PERIODIC OSCILLATING SOLITONS AND HOMOCLINIC BREATHER-WAVE SOLUTION FOR THE (3+1)-DIMENSIONAL JIMBO-MIWA EQUATION

by

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With the aid of symbolic computation, some new types of breathing wave solutions to a (3+1)-D Jimbo-Miwa equation are obtained by the extended homoclinic test method. Its homoclinic breather-wave solution, periodic oscillating soliton and doubly-soliton solution are investigated.

Key words: *homolinic test, breather-wave, periodic oscillating soliton, doubly-soliton*

Introduction

It is well-known that many important phenomena in thermal science and other fields are described by non-linear PDE. With the rapid development of symbolic computation, studies on various physical structures of solutions of non-linear evolution equations had attracted much attention in connection with the important problems that arise in scientific applications. Soliton, multi-soliton or solitary wave, oscillating soliton and breather wave are new types of non-linear and localized waves with distinct dynamical and physical characteristics in non-linear systems [1]. The high amplitude wave produced during the collision between soliton and breather can be used to elaborate the generation mechanism of rouge wave [2]. Multi-soliton or solitary wave solutions of non-linear PDE may well describe various phenomena in physics and other fields [3-6]. Some oscillating solitons may be considered as a kind of non-propagation solitons [7-10]. To find new explicit solutions, some effective methods have been proposed [11-15] and the study of non-linear localized waves and interaction solutions among them is one of the important hot topics in recent years.

In this paper, we pay our attention to construct new type periodic oscillating solitons of a non-integrable (3+1)-D PDE called Jimbo-Miwa equation [16] in its potential form, *i. e.*:

$$H_{yt} - H_{xz} - 3H_{xx}H_y - 3H_{xy}H_x - H_{xxxy} = 0 \tag{1}$$

It is known that eq. (1) is non-integrable at any meaning [17]. In [18], by using the direct method due to Clarkson and Kruskalthe, Mei and his co-author found the symmetry of eq. (1) and some exact solitary-wave solutions were reported. But, as far as we know, it is different from the well-known (3+1)-D Jimbo-Miwa equation, no more solutions, especially breathers, multi-soliton to eq. (1) has been reported. In this paper, applying the method adopted

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in [19], we discover some new solutions, such as homoclinic breather-wave solutions and multiple periodic-soliton solutions to eq. (1).

New type homoclinic breather wave solutions for the (3+1)-D

In this section, the homoclinic test technique is applied to study eq. (1). Let's begin with a Painleve-Backlund transformation:

$$H = H_0 + 2(\ln\varphi)_x \tag{2}$$

where $\varphi = (x, y, z, t)$ is an arbitrary function of variables x, y, z, and t to be determined later, and H_0 – an arbitrary seed solution of eq. (1). Substituting eq. (2) into eq. (1), an identical equation of bi-linear form is yielded:

$$(D_y D_t - D_x D_z - D_x^3 D_y)\varphi\,\varphi = 0 \tag{3}$$

where the bi-linear operator D is defined:

$$D_x^m D_y^n D_z^r D_t^s f\ g = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^r \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^s f\ g\Big|_{(x,y,z,t)=(x',y',z',t')}$$

Then, by choosing test function $\varphi = (x, y, z, t)$:

$$\varphi(x,y,z,t) = \exp(-\eta_1) + \tau\cos\eta_2 + \rho\exp(\eta_1) \tag{4}$$

where $\eta_i = a_i x + b_i y + c_i z + d_i t + r_i$, $(i = 1, 2)$ and a_i, b_i, c_i, and d_i are constants to be determined later, r_i – the arbitrary constants, and substituting (4) into eq. (3), and equating all the coefficients of exp $(-\eta_1)$, exp (η_1), $\cos\eta_2$ and $\sin\eta_2$ to zero yields a set of algebraic equations:

$$4\rho(4a_1^3 b_1 + a_1 c_1 - b_1 d_1) + \tau^2(4a_2^3 b_2 - a_2 c_2 + b_2 d_2) = 0$$

$$\tau(3a_1^2 a_2 b_1 - a_2^3 b_1 + a_1^3 b_2 - 3a_1 a_2^2 b_2 + a_2 c_1 + a_1 c_2 - b_2 d_1 - b_1 d_2) = 0$$

$$\rho\tau(-3a_1^2 a_2 b_1 + a_2^3 b_1 - a_1^3 b_2 + 3a_1 a_2^2 b_2 - a_2 c_1 - a_1 c_2 + b_2 d_1 + b_1 d_2) = 0$$

$$\tau(-a_1^3 b_1 + 3a_1 a_2^2 b_1 + 3a_1^2 a_2 b_2 - a_2^3 b_2 - a_1 c_1 + a_2 c_2 + b_1 d_1 - b_2 d_2) = 0$$

$$\rho\tau(-a_1^3 b_1 + 3a_1 a_2^2 b_1 + 3a_1^2 a_2 b_2 - a_2^3 b_2 - a_1 c_1 + a_2 c_2 + b_1 d_1 - b_2 d_2) = 0$$

Solving the resulting equations simultaneously, we obtain the following set of algebraic equations:

– *The first set is:*

$$a_1 = a,\quad a_2 = 0,\quad b_1 = 0,\quad b_2 = b,\quad c_1 = c_2 = -a^2 b$$

$$d_1 = 0,\quad d_2 = a^3,\quad r_1 = r_2 = 0,\quad \rho = \frac{\tau^2}{4} \tag{5}$$

Substituting eq. (4) with eq. (5) into eq. (2), gives a periodic breather solitary wave solution:

$$H_1(x,y,z,t) = H_0 + a\frac{\sinh[a(x - abz) + r]}{\cos[b(y - a^2 z) + a^3 t] + \cosh[a(x - abz) + r]}$$

with $r = \ln/(\tau/2)$, where a, b, and τ are arbitrary constants.

Solution $H_1(x, y, z, t)$ shows a new family of two-wave, breather solitary wave, which is a solitary wave in x-z direction and meanwhile is a periodic wave in y-z direction whose amplitude periodically oscillates with the evolution of time, see fig. 1(a).

Taking $a = ia$, $b = ib$, and $\tau = 2$ in $H_1(x, y, z, t)$, solution $H_1(x, y, z, t)$ can be rewritten:

$$H_2(x,y,z,t) = H_0 + a\frac{\sin[a(x+abz)]}{\cosh[b(y+a^2 z)-a^3 t]+\cos[a(x+abz)]}$$

where $i^2 = -1$, and a and b are arbitrary real constants.

Solution $H_1(x, y, z, t)$ shows a periodic soliton structure for eq. (1), see fig. 1(b).

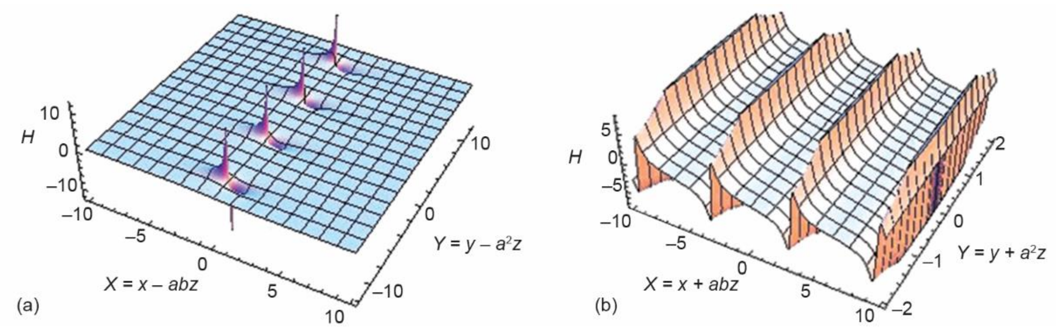

Figure 1. (a) The spatial structure of breather wave of H_1 with t = 1, (b) the spatial structure of periodic soliton of H_2 at t = 0.

– *The second set is:*

$$a_1 = -a_2 = -a, \quad b_1 = b_2 = b, \quad c_1 = c_2 = c$$

$$d_1 = d_2 = 2a^3 - \frac{ac}{b}, \quad r_1 = r_2 = 0, \quad \rho = \frac{1}{4}\tau^2 \tag{6}$$

Then, substituting eq. (4) with eq. (6) into eq. (2), we have another breather type of solitary wave solution:

$$H_3 = H_0 + 2a\frac{\sinh\left[X - by - a\left(2a^2 - \frac{c}{b}\right)t + r\right] - \sin\left[X + by + a\left(2a^2 - \frac{c}{b}\right)t\right]}{\cosh\left[X - by - a\left(2a^2 - \frac{c}{b}\right)t + r\right] + \cos\left[X + by + a\left(2a^2 - \frac{c}{b}\right)t\right]}$$

where $X = ax + cz$, $r = -\ln(\tau/2)$.

Solution represented by H_3 is a periodic kink solitary wave, whose amplitude also periodically oscillates with the evolution of time. It shows elastic interaction between a left propagation periodic wave and homoclinic wave of different direction, see fig. 2.

Multiple soliton structure for the (3+1)-D

In this section, the extended homoclinic test method is applied to study eq. (1). Multiple soliton structures for the (3+1)-D eq. (1) are derived.

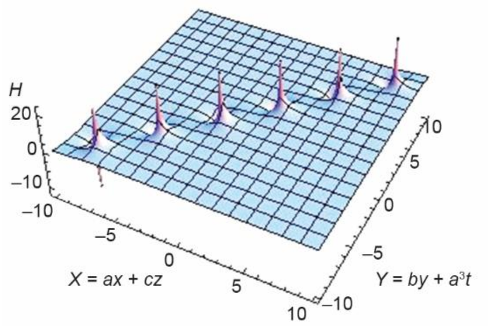

Figure 2. The spatial structure of breather wave of H_3

Let's choose the test function:

$$\varphi(x, y, z, t) = \exp(-\eta_1) + k\text{expt}(\eta_1) + m\cos(\eta_2) + h\cosh(\eta_3) \tag{7}$$

where $\eta_j = a_j x + b_j y + c_j z + d_j t$, $(j = 1, 2, 3)$.

Substituting eq. (7) into eq. (3) and equating each corresponding coefficients of $\exp(-\eta_1)$, $\exp(\eta_1)$, $\cos(\eta_2)$, $\sin(\eta_2)$, $\cosh(\eta_3)$, $\sinh(\eta_3)$ to zero, yields an algebraic system of a_j, b_j, c_j, d_j, k, m, and h. Solving the algebraic system simultaneously, we obtain the following set of algebraic equations:

$$a_1 = a, \quad a_2 = 0, \quad b_1 = 0, \quad b_2 = b, \quad c_2 = 0, \quad c_2 = c, \quad d_1 = a^3, \quad d_2 = -\frac{ac}{b}$$

$$a_3 = \Gamma, \quad b_3 = -\frac{\Gamma b^2(\Delta - 3a^2)}{4ac}, \quad c_3 = \frac{\Gamma c}{a}, \quad d_3 = \frac{\Gamma(3a^2 - \Delta)}{12}$$

$$k = \frac{1}{4}\left(m^2 + \frac{\Delta h^2}{3a^2}\right), \quad \Gamma = \frac{1}{6}\sqrt{6(3a^2 + \Delta)}, \quad \Delta = \frac{\sqrt{9a^4b^2 + 16c^2}}{b} \tag{8}$$

where a, b, c, m, and h are arbitrary constants.

Inserting eq. (8) into eq. (7) and eq. (2), a periodic breather two-solitary wave solution of eq. (1) is obtained:

$$H_4(x, y, z, t) = H_0 + 2a\frac{2\sqrt{k}\sinh(ax + cz + a^3t + r) + \dfrac{h\Gamma}{a}\sinh(\eta_3)}{2\sqrt{k}\cosh(ax + cz + a^3t + r) + m\cos\left(by - \dfrac{ac}{b}t\right) + h\cosh(\eta_3)}$$

where $\eta_3 = a_3x + b_3y + c_3z + d_3t$, and $r = \ln\sqrt{k}$.

Solution represented by $H_4(x, y, z, t)$ shows a new breather two-solitary wave, which posses two-solitary wave and meanwhile is a periodic wave whose amplitude periodically oscillates with the evolution of time, see fig. 3(a).

Especially, taking $b = ib(i^2 = -1)$ in $H_4(x, y, z, t)$, a doubly-soliton structure is derived which reads:

$$H_5(x,y,z,t) = H_0 + 2a\frac{2\sqrt{k}\sinh(ax+cz+a^3t+r)+\dfrac{h\Gamma_1}{a}\sinh(\eta_3)}{2\sqrt{k}\cosh(ax+cz+a^3t+r)+m\cosh\left(by+\dfrac{ac}{b}t\right)+h\cosh(\eta_3)}$$

with $\eta_3 = a_3x + b_3y + c_3z + d_3t$, and a_3, b_3, c_3, and d_3 satisfying the following conditions:

$$a_3 = \Gamma_1,\quad b_3 = \frac{\Gamma_1 b^2(\Delta_1 - 3a^2)}{4ac},\quad c_3 = \frac{\Gamma_1 c}{a},\quad d_3 = \frac{\Gamma_1(3a^2 - \Delta_1)}{12}$$

$$\Gamma_1 = \frac{\sqrt{6(3a^2+\Delta_1)}}{6},\quad \Delta_1 = \frac{9a^4b^2 - 16c^2}{b}),\quad k = \frac{1}{4}\left(m^2 + \frac{\Delta_1 h^2}{3a^2}\right)$$

where a, b, c, m, and h are arbitrary real constants.

Solution $H_5(x, y, z, t)$ shows a multiple kink-solitary wave structure of eq. (1). Figure 3 shows a kink soliton interacting with a breather structure, see fig. 3(b).

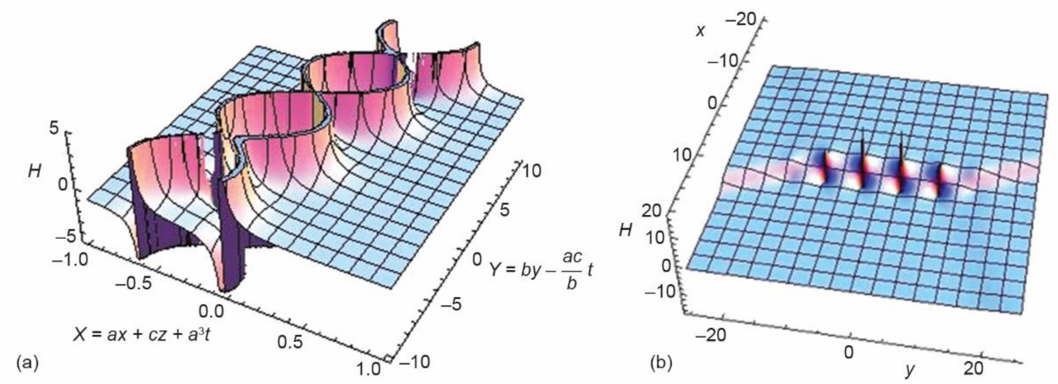

Figure 3. (a) The spatial structure of breather two-solitary wave of $H_4(x, y, z, t)$, (b) the spatial structure of multiple kink-solitary wave of $H_5(x, y, z, t)$

Conclusion

In this paper, the extended homoclinic test approach is applied to solve a (3+1)-D Jimbo-Miwa equation, new exact solutions, such as homoclinic breather-wave solutions and doubly periodic wave soliton solutions are obtained. All the presented solutions show the remarkable richness of the solution space of the (3+1)-D Jimbo-Miwa eq. (1). It is also shown that the method is concise and effective, it can be used to treat many other types of non-linear evolution equations.

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