Impact of wavy texture and hybridity of nanofluid on heat transfer augmentation over the frustum of cone geometry

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Abstract

In this article, the impact of water-based hybrid nanofluid on heat transfer characteristics along the wavy frustum of the cone is examined. We considered hybrid nanofluid containing Cu and TiO₂ nanoparticles. Non-similar form of the constitutive equations is obtained by using an appropriate set of transformations and results are achieved by employing transformed into compact non-similar form and are solved by the famous numerically implicit finite difference scheme known as Keller-box technique. The influence of the hybrid nanoparticles’ volume fraction, frustum of cone half-angle γ, and the wavy texture parameters on the Nusselt number and skin friction are scrutinized and comparison is made between the wavy frustum of the cone and flat frustum of the cone through numerical data. It is observed that the rise in the truncated cone half-angle leads to an increase in skin friction and Nusselt number. Titania – water nanofluid has lower heat transfer rates as compared to copper-titania hybrid nanofluid. The increasing of the truncated cone half-angle enhances the heat transfer rates. Generally, the results established from this analysis can be used as a benchmark for improving the natural convection heat transfer performance along the frustum of cone wavy texture.

Keywords: Wavy surface; Frustum of cone; Keller box scheme; Non-similar equations

1. Introduction

The science and phenomena of heat transfer enhancement, or intensification, or increase, have been established into the significant aspect of various topographies of engineering and thermal science. The collected literature in the enhancement of heat transfer contains thousands of references, and it continues
to grow continuously. It is a substantial challenge for the dissemination of applicable design and relevant information.

Natural convection heat transfer phenomena are encountered in many fields of industry and engineering. Due to economic advantage, low noise, and simple technique several investigators debated the natural convective phenomena about the vertical frustum of a cone in Newtonian fluids. The process of heat transfer due to natural convection phenomena over a frustum of a cone is appropriate in many types of thermal design apparatus, for example, nuclear reactor, dehydration process in chemical and food process, steam generators, cooling solar energy plants, heat exchangers, drying and geothermal reservoirs. Pioneer work on flat frustum of a cone geometry for natural convection for CWT and CHF cases were conducted by Na and Chiou [1-2]. Singh et al. [3-4] discussed the constant wall heat flux case, using the local non-similarity method. Pop and Na [5] studied natural convection over a vertical frustum of the cone in a porous medium. Yih [6] studied the effect of radiation on free convection about a truncated cone. Chamkha [7] investigated MHD free convection flow about a truncated cone in the presence of radiation effects. Hossain et al [8] studied the impact of variable thermophysical properties on the natural convection flow of a truncated cone. Postelnicu [9] explored the free convection about a vertical frustum of a cone in a micropolar fluid. Cheng [10-12] studied the influence of variable properties and power-law variation in surface temperature on natural phenomena along vertical truncated cone for micropolar fluid in the porous regime. Srinivasa and Eswara [13] explored the influence of heat generation/absorption and MHD on convective flow over the truncated cone and seen that increasing heat generation/absorption parameter corresponds to a decrease in the heat transfer coefficient.

Convective heat transfer is an essential phenomenon in several engineering processes such as electronic cooling, heat exchangers, and thermal power plants. Base fluids like water, ethylene glycol mixture, and oil have low thermal conductivity, consequently the weak heat transfer fluids. By the addition of nanophase particles in these fluids, the heat transfer phenomena of the base fluid can be boosted. This is because the addition of nanoparticles upturns the heat capacity and surface area of the fluid, the random motion of nanoparticles increases the temperature gradient of the fluid.

In the current two decades, several authors discussed convective heat transfer phenomena with nanofluids. Cheng [14] investigated free flow in a bi-disperse porous regime and shown that increasing the modified thermal conductivity ratio increases the heat transfer phenomena. Patrulescu et al. [15] studied mixed convection nanofluid flow over a vertical frustum of a cone and shown that an increase in the mixed convection parameter decreases the heat transfer characteristics. Siddiqa et al. [16] discussed two-phase dusty fluid flow over a vertical frustum of a cone and concluded that the rate of heat transfer
rate significantly increases by the inclusion of nanoparticles. Amanulla et al. [17] examined Casson nanofluid past a truncated cone texture and analyzed that the buoyancy ratio parameter increases the fluid flow. Ahmed and Mahdy [18] analyzed variable physical properties for MHD natural convection nanofluid flow past a truncated cone and found that by increasing MHD leads to an increase in the rate of heat transfer. Siddiqa et al. [19] reported the radiative heat transfer for two-phase Casson dusty fluid flow over truncated vertical cone and shown that the radiation strongly affects the heat transport phenomenon.

The wavy texture corresponds to the increased surface area which supports the convective heat transfer phenomena. The investigation of the heat transfer process with non-flat texture is particularly significant in many usages. The heat transfer phenomenon is strongly influenced by the form of the apparatus. The periodic functions like sine and cosine functions support the mathematical formation. Moreover, fluids having high conductivity such as nanofluid are the best suitable way to enhance the convective phenomena. Siddiqa et al. [20] reported gray dusty fluid flow along a vertical wavy frustum of a heated cone and analyzed that the rate of heat transfer coefficient considerably upturns for contaminated air. Siddiqa et al. [21] analyzed non-linear radiative flow along the wavy cone surface and shown that It is found that the rate of heat transfer reduces with increasing surface radiation parameters.

Mahdy and Elshehabe [22] studied nanofluid convective flow due to gyrotactic microorganisms past vertical truncated cone in a porous regime and concluded that rescaled velocity depends significantly on the bio-convection parameter. Reddy and Rao [23] explored that nanofluid flow over the frustum of a cone in the doubly stratified non-Darcy porous regime and found that reduction in velocity of fluid flow is observed as the non-Darcy parameter increases.

In this article, the heat transfer analysis is investigated for free convection past a wavy vertical frustum of a cone in hybrid nanofluid using Tiwari and Das model. The flow is incompressible, steady, and laminar. By taking frustum of cone half-angle so large that curvature effects are neglected. Using the above-mentioned assumptions, constitutive PDEs are formulated and converted to non-similar parabolic PDEs by an appropriate set of transformations. A direct numerical solution is obtained by the Keller-box method. Tables and graphs are displayed are for flow characteristics and heat transfer rates. Before analysis, for conformation of code calculated results are matched with the available data and seen that these values are in superb matching.

Important physics, primary understanding, and beneficial contributions to the analysis of convection phenomena in fluid flow along wavy texture are given in ref. [24-30].

The objective of this research article to consider the impact of hybrid nanofluid on the flow and heat transfer characteristics in a natural convection flow over an isothermal wavy truncated cone.
2. Problem statement and mathematical formulation

Two dimensional Newtonian free convective flow past vertical truncated cone of wavy texture. The nature of wavy texture is sinusoidal and its mathematical form is taken as

\[ \bar{y}_w = \bar{S}(\bar{x}) = \bar{a} \sin \left( \frac{\pi \bar{x} - \bar{x}_0}{l} \right) \]

slant height at the lower end of the truncated cone is \( \bar{x}_0 \), the wave length is \( l \), and fixed amplitude is \( \bar{a} \). The geometry of the truncated cone of wavy texture is displayed in Figure 1. \( \bar{O}(\bar{x} = \bar{y} = 0) \) is the leading edge (apex), \( \bar{x} \) and \( \bar{y} \) is along and perpendicular to truncated cone surface Flat surface and local radius \( \bar{r} \) of the truncated cone and related through

\[ \bar{r}(\bar{x}) = (\bar{x} + \bar{x}_0) \sin \beta; \beta \text{ is the truncated cone half-angle.} \]

The valid boundary constraints for this problem are

\[ \bar{y} = \bar{S}(\bar{x}); T = T_w, \quad \bar{u} = 0, \quad \bar{v} = 0, \quad \bar{y} \to \infty; \bar{u} = 0, \quad T = T_\infty, \quad \bar{p} = p_\infty, \]

\[ \bar{x} = 0; T = T_\infty, \quad \bar{p} = p_\infty, \text{ for all } \bar{y} \neq 0. \]

where \( p_\infty, T_\infty \) are the ambient pressure and temperature of fluid, and \( T_w \ (T_w > T_\infty) \) is the surface temperature.

Here we are using Tiwari and Das model [31] because it is the most appropriate that during convective transport in nanofluid it explains the nanoparticle contribution in heat transfer phenomena. This model incorporates the thermophysical properties of the nanofluid and the relations for these properties are listed in Table1. The nanoparticles used in this analysis are \( Cu \) and \( TiO_2 \) and their numerical values are recorded in Table 2. The governing consecutive equations according to Tiwari and Das model, the two-dimensional mass, momentum, and energy conservation laws are given by

![Figure 1: Physical model and coordinate system.](image-url)
\[
\frac{\partial (\overline{ru})}{\partial \overline{x}} + \frac{\partial (\overline{rv})}{\partial \overline{y}} = 0, \tag{2}
\]

\[
\overline{u} \frac{\partial u}{\partial \overline{x}} + \overline{v} \frac{\partial u}{\partial \overline{y}} = -\frac{1}{\rho_{nf}} \frac{\partial \overline{p}}{\partial \overline{x}} + v_{nf} \nabla^2 \overline{u} + \frac{1}{\rho_{nf}} g(\rho \beta^*)_{nf}(T - T_\infty) \sin \beta, \tag{3}
\]

\[
\overline{u} \frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v} \frac{\partial \overline{v}}{\partial \overline{y}} = -\frac{1}{\rho_{nf}} \frac{\partial \overline{p}}{\partial \overline{y}} + v_{nf} \nabla^2 \overline{v} + \frac{1}{\rho_{nf}} g(\rho \beta^*)_{nf}(T - T_\infty) \cos \beta, \tag{4}
\]

\[
\overline{u} \frac{\partial T}{\partial \overline{x}} + \overline{v} \frac{\partial T}{\partial \overline{y}} = \alpha_{nf}^* \nabla^2 T, \tag{5}
\]

where \(\beta^*_{nf}, \rho_{nf}, \alpha_{nf}^*, (\rho c_p)_{nf}, \rho \kappa_{nf}\) are, respectively, the thermal expansion coefficient, density and thermal diffusivity of the nanofluid, heat capacitance, thermal diffusivity, and thermal conductivity of the nanofluid which are directly related to the nanoparticle volume fraction. The relations transformed for regular fluid by selecting nanoparticle volume fraction equal to zero. Moreover, \(\nabla^2\) is the Laplacian operator, \((\overline{u}, \overline{v})\) is velocity vector along \((\overline{x}, \overline{y})\) directions and \(g\) be the gravitational acceleration.

Introducing a group of proper dimensionless variables presented by

\[
\xi = \frac{x}{L}, \quad y = \frac{y - S(x)}{l}, \quad u = \frac{\rho_{fl}}{\mu_f} G_r \frac{1}{l} \overline{u}, \quad v = \frac{\rho_{fl}}{\mu_f} G_r \frac{1}{l} (\overline{v} - S \xi \overline{u}), \quad u = \frac{\partial \psi}{\partial x},
\]

\[
\nu = -\frac{\partial \psi}{\partial y} S_\xi \overline{S}(\xi), \quad \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \psi(\xi, \eta) = \frac{3}{\pi} \int f(\xi, \eta), \quad \eta = \xi^{-1} \nu, \tag{6}
\]

\[
Pr = \frac{\nu_f}{\alpha_f}, \quad Gr = \frac{g \beta_f l^3}{\nu^2 \rho_f}, \quad p = \frac{l^2}{\nu^2 \rho_f} G_r^{-1/4} \overline{p}, \quad r(\xi) = (\xi + \xi_0) \sin \beta.
\]

By considering surface waves so small as compared to boundary layer thickness and utilizing the transformations given in Eq. (6), The Eqs., (3-5) written in compact form as

\[
\frac{\mathcal{H}^2 d^f f}{\mathcal{Q}_1 d \overline{n}^3} + \left(3 + \frac{\xi}{\xi + \xi_0}\right) f d^f f d \overline{n}^2 - \left(1 + \frac{\mathcal{H} \xi}{\mathcal{H}}\right) (d^f f) \frac{2}{d \overline{n}} + \frac{Q_4 (1 - S \xi \tan \beta)}{Q_2 \mathcal{H}^2} \theta = \xi \left[\frac{df}{d \xi} \frac{d^f f}{d \xi} - \frac{d^2 f}{d \xi^2} \frac{d \xi}{d \xi}\right], \tag{7}
\]

\[
\frac{\mathcal{Q} \mathcal{H}^2 d^2 \theta}{\mathcal{Q}_2 Pr d \overline{n}^2} + \left(3 + \frac{\xi}{\xi + \xi_0}\right) f d \theta d \overline{n} = \xi \left[\frac{d \theta}{d \xi} \frac{d^f f}{d \xi} - \frac{d^2 \theta}{d \xi^2} \frac{d \xi}{d \xi}\right]. \tag{8}
\]

And boundary constraints in final solid form shown as
\[ \begin{align*}
\theta(\xi, 0) &= 1, \quad f(\xi, 0) = 0, \quad \frac{df}{d\eta}(\xi, 0) = 0, \\
\theta(\xi, \infty) &= 0, \quad \frac{df}{d\eta}(\xi, \infty) = 0.
\end{align*} \tag{9} \]

where \( Q, Q_1, Q_2, Q_3 \) and \( Q_4 \) are physical constant parameters recorded in Table 3, \( \mathcal{H} = \sqrt{1 + S^2_{\xi}} \) and \( \mathcal{H}_{\xi} \) designate the wavy input and its derivative and \( Pr \) is the Prandtl number. The superscript ‘\( \ast \)’ and subscript ‘\( \xi \)’ state differentiation w.r.t \( \eta \) and ‘\( \xi \)’ respectively. The local skin friction coefficient and the local heat transfer rate are defined as

\[ C_{f_\xi} = \frac{\tau_w}{\rho_f U^2}, \quad Nu_x = \frac{\bar{x} q_w}{\kappa_f (T_w - T_\infty)}, \tag{10} \]

for wavy texture \( q_w \) and \( \tau_w \) are described as

\[ \tau_w = \mu_{nf} (\nabla \bar{u} \cdot \hat{n})_{y=0}, \quad q_w = -\kappa_{nf} (\nabla T \cdot \hat{n})_{y=0}, \tag{11} \]

where wavy texture normal vector is \( \hat{n} = (\frac{-S_{\xi}}{\mathcal{H}}, \frac{1}{\mathcal{H}}) \) and final form for \( C_{f_\xi} \) and \( Nu_x \) take the form

\[ C_f = C_{f_\xi} (Gr/x)^{\frac{1}{2}} = \frac{\mathcal{H}}{(1 - \phi)^{2.5}} f''(\xi, 0), \quad Nu = Nu_x (Gr x^3)^{-1/4} = \mathcal{H} \frac{\kappa_{nf}}{\kappa_f} \theta'(\xi, 0). \tag{12} \]

### 3. Solution methodology and code validation

An elegant, implicit finite difference procedure to solve the coupled PDEs (7) and (8) along with BCs (9). The detailed procedure can be seen in the refs. [32, 33]. This scheme is particularly suitable for parabolic nature boundary layer fluid flow problems and extensively used in computational fluid dynamics. It is unconditionally stable and fast second-order convergence for highly coupled fluid flows. It includes the following main four steps:

1) Reduction of the \( N^{th} \) order PDEs system to \( N \) first-order ODEs.
2) Use finite difference approximation to transform finite difference equations.
3) Newton method for linearization of non-linear algebraic equations.
4) Block-tridiagonal elimination technique is adopted for solving linearized algebraic equations.

The linearized algebraic equations are solved iteratively. The same process is repeated until the relative difference between the current and the previous iteration up to \( 10^{-5} \). The solution is supposed to have converged and the iterative process is stopped. The flow chart of the Keller-box scheme is demonstrated below:
In order to authenticate the accuracy of the current procedure, we have compared our results for the $C_f$ and $Nu$, for various Prandtl numbers are listed in Tables 1 and 2. The matching in all the overhead cases are shown to be superb with the results established by Alim et al. [34] and Hossain et al. [35].
4. Result discussion

It is worth mentioning that the current flow problem transformed into viscous fluid and vertical wavy texture by assuming $\phi = 0$ and $\beta = 0$ converted to vertical wavy texture problem. Figures 2, 4, and 6 illustrate the influence of wavy amplitude, the concentration of nanoparticles, and cone half-angle on $C_f$. Figure 2 shows that $C_f$ decreases with the increase of wavy amplitude. $C_f$ is minimum for $TiO_2$, maximum for $Cu-TiO_2$ hybrid nanoparticle. Figure 4 reveals that $C_f$ is the minimum for plain fluid and rises with the inclusion of nanoparticles. $C_f$ is minimum for $TiO_2$, maximum for $Cu-TiO_2$ hybrid nanoparticle. Figure 6 represents that $C_f$ enhances with the decrease of cone half-angle. Once again $C_f$ is minimum for $TiO_2$ then for $Cu-TiO_2$ hybrid nanoparticle. The reason behind that the nanoparticles in the base fluid cause some change in the flow characteristics in terms of shear stress as observed from these figures. This is due to the increase in the viscosity of the
base fluid. By considering the proportion of the volumetric fraction of the nanoparticles in the hybrid nanofluid the friction factor can be adjusted to increase the heat transfer rate for the practical situations.

The heat transfer rate in terms of Nusselt number $Nu$ for various parameters namely, wavy amplitude, the concentration of nanoparticles, and cone half-angle are plotted in figures 3, 5, and 7. It is seen from Figure 3 that $Nu$ reduces with the increase of wavy amplitude. $Nu$ is minimum for $TiO_2$, maximum for $Cu-TiO_2$ hybrid nanofluid. Figure 5 shows that $Nu$ is minimum for plain fluid and increases for $Cu-TiO_2$ hybrid nanofluid. Figure 7 depicts that $Nu$ enhances with the reduction of the cone half-angle. $Nu$ is minimum for $TiO_2$ and $Cu-TiO_2$ hybrid nanoparticle at ratio 5%:5% concentration of nanoparticle.

This is because Nusselt number is depending upon different factors such as the concentration of nanoparticles, random movement of particles due to the Brownian diffusion and thermophoresis rises the viscosity and thermal conductivity of nanofluid. The Figures reveals that the rate of heat transfer in terms of Nusselt number enhances for hybrid nanofluid in comparison to water and $TiO_2$/water nanofluid by adding a small number of copper nanoparticles.
\(C_f\) against wavy amplitude and concentration of nanoparticles are shown in Figures 8 and 10. Figure 8 follows that \(C_f\) reduces with the increase of wavy amplitude. Figure 10 depicts \(C_f\) upturns with incrementing the concentration of nanoparticle. The same reason that inclusion of nanoparticle strengthens the viscosity of the base fluid and hence fluid friction signifies. The graph of \(Nu\) versus wavy amplitude, the concentration of nanoparticles is plotted in Figures 9 and 11. It can be seen from Figure 9 that follows that \(Nu\) varies linearly with the increase of wavy amplitude. Figure 10 depicts \(C_f\) upturns with incrementing the concentration of nanoparticle. The same reason that inclusion of nanoparticle strengthens the viscosity of the base fluid and hence fluid friction increases. \(C_f\) is minimum for \(TiO_2\), it increases for \(Cu - TiO_2\) hybrid nanofluid. Variation of \(Nu\) concerning \(\alpha\) and \(\phi\) are shown in Figures 9 and 11. \(Nu\) varies linearly with the increase of \(\alpha\) whereas exponentially increase is depicted in figure 11 in the case of \(\phi\). Heat transfer is minimum for \(TiO_2\), almost equal in the case of \(Cu - TiO_2\) hybrid nanofluid.

Fig. 6: Graph of skin friction influenced by \(\gamma\).

Fig. 7: Graph of Nusselt number influenced by \(\gamma\).

<table>
<thead>
<tr>
<th>Properties</th>
<th>(C_p) (J/kgK)</th>
<th>(\rho) (kg/m3)</th>
<th>(k) (W/mK)</th>
<th>(\beta \times 10^{-5}) (1/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid (water)</td>
<td>4179</td>
<td>997.1</td>
<td>0.613</td>
<td>21.00</td>
</tr>
<tr>
<td>(Cu)</td>
<td>383.1</td>
<td>8954</td>
<td>386</td>
<td>1.67</td>
</tr>
<tr>
<td>(TiO_2)</td>
<td>686.2</td>
<td>4250</td>
<td>8.9538</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Fig. 8: Skin friction graph verses $\alpha \cdot q$

Fig. 9: Nusselt number against $\alpha$.

Fig. 10: Skin friction variation against $\phi$.

Fig. 11: Nusselt number variation for different $\phi$.

Conclusions

In this research article, we have investigated natural convection heat transfer of $Cu – TiO_2$ hybrid nanofluid past wavy truncated cone wavy texture. Using suitable transformations the PDEs are converted to compact non-similar nature system of PDEs. The system is solved by an appropriate and accurate Keller-box technique having second degree of convergence. The results show accurate matching with the previous results which confirm the rationality of the code and presented data. The key outcomes of this analysis are

(i) The local skin friction increases and the Nusselt number decreases by increasing the values of wavy amplitude.

(ii) An increase in the concentration of nanoparticles in the base fluid produces an increase in the skin friction coefficient and the local Nusselt number.

(iii) A rise in the truncated cone half-angle leads to the increase in skin friction and Nusselt number.
(iv) Titania – water nanofluid has lower heat transfer rates as compared to copper-titania hybrid nanofluid.
(v) The increasing of the truncated cone half-angle enhances the heat transfer rates.
(vi) Additionally, this analysis illustrates that the hybrid nanofluid will be very significant in the cooling and heating processes.

References


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