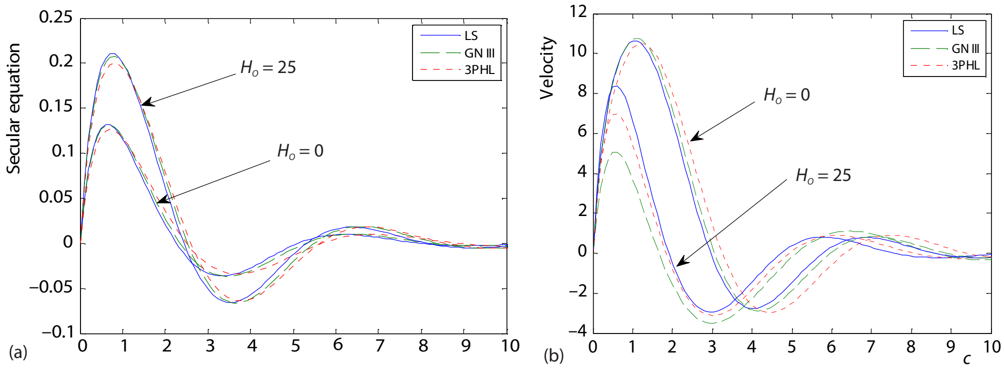


Figure 2. Rayleigh wave secular equation, velocity, and attenuation concern to phase velocity, *c*, in the presence and absence of gravity in the context of three theories



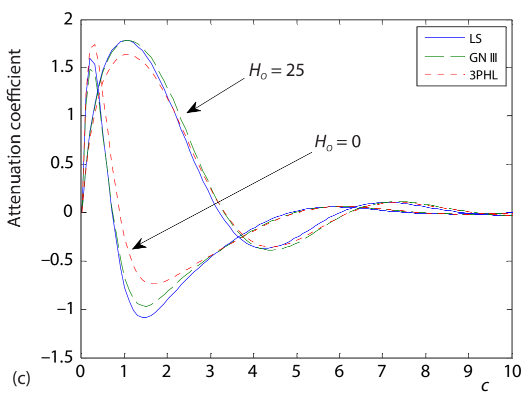
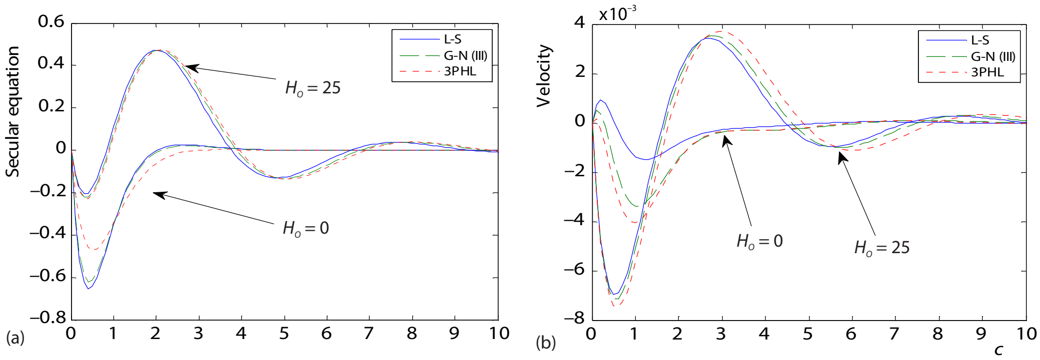


Figure 3. Rayleigh wave secular equation, velocity, and attenuation concern to phase velocity, *c*, in the presence and absence of magnetic field in the context of three theories

efficient but negatively on the Rayleigh wave velocity. From fig. 3, it obvious that magnetic field affects positively in frequency equation but negatively on Rayleigh wave velocity and



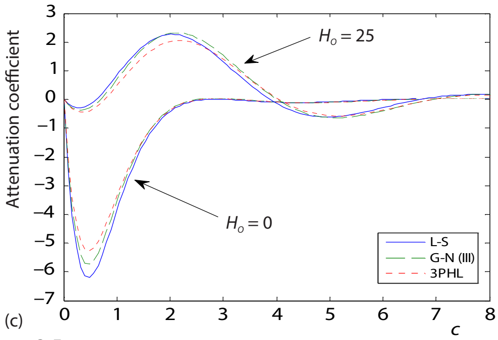
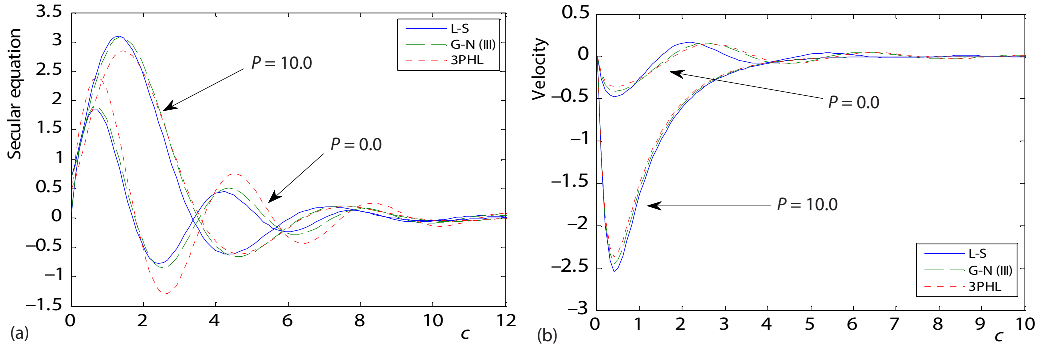


Figure 4. Stoneley wave secular equation, velocity, and attenuation concern to phase velocity, *c*, in the presence and absence of magnetic field in the context of three theories



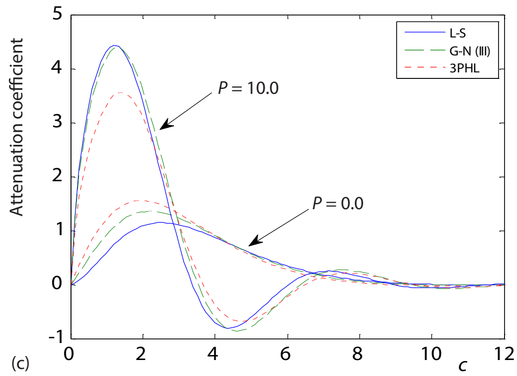


Figure 5. Stoneley wave secular equation, velocity, and attenuation concern to phase velocity, *c*, in the presence and absence of initial stress in the context of three theories

attenuation coefficients with the smallest values of phase velocity c and take inverse manner periodically with the increased values of c tend to zero with the greatest values.

Figures 4 and 5 show variation of secular equation, velocity and attenuation coefficient for Stoneley wave with respect to phase velocity, c, respectively, under influence of magnetic field and initial stress in the context of three theories (L-S), (G-N III), and (3PHL). From fig. 4, it is shown, with the presence of magnetic field, it obvious that the secular equation and attenuation coefficient increase but Stoneley wave velocity decreases with the smallest values of c, but with the greatest values take inverse manner periodically in presence of H. From fig. 5, we concluded that presence of P makes increasing of attenuation coefficients of Stoneley wave with the smallest values of c, but secular equation and Stoneley wave velocity decrease with presence of initial stress.

Conclusions

With the presence and absence of magnetic field, gravity and initial stress, considering theories: Lord and Shulman, Green and Naghdi, and three phase lag, we concluded the following remarks.

- Analytical solutions in the context of normal mode analysis for themoelastic problem in solids have been developed and utilized.
- The curves of the physical quantities with 3PHL theory in most of the figures are strongly appear comparing with L-S and GN III. in comparison with those under GN theory (type II).
- All the functions calculated and shown graphically are continuous and the value of all physical quantities converge to zero at phase velocity ($c = 0$) and with an increasing of phase velocity converge to zero.
- The presence of gravity, magnetic field and initial stress have a significant role in all the physical quantities.
- All external parameters have a significant role on the phenomena of surface waves and applicable in diverse field, especially, earthquakes, volcanoes, geophysics and geology.

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Nomenclature

a – parameter of two temperature
C_e – specific heat per unit mass
e – cubical dilatation
e_{ij} – strain tensor
$\vec{E}$ – electric intensity
$\vec{F}$ – vector of Lorentz's body forces
$\vec{h}$ – vector of perturbed magnetic field
$\vec{H}'_0$ – vector of primary magnetic field
$\vec{J}$ – electric current density
K – thermal conductivity
P – initial stress
T – absolute temperature
T_0 – reference temperature of the medium, $|(T - T_0)/T_0| < 1$
u_i – displacement vector

Greek symbols

α_t – alinear thermal expansion
δ_{ij} – kronecker delta function
ε_0 – electric permittivity
η – hydrostatic initial stress parameter
θ – thermodynamical temperature, $(= T - T_0)$
λ, μ – counterparts of Lame's parameters
μ_0 – magnetic permeability
ρ – density of the medium
σ_{ij} – stress tensor
τ_{ij} – Maxwell stress tensor
τ_q, τ_T, τ_v, τ_v^* – relaxation times
φ – temperature of conductivity, $(= \varphi_0 - T)$

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