# ON ZERO-DIMENSIONAL OCEAN DYNAMICS

by

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*How to study the effect of the Sun or the Moon's gravity on ocean motion? Of course, Newton's gravity should be considered. However, Newton's law considers the Earth as a 0-D point, the ocean motion inside of a 0-D point of the Earth is negative 3-D, and Newton's law becomes invalid in a negative space. In order to solve the problem, we divide the Earth into two parts, one part is the studied ocean, the other is the left Earth without the ocean. A mechanics model can be then established for the 0-D ocean dynamics.*

Key words: *Newton's gravity, negative space, variational principle, three-spring system, three-body system, tsunami-like wave, resonance*

## Introduction

The ocean motion is extremely complex [1], it is affected by various factors. This paper focuses itself on the effect of the Sun and the Moon's gravities on ocean dynamics. The Qianjiang tide [2] occurs from the $16^{th}$ to $18^{th}$ of the eighth Chinese lunar month when the Earth, the Sun, and the Moon is in a straight line. It is generally considered that the tide is produced by the combination of gravitational effect due to the Sun and the Moon and the rotation effect of the Earth and the Moon.

According to Newton's law, the gravitational attraction between two bodies can be expressed:

$$F = G\frac{M_1 M_2}{R^2} \tag{1}$$

where $M_1$ and $M_2$ are masses for the two bodies and $R$ – the distance. Equation (1) considers the two bodies as 0-D points, that means the Earth is a 0-D point, so the ocean motion, if Newton's law is applied, is of negative 3-D [3-8].

## Invalid Newton's law in a negative space

Dimension is everything, at different scales, we have different physical laws [9, 10]. For example, the ocean motion can be modeled by the continuum mechanics on a large scale, but when the scale becomes small and small until to a molecule scale, all laws in continuous fluid dynamics becomes invalid, and many phenomena have to be explained in a fractal space, where fractal calculus has to be used [11-19].

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Though the Earth is large enough and can be considered as a 3-D object on any observation scales, it is a 0-D point with a huge mass according to Newton's gravity. The water in a sea is on the surface of the 0-D Earth is in a negative space when the Earth is considered as a point [6-8]. To elucidate this, we first consider the boundary dimensions of an object.

The boundaries of a 3-D cube, 2-D surface, and a 1-D line have, respectively, 2-dimensions, 1-dimension, and zero-dimensions. The boundary dimensions can be easily derived:

$$\begin{gathered}\text{3-D (cube)} - 1 \to \text{2-D (surface)}\\ \text{2-D (surface)} - 1 \to \text{1-D (line)}\\ \text{1-D (line)} - 1 \to \text{0-D (point)}\end{gathered} \tag{2}$$

Then what is the dimension of the boundary of a 0-D point? According to the derivation, the boundary of the 0-D point is negative one dimension:

$$\text{0-D (point)} - 1 \to -\text{1-D (negative space)} \tag{3}$$

Continuing the derivation, we have the negative three-dimensions in a 0-D point.

If the Earth is considered as a 0-D point, the 3-D ocean motion on any observation scales is negative 3-D in view of Newton's law, which cannot be directly applied to a negative space.

## Effect of the Sun and the Moon's gravities on ocean motion

In order to apply Newton's law to studying the effects of the Sun and the Moon's gravities on the ocean motion, we must consider the studied ocean as a 0-D point, and the other part of the Earth is also a 0-D point as illustrated in fig. 1.

The 0-D ocean is attracted by the Earth (point B), the Sun and the Moon:

$$F_S = G\frac{M_{Sun}M_{sea}}{R^2_{Sun\text{-}sea}} \tag{4}$$

$$F_M = G\frac{M_{Moon}M_{sea}}{R^2_{Moon\text{-}sea}} \tag{5}$$

$$F_E = G\frac{M_{Earth}M_{sea}}{R^2_{Earth\text{-}sea}} \tag{6}$$

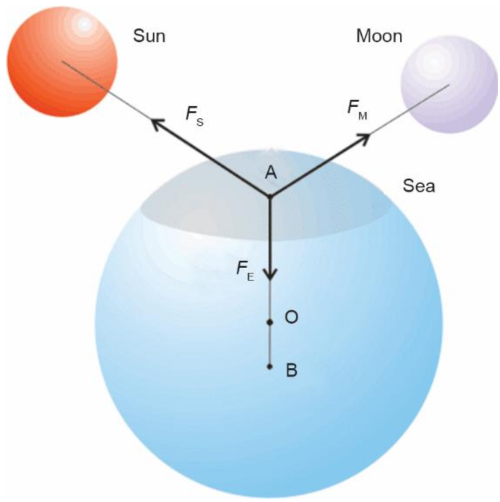


**Figure 1. The Earth system consisting of two parts: one is the studied ocean, the other is left part without the ocean;** *O is the center of the Earth, A is the mass center of the ocean, and B is the mass center of the part without the ocean*

where $M_{Sea}$ is the total mass of the studied ocean, $M_{Earth}$ – the mass of the Earth without the studied ocean, $M_{Sun}$ and $M_{Moon}$ – the masses of the Sun and the Moon, respectively. The $R_{Earth\text{-}sea}$ is the distance between $A$ and $B$ as illustrated in fig. 1, $R_{Sun\text{-}Sea}$ and $R_{Moon\text{-}sea}$ are the distances of the Sun and the Moon from the point $A$, respectively.

**Ocean dynamics**

In this section, we use the variational principle [20-23] to establish a mathematical model for the point *A* illustrated in fig. 1.

The potentials acted by the Sun, the Moon and the Earth without the studied ocean can be expressed, respectively:

$$E_{\mathrm{S}} = -G\frac{M_{\mathrm{Sun}}M_{\mathrm{sea}}}{R_{\mathrm{Sun\text{-}sea}}} \tag{7}$$

$$E_{\mathrm{M}} = -G\frac{M_{\mathrm{Moon}}M_{\mathrm{sea}}}{R_{\mathrm{Moon\text{-}sea}}} \tag{8}$$

$$E_{\mathrm{E}} = -G\frac{M_{\mathrm{Earth}}M_{\mathrm{sea}}}{R_{\mathrm{Earth\text{-}sea}}} \tag{9}$$

The kinetic energy consists of three parts: the ocean's kinetic energy, $1/2M_{\mathrm{sea}}u^2$, the kinetic energy due to Earth's self-rotation, $1/2J_{\mathrm{Earth}}\omega^2$, and the kinetic energy due to Earth's rotation around the Sun $1/2J_{\mathrm{Sun}}\Omega^2$:

$$K = \frac{1}{2}M_{\mathrm{Sea}}u^2 + \frac{1}{2}J_{\mathrm{Earth}}\omega^2 + \frac{1}{2}J_{\mathrm{Sun}}\Omega^2 \tag{10}$$

where $u$ is total velocity of the point *A* in fig. 1, $J_{\mathrm{Earth}}$ and $J_{\mathrm{Sun}}$ – the rotational inertias of Earth's self-rotation with frequency of $\omega$, and Earth's rotation around the Sun with frequency of $\Omega$, respectively.

The Lagrange function can be easily obtained:

$$L = \frac{1}{2}M_{\mathrm{sea}}u^2 + \frac{1}{2}J_{\mathrm{Earth}}\omega^2 + \frac{1}{2}J_{\mathrm{Sun}}\Omega^2 + G\frac{M_{\mathrm{Sun}}M_{\mathrm{sea}}}{R_{\mathrm{Sun\text{-}sea}}} + G\frac{M_{\mathrm{Moon}}M_{\mathrm{sea}}}{R_{\mathrm{Moon\text{-}sea}}} + G\frac{M_{\mathrm{Earth}}M_{\mathrm{sea}}}{R_{\mathrm{Earth\text{-}sea}}} \tag{11}$$

We can use spherical co-ordinates $(r, \theta, \phi)$ to derive the governing equations:

$$\frac{\partial L}{\partial r} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{r}}\right) = 0 \tag{12}$$

$$\frac{\partial L}{\partial \theta} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0 \tag{13}$$

$$\frac{\partial L}{\partial \phi} - \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0 \tag{14}$$

We obtain a coupled non-linear vibration system, which can be solved by various analytical methods [24-38].

## Conclusions

Newton's gravity due to the Sun or the Moon will greatly affect the ocean motion. In this paper, for the first time ever, we consider the studied ocean as a 0-D point, so that Newton's gravity can be effectively applied.

According to Newton's gravity, the Moon's attraction to the sea water is almost twice of that from the Sun. As a preliminary study, the Sun's effect can be ignored. On the other hand, if the effects of wind and water temperature are considered, the model will be much more perfect though it becomes much more complex.

The following conclusions are made:

- The Earth is divided into two parts, one part is the studied ocean, the other is the left Earth without the ocean.
- The whole studied ocean is considered as a 0-D point so that Newton's gravity can be applied.
- A 0-D ocean dynamics is preliminary established by the variational principle.

When the frequency of the ocean vibration is closed to that of the Earth's rotation, orbital frequencies of the Earth and the Moon's motions, tsunami-like waves [12, 17, 34] can be predicted due to the resonance.

## References

[1] Yuan, Y. L., *et al.*, Establishment Of The Ocean Dynamic System With Four Sub-Systems And The Derivation Of Their Governing Equation Sets, *J. Hydrodynam. B, 24* (2012), 2, pp. 153-168
[2] Sun, J. Q., *et al.*, Levels And Distribution Of Dechlorane Plus And Related Compounds In Surficial Sediments Of The Qiantang River In Eastern China: The Results Of Urbanization And Tide, *Sci. Tot. Environ.*, *443* (2013), Jan., pp. 194-199
[3] El Naschie, M. S., On Certain 'Empty' Cantor Sets and Their Dimensions, *Chaos, Solitons & Fractals*, *4* (1994), 2, pp. 293-296
[4] El Naschie, M. S., A Review of E Infinity Theory and The Mass Spectrum of High Energy Particle Physics, *Chaos, Solitons & Fractals*, *19* (2004), 1, pp. 209-236
[5] El Naschie, M. S., On the Philosophy of Being and Nothingness in Fundamental Physics, *Non-Linear Science Letters B*, *1* (2011), 1, pp. 4-5
[6] He, J. H., An Elementary Introduction to Recently Developed Asymptotic Methods and Nanomechanics in Textile Engineering, *Int. J. Nonl. Sci. Num. Sim.*, *22* (2008), 21, pp. 3487-3578
[7] He, J. H., Hilbert Cube Model For Fractal Spacetime, *Chaos Solitons & Fractals, 42* (2009), 5, pp. 2754-2759
[8] He, J. H., A Note on Elementary Cobordism and Negative Space, *Int. J. Nonl. Sci. Num. Sim., 11* (2010), 12, pp. 1093-1101
[9] Ain, Q. T., He, J. H., On Two-Scale Dimension and Its Applications, *Thermal Science*, *23* (2019), 3B, pp. 1707-1712
[10] He, J. H., Ji, F. Y., Two-Scale Mathematics and Fractional Calculus for Thermodynamics, *Thermal Science*, *23* (2019), 4, pp. 2131-2133
[11] He, J. H., Fractal Calculus and Its Geometrical Explanation, *Results Phys*., *10* (2018), Sept., pp. 272-276
[12] Wang, Y., Fractal Derivative Model for Tsunami Travelling, *Fractals*, *27* (2019), 2, 1950017
[13] He, J. H., A New Fractal Derivation, T*hermal Science, 15* (2011), Suppl. 1, pp. S145-S147
[14] He, J. H., Fractal Calculus and Its Geometrical Explanation, *Results in Physic, 10* (2018), Sept., pp. 272-276
[15] He, J. H., A Simple Approach to One-Dimensional Convection-Diffusion Equation and Its Fractional Modification for E Reaction Arising in Rotating Disk Electrodes, *Journal of Electroanalytical Chemistry*, *854* (2019), Dec., ID 113565
[16] He, J. H., Variational Principle for the Generalized KdV-Burgers Equation with Fractal Derivatives for Shallow Water Waves, *J. Appl. Comput. Mech.*, *6* (2020), 4, pp. 735-740
[17] Wang, Y., *et al.*, A Fractional Whitham-Broer-Kaup Equation and Its Possible Application to Tsunami Prevention, *Thermal Science, 21* (2017), 4, pp. 1847-1855

[18] Wang, Y., *et al.* A Variational Formulation for Anisotropic Wave Traveling in a Porous Medium, *Fractals*, *27* (2019), June, 1950047
[19] Wang, K. L., He, C. H., A Remark on Wang's Fractal Variational Principle, *Fractals*, *27* (2019), 8, ID 1950134
[20] He, J. H., A Modified Li-He's Variational Principle for Plasma, *International Journal of Numerical Methods for Heat and Fluid Flow*, On-line first, https://doi.org/10.1108/HFF-06-2019-0523, 2019
[21] He, J. H., Lagrange Crisis and Generalized Variational Principle for 3D unsteady flow, *International Journal of Numerical Methods for Heat and Fluid Flow*, On-line first, https://doi.org/10.1108/HFF-07-2019-0577, 2019
[22] He, J. H., Sun, C., A Variational Principle for a Thin Film Equation, *Journal of Mathematical Chemistry*, *57* (2019), 9, pp. 2075-2081
[23] He, J. H., Variational Principles For Some Non-Linear Partial Differential Equations With Variable Coefficients, *Chaos Solitons & Fractals*, *19* (2004), 4, pp. 847-851
[24] He, J. H., Some Asymptotic Methods for Strongly Non-Linear Equations, *Int. J. Mod. Phys. B*, 20 (2006), 10, pp. 1141-1199
[25] He, J. H., Ji, F. Y., Taylor Series Solution for Lane-Emden Equation, *Journal of Mathematical Chemistry*, *57* (2019), 8, pp. 1932-1934
[26] Ren, Z. F., Wu, J. B. He's Frequency-Amplitude Formulation for Non-Linear Oscillator with Damping, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1045-1049
[27] Ren, Z. F., Hu, G. F., He's Frequency-Amplitude Formulation with Average Residuals for Non-Linear Oscillators, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1050-1059
[28] Ren, Z. F., Hu, G. F. Discussion on the Accuracies of He's Frequency-Amplitude Formulation and Its Modification with Average Residuals, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1713-1715
[29] Wang, Q. L., *et al.*, A Short Remark on Ren-Hu's Modification of He's Frequency-Amplitude Formulation and the Temperature Oscillation in a Polar Bear Hair, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1374-1377
[30] Hu, G. F., Deng, S. X., Ren's Frequency-Amplitude Formulation for Non-Linear Oscillators, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1681-1686
[31] Tao, Z. L., *et al.* Approximate Frequency-Amplitude Relationship for a Singular Oscillator, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1036-1040
[32] He, C. H., *et al.*, A Complement to Period/Frequency Estimation of a Non-Linear Oscillator, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 992-995
[33] Tao, Z. L., *et al.*, Frequency and Solution of an Oscillator with a Damping, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1699-1702
[34] Wang, Y., An, J. Y., Amplitude-Frequency Relationship to a Fractional Duffing Oscillator Arising in Microphysics and Tsunami Motion, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1008-1012
[35] He, J. H., The Simpler, the Better: Analytical Methods for Non-Linear Oscillators and Fractional Oscillators, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1252-1260
[36] He, J. H., The Simplest Approach to Non-Linear Oscillators, *Results in Physics*, *15* (2019), 102546
[37] Ren, Z. F., *et al.*, He's Multiple Scales Method for Non-Linear Vibrations, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1708-1712
[38] Yu, D. N., *et al.*, Homotopy Perturbation Method with an Auxiliary Parameter for Non-Linear Oscillators, *Journal of Low Frequency Noise, Vibration and Active Control*, *38* (2019), 3-4, pp. 1540-1554

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