A THERMODYNAMIC MODEL FOR A PACKING DYNAMICAL SYSTEM

by

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So far shock and vibration are the inherent properties of all packing systems, and various mitigation measures have been considered using packing materials, which result in a large packing size. Now the thing changes, this paper gives an effective thermodynamic technology that can completely avoid shock and vibration. An aerial delivery system is used as an example to elucidate the novel packing system, which requires zero loading velocity when loaded. A thermodynamic model is established to reveal the main factors affecting the packing system.

Key words: *non-linear vibration, tangent oscillator, flexible sensor, gecko, thermodynamics, packing size, porous materials, nanofiber, two-scale thermodynamics*

Introduction

A safe paradrop is of great importance for both the goods and parachutists in emergency circumstances. For example, an accurate and safe aerial delivery of an extremely precise instrument is remarkably important for the case when traditional deliveries are failed. Many tries have been tried to mitigate the vibration of the goods when loaded on the earth, however, due to the heavy impulse, oscillation of the packing system occurs. Shock and vibration cannot be avoided for almost all packaged products, additional weight and enlarged volume have to be considered for shock absorption by a thick cushion or a porous medium. The cushion might take up 95% volume for some packing systems. In some urgent cases, goods and persons have to be delivered together, this makes the design of a packing system ever more difficult if not impossible.

The vibration can be described by the well-known hyperbolic tangent oscillator [1], and it is easy to predict accurately the frequency-amplitude relationship [2-12], which is the main factor affecting the safe loading. Though we have many technologies to absorb the kinetic energy of the loading goods, a safe loading is almost impossible unless the loading kinetic energy is zero when loaded. In this paper, we will design a thermodynamic system to guarantee the loading velocity is zero.

Packing dynamical system

A packaging system will oscillate when it is loaded on the earth due to the impact force, the hyperbolic tangent oscillator is always used to describe the non-linear vibration property, which reads [1]:

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$$m\ddot{x} + F_0 \tanh\left(\frac{x_0}{F_0}x\right) = 0, \quad x(0) = 0, \quad \dot{x}(0) = \sqrt{2gH} \tag{1}$$

where F_0 is the impulsion force when the packing system is loaded on the Earth, m – its mass, H – the dropping height, and x_0 – the maximal displacement due to the cushioning material.

Equation (1) can be effectively solved by some analytical methods, for examples, the variational iteration method [13-16], the variational approach [17-19], the homotopy perturbation method [20-28], He's frequency formulation [2-12], and Taylor series method [29]. Though the analytical methods can reveal the basic property of the vibration, it is impossible to present the packing system from vibration.

Vibration is therefore the intrinsic property of the packing system, and so far there is no way to stop the vibration, the frequency-amplitude relationship is the main factor for designing a packing system. However, if the initial conditions are:

$$x(0) = 0, \quad \dot{x}(0) = 0 \tag{2}$$

The packing system's vibration can be completely avoided. That implies when the loading velocity is zero, as shown in eq. (2), shock and vibration can be completely avoided. This paper is to achieve the zero-loading velocity thermodynamically, and we call the zero loading velocity as an inverse problem for a safe loading.

Thermodynamic model for the inverse problem

Gecko has a unique loading system, which enables the creature to load on any surface with a zero loading velocity, fig. 1 [30].

For the first time ever, this paper suggests a thermodynamic approach to achieving the zero-loading velocity when the packing system is loaded on the earth. Figure 2 shows the packing system with a parachute, which includes a height sensor, a temperature sensor, and velocity sensor. When the packing system reaches a height of h, the heating system located inside of the parachute begins to heat the air inside of the parachute.

Figure 1. Gecko's loading ability with a zero loading velocity

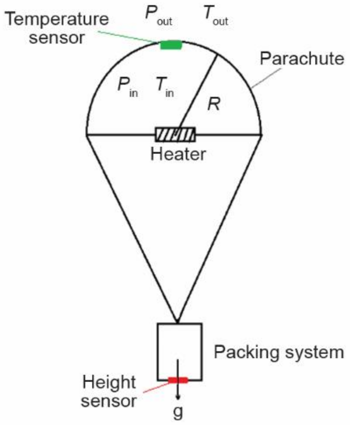

Figure 2. Parachute-packing system

We assume that the kinetic energy at the height of h is:

$$K=\frac{1}{2}Mu_0^2 \tag{4}$$

where M is the mass of the packing system and u_0 – the velocity at the height of h. When the packing system is loaded on the earth, the work done by the weigh is:

$$W=Mgh \tag{5}$$

This work will convert kinetic energy when loaded if no special measure is taken. Now the heat system is working to increase the temperature under the parachute.

We consider the air motion under the parachute is an isentropic process, the homentropic equation is:

$$P=c\rho^k \tag{6}$$

where P is the air pressure, ρ – the air density, c and k – the constants.

For an ideal gas, the station equation is:

$$\frac{P}{\rho}=\mathrm{R}T \tag{7}$$

where R is the gas constant.

From eqs. (6) and (7), we have:

$$P^{[(k-1)/k]}=c^{-1/k}\mathrm{R}T \tag{8}$$

or

$$P=c^{-1/(k-1)}\mathrm{R}^{\mu}T^{\mu} \tag{9}$$

where μ is a constant defined as:

$$\mu=\frac{k}{k-1} \tag{10}$$

We assume the temperature inside and outside the parachute are, respectively, T_{in} and T_{out}. The pressure difference between the inside and outside the parachute is:

$$\Delta P=c^{-1/(k-1)}\mathrm{R}^{\mu}(T_{\mathrm{in}}^{\mu}-T_{\mathrm{out}}^{\mu}) \tag{11}$$

This rising force due to by the pressure difference $A\Delta P$, where A is the section area of the parachute. The work done by this force is to absorb all energy when the packing system is loaded on the earth, that is:

$$A\Delta Ph=K+W=\frac{1}{2}Mu_0^2+Mgh \tag{12}$$

or

$$\Delta P=\frac{Mu_0^2+2Mgh}{2Ahc^{-1/(k-1)}} \tag{13}$$

In view of eq. (11), we have:

$$T_{\mathrm{in}}^{\mu}=T_{\mathrm{out}}^{\mu}+\frac{Mu_0^2+2Mgh}{2Ahc^{-1/(k-1)}\mathrm{R}^{\mu}} \tag{14}$$

or

$$T_{\text{in}} = \left[T_{\text{out}}^{\mu} + \frac{Mu_0^2 + 2Mgh}{2Ahc^{-1/(k-1)}\text{R}^{\mu}} \right]^{1/\mu} = \left[T_{\text{out}}^{k/(k-1)} + \frac{Mu_0^2 + 2Mgh}{2Ahc^{-1/(k-1)}\text{R}^{k/(k-1)}} \right]^{(k-1)/k} \tag{15}$$

When the temperature satisfies eq. (15), the packing system arrives at the earth with zero velocity.

Conclusion

For the first time ever, we suggest a safe packing system with zero loading velocity when it is loaded on the earth. When the packing system is loaded on the earth, All kinetic energy is absorbed by the parachute. The temperature in the parachute can be easily controlled, and the present technology can be directly applied for practical applications:

$$T_{\text{in}} = \left[T_{\text{out}}^{k/(k-1)} + \frac{MgH}{Ahc^{-1/(k-1)}\text{R}^{k/(k-1)}} \right]^{(k-1)/k} \tag{16}$$

where H is the total dropping height. We can use insulated canvas which is filmed by ZrO_2 nanofibers [31, 32], its thermal property can be analyzed by the two-scale thermodynamics [33, 34].

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