

## ON FABRICATION OF NANOSCALE NON-SMOOTH FIBERS WITH HIGH GEOMETRIC POTENTIAL AND NANOPARTICLE'S NON-LINEAR VIBRATION

by

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*Non-smooth surface of a nano or micro-scale fiber has an extremely large surface area and a tremendously high surface energy (geometric potential). This paper focuses on the formation mechanism of fabrication of a non-smooth surface by electrospinning through controlling solvent evaporation and nanoscale adhesion of nanoparticles on the surface. Poly(vinylidene fluoride), multi-wall carbon nanotubes and a binary solvent system are adopted in the experiment to elucidate how to fabricate nanoscale porous nanofibers and lotus-surface-like nanofibers. A nanoparticle's vibration near its equilibrium is also discussed, which also affects greatly the surface morphology.*

Key words: *lotus effect, antifouling, self-cleaning, Bernoulli equation, non-linear oscillation, frequency-amplitude relationship*

### Introduction

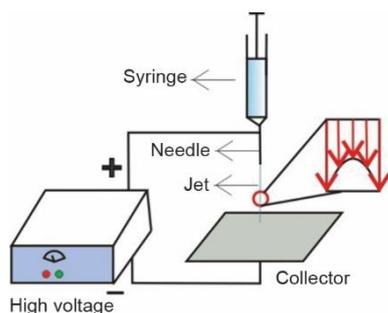
Nanofibers have a large surface area, which produces a high geometric potential (surface energy) [1-7], as a result, nanofibers can find wide applications in various areas. If we want to have an even higher surface energy for an advanced application, smaller nanofibers are needed, but its fabrication process becomes impossible when fiber diameter reaches a threshold, and its cost becomes higher. So we should search for fabrication processes to remarkably increase fiber's surface area, among which the nanoscale porous fibers [8-12] have been caught much attention, Liu *et al.* [13] used a multiple solvent system to control the solvent evaporation to fabricate controllable nanoporous fibers. Zhao *et al.* [14] found that the sudden solvent evaporation was an efficient way to produce nanoporous fibers. Peng *et al.* [15] suggested a Rachford-Rice-like equation for estimation of solvent evaporation. The solvent's fast evaporation always results in a porous fiber with a concave surface during the electrospinning process. In this paper we will try to fabricate lotus-surface-like nanofibers with convex surface

### Experimental design

Poly(vinylidene fluoride) (PVDF), and multi-wall carbon nanotubes (MWCNT) with diameter ranging from 10 nm to 20 nm and length ranging from 30  $\mu\text{m}$  to 100  $\mu\text{m}$ ) were

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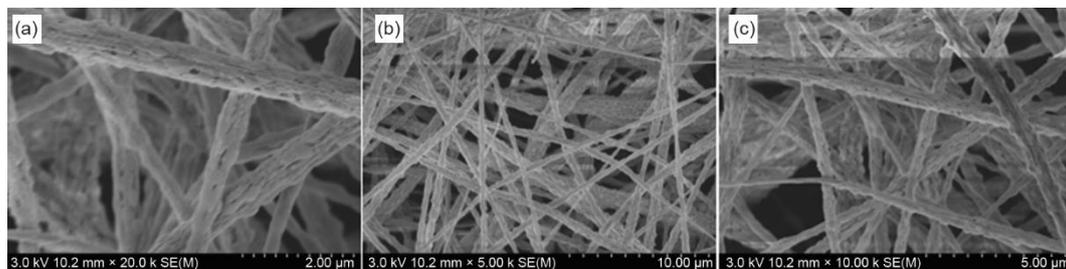
used in our experiment. A binary solvent system was prepared using N,N-dimethylformamide (DMF) and acetone with 7:3 mole ratio, and a 10% PVDF solution was prepared as a similar way in our previous publications [16-19]. In order to study the effect of MWCNT on the surface property of nanofiber, 0.5 wt.% MWNT were added to the 10% PVDF solution.



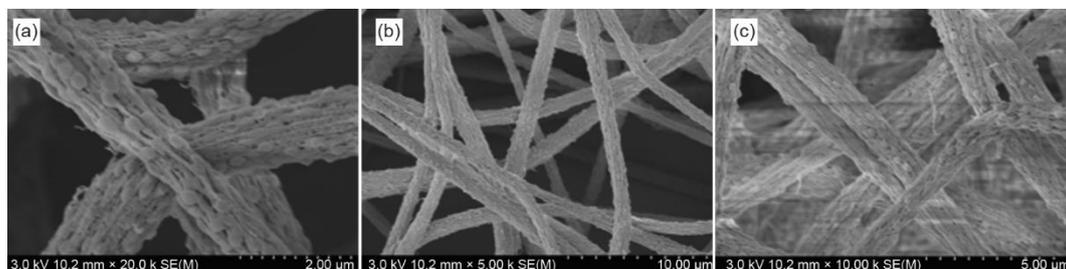
**Figure 1. Experimental set-up**

The experimental set-up was illustrated in fig. 1, the solution was placed into a 10 mL plastic syringe installed with a stainless steel-needle as the spinneret. The plastic syringe was connected to a 20 KV voltage source while a collector was grounded. The distance between the needle tip and the collector was 18 cm for PVDF solution and 15 cm for PVDF/MWNT solution, respectively. The feeding rate of the solution in the syringe was controlled at 1.0 mL/h.

Surface morphologies of PVDF fibers and PVDF/MWCNT fibers were given, respectively, in figs. 2 and 3. The surface of pure PVDF fiber is porous, while that of PVDF/MWCNT is lotus-surface-like structure with nanoparticles on the surface. The binary solvent system was discussed in [14], where the formation mechanism of nanoscale porous surface was also elucidated. The fast solvent evaporation is the key factor for fabricating nanoscale porous fibers. During the spinning process, a moving jet becomes gradually solidified, while the solvent evaporation can effectively control the solidification process, smooth and small fibers can be obtained when the solvent evaporation is slow. However, when some a solvent in the binary solvent system, *e. g.*, acetone, has a faster evaporation ratio, nanoscale porous fibers with larger diameter are predicted.



**Figure 2. The SEM images of unsmooth PVDF fibers with average fiber diameter of  $396 \pm 79$  nm**



**Figure 3. The SEM images of unsmooth PVDF/MWNT fibers with average fiber diameter of  $1220 \pm 286$  nm, the average size of the nanoparticles on the surface is  $293 \pm 39$  nm X  $180 \pm 45$  nm**

### Theoretical analysis

Fiber diameter can be predicted by the mass equation of 1-D fluid:

$$\pi r^2 \rho u = Q \quad (1)$$

where  $r$  is the radius of the moving jet,  $u$  – the velocity,  $\rho$  – the density, and  $Q$  – the flow ratio, which is a constant in our experiment.

For the PVDF/MWNT solution, the section area of the jet can be written in the form:

$$\pi r^2 = A_{\text{PVDF}} + A_{\text{MWNT}} \quad (2)$$

where  $A_{\text{MWNT}}$  and  $A_{\text{PVDF}}$  are section areas occupied by the PVDF solution and MWNT particles, respectively. During the spinning process  $A_{\text{MWNT}}$  keeps unchanged as shown in fig. 4. So we can predict a larger fiber size of the PVDF/MWNT fibers than that of PVDF fibers.

The PVDF/MWNT jet has higher viscous force than that of the PVDF jet, the MWNT particles hinder the fluid, so we need a higher force in the spinning process. In our experiment, we shorten collector's distance from 18 cm to 15 cm.

In order to elucidate the surface morphology of PVDF/MWNT fibers. We consider Bernoulli equation:

$$\frac{1}{2} u^2 + \frac{P}{\rho} = B \quad (3)$$

where  $P$  is the pressure and  $B$  – the Bernoulli constant.

Tian *et al.* [20-22] suggested a macromolecular electrospinning using a long and narrow tube, so that macromolecule orientation can be easily controlled by the Bernoulli equation. The same principle can be also applied to control the nanoparticles distribution. We assume that nanoparticles are distributed uniformly on the section of the spinneret as illustrated in fig. 5. As the electrostatic force acts on the surface of the moving jet, the velocity distribution is not as that in a tube flow. The velocity is maximal on the surface of the boundary, and it is minimal in the center of the jet, fig. 5.

Considering two points on the section of the moving jet, one is at the center, the other is on the boundary, according to the Bernoulli equation, there exists a pressure gradient, which is:

$$P_{\text{center}} - P_{\text{boundary}} = \frac{1}{2} [\rho (u_{\text{boundary}})^2 - (u_{\text{center}})^2] > 0 \quad (4)$$

This pressure gradient is the acting force that pushes gradually the particles in the center to the boundary, and finally all particles will be all distributed on the surface.

According to MWNT producer, MWNT are nanofibers with diameter ranging from 10 nm to 20 nm and length ranging from 30  $\mu\text{m}$  to 100  $\mu\text{m}$ , see the producer label in fig. 6.

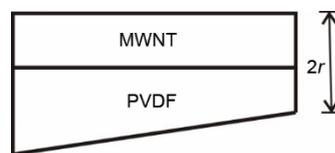


Figure 4. The spinning process of PVDF/MWNT jet

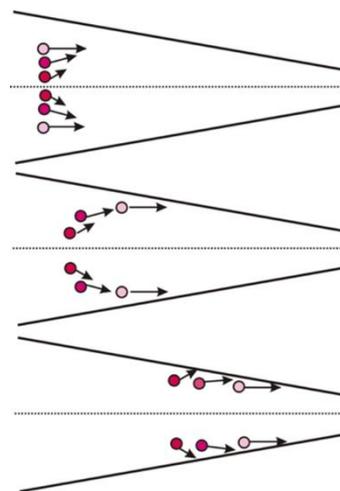


Figure 5. Velocity distribution and particle motion in the moving jet



**Figure 6.** Producer label describing MWNT properties

According to our experimental observation given in fig. 3, the used MWNT should have particle-like geometry with length of  $293 \pm 39$  nm and width of  $180 \pm 45$  nm.

### Particle oscillation

When a particle moves onto the surface of the moving jet, a small perturbation will arouse its vibration. We assume that the particle moves a distance radially from the boundary to the center, due to the pressure difference, the particle will push back to its equilibrium position, the force can be expressed:

$$F = A \frac{\Delta P}{\Delta x} \quad (5)$$

where  $A$  is section area of the particle. Ignoring the viscous force, we have the following motion equation:

$$F = m\ddot{x} \quad (6)$$

where  $m$  is the mass of the particle and  $x$  – the radial co-ordinate. We assume the velocity distribution has the form:

$$u = u_0 + \frac{u_b - u_0}{r^2} x^2 \quad (7)$$

where  $u_0$  and  $u_b$  are the velocities at the center ( $x = 0$ ) and the boundary ( $x = r$ ), respectively.

It is obvious that:

$$\Delta u = \frac{2(u_b - u_0)}{R^2} x \Delta x \quad (8)$$

According to eq. (3), we have:

$$\Delta P = -\rho u \Delta u = -\frac{2\rho(u_b - u_0)}{R^2} \left( u_0 + \frac{u_b - u_0}{R^2} x^2 \right) x \Delta x \quad (9)$$

In view of eq. (9), we re-write eq. (6) in the form:

$$m\ddot{x} + A \frac{2\rho(u_b - u_0)}{R^2} \left( u_0 + \frac{u_b - u_0}{R^2} x^2 \right) x = 0 \quad (10)$$

We re-write eq. (10) in the form:

$$\ddot{x} + \omega^2 x + \varepsilon x^3 = 0 \quad (11)$$

where

$$\omega^2 = \frac{2\rho A u_0 (u_b - u_0)}{m R^2} \quad (12)$$

$$\varepsilon = \frac{2\rho A (u_b - u_0)^2}{m R^4} \quad (13)$$

Equation (11) is the well-known Duffing equation, it can be solved by the Taylor series method [23], the variational iteration method [24, 25], the variational method [26-29] and the homotopy perturbation method [30]. Here we use He's formulation [31, 32] to find the frequency-amplitude relationship:

$$\Omega^2 = \left. \frac{df(x)}{dx} \right|_{x=A/2} = (\omega^2 + 3\varepsilon x^2) \Big|_{x=A/2} = \omega^2 + \frac{3}{4} \varepsilon A^2 \quad (14)$$

where  $\Omega$  and  $A$  are the frequency and amplitude, respectively,  $f(x)$  – the force,  $f(x) = \omega^2 x + \varepsilon x^3$ . In an energy form, eq. (11) can be deduced as a stationary condition of the following variational principle:

$$J(x) = \int_0^A \left[ \frac{1}{2} \dot{x}^2 - \left( \frac{1}{2} \omega^2 x^2 + \frac{1}{4} \varepsilon x^4 \right) \right] dx \quad (15)$$

where  $\dot{x}^2/2$  is the kinetic energy and  $(\omega^2 x^2)/2 + (\varepsilon x^4)/4$  is the potential energy.

The oscillation of the nanoparticle on the boundary will greatly affect the surface morphology of the fibers.

## Conclusions

This paper suggests two methods to fabricate unsmooth fibers, one is to use the multiple solvent system to fabricate porous fibers with concave surface, the other is to use nanoparticles as additives to fabricate lotus-surface-like nanofibers with convex surface. The unsmooth surface has many attractive properties, e. g., antifouling, self-cleaning, superwettability, high surface energy (geometric potential). The Bernoulli equation is used to reveal the formation mechanism during the spinning process. Additionally this paper also suggests an experimental identification of the particle size by the solvothermal method [33] or the molecular beam method [34].

To exactly describe the jet motion during the spinning process, we need two-scale thermodynamics [35, 36], e. g., we need two scales to describe the spinning process. One scale is large where the Bernoulli equation or more accurately the laws in fluid mechanics works, and the other scale is much smaller, where the solvent evaporation should be considered, and fractal calculus [37-39] should be adopted.

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