A FRACTAL MODEL FOR PRESSURE DROP THROUGH A CIGARETTE FILTER

by

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A fractal model for pressure drop through a cigarette filter is suggested, the fractal dimensions of both a single fiber and the filter's cross-sections are calculated, which are two main factors affecting the pressure drop. The two-scale transform is made to convert the fractal derivative model on a smaller scale to an approximate continuous model on a larger scale, so that the model can be easily solved. An optimal filter structure is suggested for minimal pressure drop.

Key words: *fractal calculus, local fractional calculus, two scale dimension, two scale transform, cellulose acetate, trilobal-shaped fiber, Y cross-sectional shape, X cross-sectional shape, Darcy's law, hierarchical structure*

Introduction

A cigarette filter [1] plays an important role to make a smoker less unhealthy, it is generally made from cellulose acetate fibers [2, 3]. The pressure drop through a cigarette filter is the main factor affecting consumer's feeling and attitude to the quality of cigarette. Cellulose acetate (CA) fibers with different cross-sectional shapes have widely used in the cigarette filter. The Y-shaped CA fibers are widely used in cigarette industry, and there is a trend to use X-shaped CA fibers to ameliorate the filter's properties including their high air permeability, low filtration resistance, high filtration efficiency, and good adsorption ability due to high surface-to-volume ratio compared with Y-shaped one. Chen *et al.* [4] elucidated for the first time ever the mechanism of fabrication of shaped fibers by the geometric potential theory. A geometric potential produces on a surface, and it can explain many phenomena from capillary motion, cell orientation to nanoscale adhesion, surface's wettability property, highly selective adsorption property and Hall-Petch effect (nano-effect) [5-12]. The geometric potential theory gives a good theoretical tool to the design of a needed irregular shaped fiber.

This paper is to establish a fractal model to elucidate the main factors affecting the pressure drop though the filter. Fractal models are widely used to study various phenomena arising in porous media, including thermal property, flow resistance, electroosmotic property, gas diffusion and moisture diffusion [13-16]. Fractal analysis is also a powerful tool to revealing the

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bio-functional mechanism of a natural fiber, for examples, polar bear hair [17-19], cocoon [20, 21]. When the last level of a hierarchy tends to be a nanoscale, extremely fascinating properties can be predicted [22], this is because when a fractal-like hierarchy tends to a nanoscale, the surface area increases remarkably with an extraordinarily high geometric potential [5-12], as a result, many fascinating properties can be seen in a nanofiber membrane [23-28], for examples, remarkably enhanced mechanical properties, extremely high surface reactivity, excellent thermal and electric conductivity, high adsorption or separation, independent of their bulk materials.

Hagen-Poiseuille equation and its modification

The well-known Hagen-Poiseuille equation is widely used to predict pressure drop through a cylindrical pipe, it can be written in the form [29]:

$$\Delta p = \frac{128\mu QL}{\pi d^4} \tag{1}$$

where Δp is the pressure drop though the pipe with length of L and diameter of d, μ – the dynamic viscosity, and Q – the volumetric flow rate.

Equation (1) becomes invalid for non-smooth boundary and seepage and flows through porous media in a pipe. Majumder *et al.* [23] found that a liquid flow on a few nanoscale through aligned carbon nanotubes is more than 10000 times faster than would be predicted eq. (1). The pressure drop through a porous medium can be predicted approximately by Darcy's law:

$$\Delta p = \frac{\mu QL}{\lambda A} \tag{2}$$

where A is the section area, and λ is a constant.

According to fractal theory, a fractal modification of Darcy's law was proposed in [29], which reads:

$$\Delta p = k\frac{QL^{\gamma}}{R^{2\alpha}} \tag{3}$$

where R is the radius of the pipe, α and γ – the values of fractal dimensions for the pipe section and the path line (trajectory) of fluid as illustrated in fig. 1.

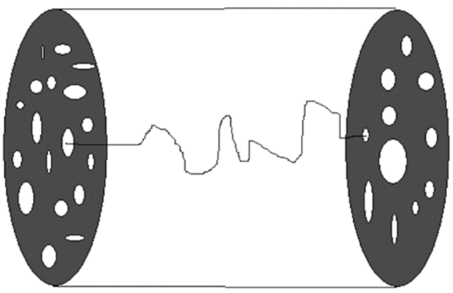

Figure 1. Pressure drop through a porous medium

When $\alpha = \gamma = 1$, eq. (3) is the Darcy's law, and when $\alpha = 2$ and $\gamma = 1$, eq. (3) turns out to be the Hagen-Poiseuille equation, for a porous medium it follows that $1 < \alpha < 2$ and $\gamma > 1$. In this paper we will study the pressure drop through a cigarette filter by the fractal theory.

Fiber section and fractal dimensions

A country's coastline is discontinuous, and its length depends on the scale used for the measure. The coastline is 100 km if the scale is 1 km, however, it becomes, for example, 1000000 km if our measure uses a scale of 1 cm. The coastline paradox led Mandelbrot to the establishment of the well-known fractal geometry. Though the coastline length is scale-dependent, its fractal dimension can be used to describe coastline's property. Generally we need two scales to measure the fractal property, one is large, and the other is small. On the large scale, the studied problem can be considered as a continuum, but when we measure it on a smaller scale, discontinuous property appears [30, 31]. This is also true for a cigarette filter, which is a cylinder on a large scale, but on a smaller scale, it consists of many small fibers. If we want to study the effect of fiber size on the filter's property, we have to use the scale of the fiber size. If an ever smaller scale is used, we find the small fibers are not cylindrical. If we want to study the effect of the shaped-fiber structure on the filter's property, we have to use the scale of lobal width, h, as illustrated in fig. 2.

Fractal requires self-similarity, however, in practical applications, no fractal can be found. A hierarchical structure can be approximately considered as a fractal, a porous medium is always considered as a fractal space though it is not. In practical applications, the porous structure is considered a level of a fractal geometry, so its fractal dimension can be measured.

Fibers section plays an important role in both filtration and absorption for a cigarette filter. Figure 2 shows two fibers sections.

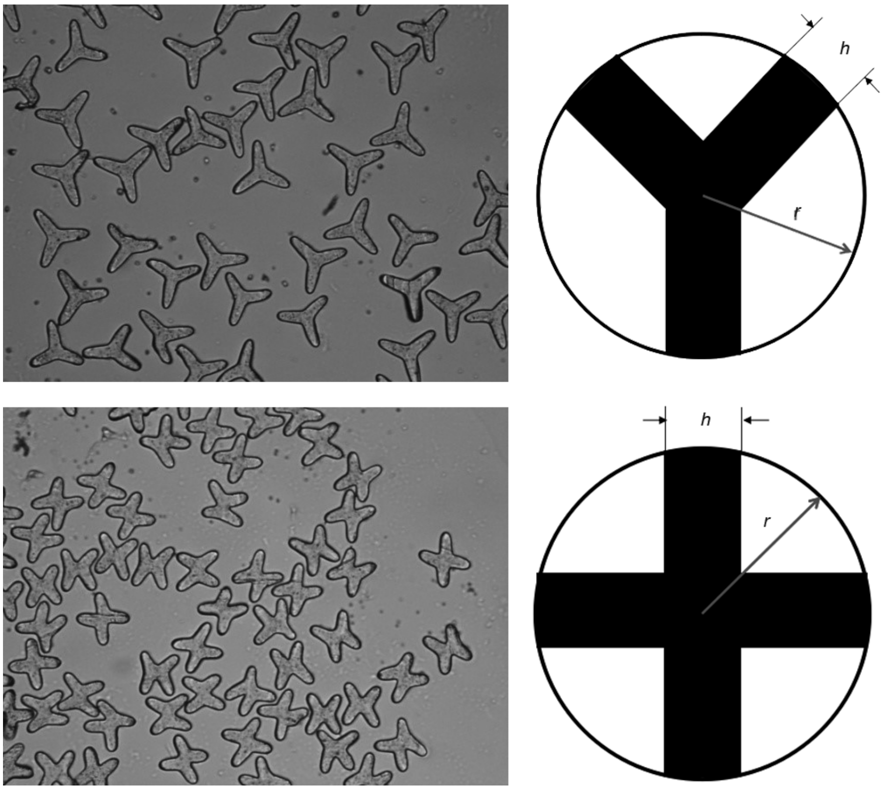

Figure 2. Different cross-sections of CA shaped fibers at ×400 magnification

The fractal dimensions can be calculated by the following mathematic formulation:

$$\alpha = \frac{\ln N}{\ln M} \tag{4}$$

where M is the number of new units with a new dimension, l, for a chosen unit with dimension, L, and $M = L/l$.

To calculate the value of the fibers fractal dimensions, we use the two scales, one is the fiber radius, r, and the other is the lobal width, h, see fig. 2. For the scale of r, we have a unit area, and when we use the scale of $h/2$, we have the real area $\pi r^2 - 3rh$ for a trilobal-shaped fiber, *i. e.*, the number of new units is:

$$N = \frac{\pi r^2 - 3rh}{\pi h^2} \tag{5}$$

The scale ratio is:

$$M = \frac{2r}{h} \tag{6}$$

The fractal dimensions for a trilobal-shaped fiber reads:

$$\alpha = \frac{\ln N}{\ln M} = \frac{\ln \dfrac{\pi r^2 - 3rh}{\pi h^2}}{\ln \dfrac{2r}{h}} \tag{7}$$

For a multi-lobal shaped fiber, the fractal dimensions can be calculated:

$$\alpha = \frac{\ln N}{\ln M} = \frac{\ln \dfrac{\pi r^2 - nrh}{\pi h^2}}{\ln \dfrac{2r}{h}} \tag{8}$$

where n is the lobal number, for the X-shaped fiber as illustrated in fig. 1 $n = 4$.

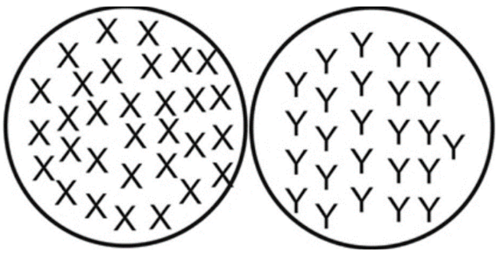

Figure 3. Cross-section of a cigarette filter

Filter fractal dimensions

A cross-section of a cigarette filter is illustrated in fig. 3, the fractal dimensions of a filter depends upon the filter radius and fiber number. We choose two scales, one is the fiber radius, R, the other is the radius of the fiber, r. The filter's dimensions can be easily calculated:

$$\beta = \frac{\ln N}{\ln \dfrac{R}{r}} \tag{9}$$

where N is the number of fibers in the filter.

Fiber bend in the filter and its fractal dimensions

We assume all fibers have a length of L_0 at a free condition, and they are compressed into a cigarette filter with length of L. Each fiber in the filter is bended, and its fractal dimensions can be calculated using the two scales, one is the filter radius, R, and the other is filter length, L. For the scale of R, the number of new units is:

$$N = \frac{L_0}{R} \tag{10}$$

and the scale ratio is:

$$M = \frac{L}{R} \tag{11}$$

The fractal dimensions can be calculated:

$$\gamma = \frac{\ln N}{\ln M} = \frac{\ln \dfrac{L_0}{R}}{\ln \dfrac{L}{R}} \tag{12}$$

Fractal derivative model for the pressure drop through the filter

The flow rate through a cigarette filter can be described by a fractal derivative model:

$$Q = \lambda A \frac{\mathrm{d}p}{\mathrm{d}x^\gamma} \tag{13}$$

where λ is a constant, A is real section area for a fluid to go through, $\mathrm{d}p/\mathrm{d}x^\gamma$ is the fractal derivative defined [32-35]:

$$\frac{\mathrm{d}p}{\mathrm{d}x^\gamma}(x_0) = \Gamma(1+\gamma) \lim_{\substack{x - x_0 \to \Delta x \\ \Delta x \neq 0}} \frac{p - p_0}{(x - x_0)^\gamma} \tag{14}$$

The physical explanation of eq. (14) is available in [30-32], hereby Δx can be the lobal width, *h*, see fig. 2.

The section area, *A*, depends on fractal geometry of the fiber and the fiber number involved in the filter. We assume that it can be written in the form:

$$A = A(r, R) \propto r^\alpha R^\beta \tag{15}$$

In [30, 31] the following transform is used:

$$s = x^\alpha \tag{16}$$

We can call it as the two-scale transform, where *x* can be considered as a small scale, while *s* is the large scale. Using the two-scale transform, eq. (13) becomes:

$$Q = \lambda A \frac{\mathrm{d}p}{\mathrm{d}s} \tag{17}$$

We can write eq. (17) in an approximate form:

$$Q = \lambda A \frac{\Delta p}{L^\gamma} \tag{18}$$

In view of eq. (15), eq. (18) becomes:

$$\Delta p = k \frac{QL^\gamma}{r^\alpha R^\beta} \tag{19}$$

In case $r = R$ and $\gamma = 1$, eq. (19) should become eq. (1), so the constant involved in eq. (19) can be identified:

$$k = \frac{8\mu\sigma^2}{\pi} \tag{20}$$

where $\sigma = R/r$.

We, finally, obtain the following pressure drop formula:

$$\Delta p = \frac{8\mu\sigma^2}{\pi}\frac{QL^\gamma}{r^\alpha R^\beta} = \frac{8\mu}{\pi}\frac{QL^\gamma}{r^{\alpha+2}R^{\beta-2}} \tag{21}$$

Conclusion

This paper gives a fractal approach to prediction of the pressure drop through a cigarette filter. The fractal dimensions for the sections of a shaped fiber and the filter and a bended fiber in the filter are the main factors affecting the pressure drop. In order to improve air permeability of the cigarette filter, a cocoon-like hierarchical structure is much needed, in the present study a hierarchy can not be formed because a hierarchy requires at least three levels, and the cigarette filter has only two levels. This paper suggests a self-contained fractal model for a complex industrial process of critical importance for the industry, especially the specialists in design of the cigarette filter. The fractal model is able to predict the pressure drop through the cigarette filter, and it does not require any further empirical or semi-empirical input, making the model much perfect in practical applications.

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