ESTIMATION OF THERMOPHYSICAL PROPERTIES OF FAR INFRARED VACUUM DRYING POTATO BY APPLICATION OF INVERSE APPROACH

by

Vangelče MITREVSKI[a*], Aleksandar DEDINAC[b], and Cvetanka MITREVSKA[c]

[a]Faculty of Technical Sciences, University St. Kliment Ohridski, Bitola, Republic of North Macedonia
[b]Research Center for Energy, Informatics, and Materials,
Macedonian Academy of Sciences and Arts, Skopje, Republic of North Macedonia
[c]Faculty of Safety Engineering, International Slavic University Gavrilo Romanovic Derzhavin, Bitola, Republic of North Macedonia

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In this paper the estimation of the moisture diffusivity, together with other thermophysical properties of a far-infrared vacuum dried potato slices by using an inverse approach were studied. In direct problem a mathematical model of the far-infrared vacuum drying process of shrinking bodies was used. The Levenberg-Marquardt method was used to solve the inverse problem. An analysis of the influence of the vacuum pressure, temperature of heaters, drying body dimension, and drying time, that enables the design of the proper experiments by using the so-called D-optimum criterion was conducted. The estimated values of moisture diffusivity of potato obtained from this study are within the range from $5.14\cdot 10^{-8}$ to $5.01\cdot 10^{-9}$ m^2/s. The experimental transient temperature and moisture content changes during the far infrared vacuum drying were compared with numerical calculated values.

Key words: *moisture diffusivity, inverse approach, potato far-infrared vacuum drying*

Introduction

Thermophysical properties of food materials are important parameters in the design of process equipment and in the mathematical modeling of drying, freezing, and canning processes. The existing mathematical models which are used to modeling the drying process were classified into several groups. In the approach proposed by Luikov the moisture and temperature fields in the drying body are expressed by a system of two coupled PDE [1]. The system of equations incorporates thermophysical properties of dried material that must be determined experimentally. All the thermophysical properties except for the moisture diffusivity can be relatively easily determined by experiments [2]. A number of methods for the experimental determination of the moisture diffusivity exist but in last few decades the application of the inverse approach is a very popular tool [3-7]. In scientific and engineering litera-

* Corresponding author, e-mail: vangelce.mitrevski@uklo.edu.mk

ture moisture diffusivity data for a one same material are differ from a reason that the authors use diverse methods for the experimental determination, different methods of analysis drying kinetics data, different mathematical models for calculation as well as because of variation in composition and structure of the dried materials [8].

The objectives of this study were:

– an analysis of the possibility of simultaneous estimation of the moisture diffusivity together with the other thermophysical properties of far-infrared vacuum dried potato slices by application of inverse approach,
– compared estimated values of moisture diffusivity with the values published in scientific and engineering literature, and
– an analysis of the influence of the vacuum pressure, temperature of heaters, drying body dimension and drying time on the moisture diffusivity estimation that enables the design of the proper experiment.

Direct problem

Mathematical model of drying

The mathematical model of far infra-red vacuum drying was presented in the paper [7]. The physical problem involves a single slice of potato of thickness, $2L$, initially at uniform temperature and uniform moisture content. The problem is symetrical relative to the mid-plane of the slice, while the thickness of the body changes during the drying from $2L_0$ to $2L_\mathrm{f}$.

In the case of an infinite flat plate, the unsteady temperature, $t(x,\ \tau)$, and moisture content, $u(x,\ \tau)$, fields in the drying body are expressed by the following system of coupled non-linear PDE for energy and moisture transport [2, 7]:

$$c\rho_\mathrm{s}\frac{\partial t}{\partial \tau}=\lambda\frac{\partial^2 t}{\partial x^2}+\varepsilon r\frac{\partial(\rho_s u)}{\partial \tau} \tag{1}$$

$$\frac{\partial(\rho_s u)}{\partial \tau}=\frac{\partial}{\partial x}\left(a_m\rho_s\frac{\partial u}{\partial x}\right) \tag{2}$$

The shrinkage effect was incorporated through the changes of the specific volume of the drying body. The linear relationship between the specific volume, v_s, and the moisture content, u, was used [2]:

$$v_\mathrm{s}=\frac{1}{\rho_\mathrm{s}}=\frac{V}{m_\mathrm{s}}=\frac{1+\beta' u}{\rho_{\beta 0}} \tag{3}$$

The problem of the moving boundaries due to the changes of the dimensions of the body during the drying was resolved by introducing the dimensionless co-ordinate:

$$\psi=\frac{x}{L(\tau)} \tag{4}$$

Substituting the expression (3) and (4) into eqs. (1) and (2) and rearranging with $\delta = 0$, the resulting system of equations for the temperature and moisture content predictions becomes [7]:

$$\frac{\partial t}{\partial \tau}=\frac{\lambda}{\rho_\mathrm{s}c}\frac{1}{L^2}\frac{\partial^2 t}{\partial \psi^2}+\frac{\psi}{L}\frac{\partial L}{\partial \tau}\frac{\partial t}{\partial \psi}+\frac{\varepsilon r}{c}\frac{\rho_\mathrm{s}}{\rho_{\beta 0}}\left(\frac{\partial u}{\partial \tau}-\frac{\psi}{L}\frac{\partial L}{\partial \tau}\frac{\partial u}{\partial \psi}\right) \tag{5}$$

$$\frac{\partial u}{\partial \tau} = a_\mathrm{m} \frac{\rho_{\beta 0}}{\rho_\mathrm{s}} \frac{1}{L^2} \frac{\partial^2 u}{\partial \psi^2} + \left[\frac{\rho_{\beta 0}}{\rho_\mathrm{s}^2} \frac{1}{L^2} \frac{\partial (a_\mathrm{m} \rho_\mathrm{s})}{\partial \psi} + \frac{\psi}{L} \frac{\partial L}{\partial \tau} \right] \frac{\partial u}{\partial \psi} \tag{6}$$

The initial conditions are:

$$\tau = 0, \quad t(\psi, 0) = t_0, \quad u(\psi, 0) = u_0 \tag{7}$$

The temperature and the moisture content boundary conditions on the surfaces of the drying slice are:

$$-\lambda \frac{1}{L} \left(\frac{\partial t}{\partial \psi} \right)_{\psi=1} + j_q - r(1-\varepsilon)\, j_m = 0 \tag{8}$$

$$u_{\psi=1} = u_\mathrm{eq}(t_{\psi=1},\, p) \tag{9}$$

The heat flux, $j_q(\tau)$, and mass flux, $j_m(\tau)$, on the surfaces of drying slice are [7]:

$$j_q = \varepsilon_r \sigma \left(T_\mathrm{h}^4 - T_{\psi=1}^4 \right) \tag{10}$$

$$j_m = -\rho_s L \frac{\mathrm{d}\bar{u}}{\mathrm{d}\tau} \tag{11}$$

The boundary conditions on the mid-plane of the drying slice are:

$$\left(\frac{\partial t}{\partial \psi} \right)_{\psi=0} = 0, \qquad \left(\frac{\partial u}{\partial \psi} \right)_{\psi=0} = 0 \tag{12}$$

Problem defined by eqs. (5)-(12) is referred to as a direct problem when initial and boundary conditions as well as all the parameters appearing in the formulation are known. The objective of the direct problem is to determine the temperature and moisture content fields in the drying body. In order to approximate the solution of eqs. (5) and (6), an explicit numerical procedure was used.

Inverse problem

In the inverse problem, the thermophysical properties and half-thickness of a drying body are regarded as unknown parameters. In the scientific literature many optimization methods used to solve inverse problems. In recent years the hybrid optimization program OPTRAN were used successfully for estimation of unknown parameters [2, 6]. From other side the deterministic methods, was successfully applied for estimation unknown parameters in drying processes [1, 4]. The estimation methodology used in this paper is based on the minimization of the ordinary least square norm:

$$\mathrm{E}(\mathbf{P}) = \left[\mathbf{Y} - \mathbf{T}(\mathbf{P}) \right]^\mathrm{T} \left[\mathbf{Y} - \mathbf{T}(\mathbf{P}) \right] \tag{13}$$

Here, $\mathbf{Y}^\mathrm{T} = [\mathrm{Y}_1, \mathrm{Y}_2, \ldots, \mathrm{Y}_\mathrm{imax}]$ is the vector of measured temperatures,

$$\mathbf{T}^\mathrm{T} = \left[\mathrm{T}_1(\mathbf{P}),\ \mathrm{T}_2(\mathbf{P}),\ \ldots\ \mathrm{T}_\mathrm{imax}(\mathbf{P}) \right]$$

is the vector of estimated temperatures at time τ_i (i = 1,2, …, imax), $\mathbf{P}^\mathrm{T} = [\mathrm{P}_1, \mathrm{P}_2, \ldots\ \mathrm{P}_\mathrm{N}]$ – the vector of unknown parameters, imax – the total number of measurements, and N – the total number of unknown parameters (imax ≥ N). For the minimization of E(**P**) representing the solution of the present parameter estimation problem the Levenberg-Marquardt method [1, 4] were utilized.

Experiments

The real experiments have been conducted on the experimental set-up that is designed to imitate an industrial far-infrared vacuum dryer [9]. Drying of approximately three millimeters thick potato slices were examined. The three micro-thermocouples inserted at the position x = 0, insert separately of each of the three drying slices of potato was used as a transient temperature reading, fig. 1. The measurement of potato mass changes with time was enabled with load cell on every one minutes in order to obtain volume average moisture content change during drying.

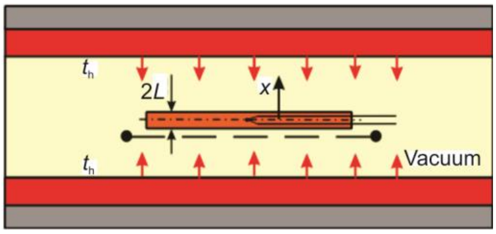

Figure 1. Scheme of drying experiment

The experiments were repeated for different temperature of heaters and vacuum pressure. The temperature of heaters was varied from 120 $^{\mathrm{o}}$C to 200 $^{\mathrm{o}}$C, while the vacuum pressure was varied from 20 kPa to 80 kPa. The relative errors of the measurements were estimated between 0.1% and 1.0% for the mass and 0.2-2.2% for the dimensions of the slices. Micro-thermocouples were calibrated, relative to each other, within 0.3 $^{\mathrm{o}}$C in the range of 120-200 $^{\mathrm{o}}$C.

The initial moisture content, u_0, and the initial potato slices thickness, $2L_0$, were measured for each of the experiments. The change of the specific volume of the drying slices was determined by a separate experiment, described in previous research [2].

Results and discussion

In this paper the possibility of the simultaneous estimation of the moisture diffusivity together with the other thermophysical properties of the potato were analyzed. The heat capacity of food materials can be taken as equal to the sum of the heat capacity of solid matter and water absorbed by that solid:

$$c = c_{\mathrm{s}} + c_{\mathrm{w}} u \tag{14}$$

Although the specific heat capacities of solid matter, c_{s}, and water, c_{w}, are functions of the temperature, constant values have been most widely used. In this paper the following values for potato were used: c_{s} = 1381 J/kgK db, and c_{w} = 4187 J/kgK db [1, 2].

From the previous research it was concluded that for practical calculations the system of the two simultaneous PDE could be used by treating the thermal conductivity, λ, as constant. For this reason a mean value from results obtained for potato = 0.40 W/m/K was used [1, 4]. The influence of the phase conversion factor ($0 \leq \varepsilon \leq 1$) on the transient moisture content and temperature profiles is very small, for this reason a mean value, ε = 0.5 was used in this paper.

Previously conducted own experimental results for potatoes, confirm the expression (3) with density of the fully body dried samples $\rho_{\beta 0}$ = 755 kg/m^3 and the shrinkage coefficient β' = 0.57, [2].

Moisture diffusivity of foods is very often considered as an Arrhenius-type temperature function [2]:

$$a_m = a_{m0} \exp\left[-E_0 / \left(\mathrm{R} T_k\right)\right] \tag{15}$$

where, a_{m0} is the Arrhenius constant, E_0 – the activation energy for moisture diffusion, R – the ideal gas constant, and T_k is the absolute temperature.

The variation in water activity with change in moisture content of samples at a specified temperature is defined by sorption isotherms. In this paper, the equilibrium moisture content, u_{eq}, was calculated in function of the temperature of the surface on dry material, t_s and pressure, p, in the vaccum chamber:

$$u_{eq} = f\left(t_s,\, p\right) \tag{16}$$

For the inverse problem, the moisture diffusivity parameters, together with other thermophysical properties and half-thickness of the potato, were treated as unknown parameters. Thus, in the inverse problem the analysed vector of the unknown parameters was:

$$\mathbf{P}^{\mathrm{T}} = [a_{m0},\, E_0, \lambda, \varepsilon, c_s, \rho_{\beta 0}, \beta^{`}, L_0] \tag{17}$$

For the simultaneous estimation of these unknown parameters, an arithmetical mean of the readings from the three micro-thermocouples was used as a transient temperature reading.

An analysis of the influence of the drying parameters and dimensions of the drying sample needed for the design of the appropriate experiment was conducted. In order to realize this analysis, the relative temperature sensitivity coefficients and the determinant of the information matrix were calculated for the characteristic drying regimes and drying body dimensions. The sensitivity coefficients analysis was carried out for an infinite flat plate model of a slice of potato with initial moisture content of $u(x, 0) = 5.20$ kg/kg and initial temperature $t(x, 0) = 20.0$ °C. The temperature of heaters, t_h, was varied between 120 °C and 200 °C, the pressure, p, in the vacuum chamber was varied between 20 and 80 kPa, and the initial thickness, $2L_0$, of the potato slice was varied between 2.2 and 3.4 mm.

In this paper from the sensitivity coefficients analysis the following experimental parameters were chosen: $t_h = 140$ °C, $p = 60$ kPa, $2L_0 = 3$ mm. In fig. 2 the relative sensitivity coefficients $P_j T_i/\partial P_j$, $i = 1, 2, \ldots,$ imax, for temperature with respect to the unknown parameters are shown.

From fig. 2 it can be seen that the relative temperature sensitivity coefficients for thermal conductivity, λ, is very small. This indicates that λ cannot be estimated in this case, so indicates that the influence of the thermal conductivity on the transient moisture content and temperature profiles is very small in this case. For these reasons, the thermal conductivity was treated as known quantities for the examination described below. The relative temperature sensitivity coefficients for the phase conversion factor, ε, is relatively high. Due to fact that in numerical calculations the mean value of the phase conversion factor was used, and fact that the used value was with high accuracy, in further consideration the value of this parameter was taken as known. The specific heat capacity of wet potato was taken as equal to the sum of the heat capacity of solid matter and absorbed water, eq. (14). Since the heat capacity of the solid matter, c_s, presents only a few percent of the overall heat capacity of the potato, the relative sensitivity temperature coefficients with respect to the heat capacity of solid matter is very small. Consequently, the value of the heat capacity of the solid matter was also taken as known.The relative temperature sensitivity coefficients with respect to the density of the

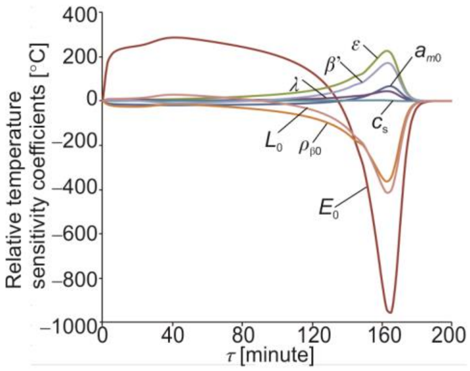

Figure 2. Relative temperature sensitivity coefficients

fully dried body, $\rho_{\beta 0}$, and the shrinkage coefficient, β', are relatively high. From fig. 2 it can be noted that these two parameters are linealy-dependent. This makes it impossible to simultaneously estimate these two parameters. Because the shrinkage effect of the drying body was incorporated through the changes of the specific volume of the drying body, these parameters were determined by separate experiments [2]. From this reason, the density of the fully dried body and the shrinkage coefficient were treated as known quantities for the examination. The relative temperature sensitivity coefficients with respect to the half-thicknes potato slice thickness is high and in further consideration this parameter is regarded as an unknown.

Thus, it appears to be possible to estimate simultaneously the moisture diffusivity parameters, a_{m0} and E_0 and the half-thicknes of the drying sampless, L_0, by a single micro-thermocouple temperature response in a far-infra red vacuum drying slice of potato.

From fig. 2 it can be seen that the relative temperature sensitivity coefficients with respect to the moisture diffusivity parameters, a_{m0} and E_0, are nearly linearly dependent. Despite this, we were able to obtain results using Levenberg-Marquardt algorithm. Thus, simultaneously were estimated the moisture diffusivity parameters, a_{m0} and E_0, and the half-thickness of the drying slices, L_0.

Table 1 shows the computationally obtained parameters without and with added normally disributed error, with zero mean and standard deviations, σ = 0.3 ºC for the one of real realized experiment E17 (t_h = 200 °C, p = 20 kPa, $2L_0$ = 3 mm, u_0 = 5.24 kg/kg and t_0 = 24.82 °C) [10]. From tab. 1 is obviously that the parameter of Arrhenius factor a_{m0} can be simultaneously estimated with the other parameters with the relative errors of 3%, while the value of activation energy, E_0, and initial half-thickness of the drying sample, L_0, can be simultaneously estimated with the other the parameters with the relative errors of 2%, in the case of where on the temperature measurements is added normally distributed error, with zero mean and standard deviations, σ = 0.3 ºC.

Table 1. Estimated values of unknown parameters

Parameters	Exact values	Estimated values		Relative error %
		σ = 0	σ = 0.3 °C	
$a_{m0} \cdot 10^3$ [m^{2}s^{-1}]	4.8006	4.8006	4.6619	2.98
E_0 [kJmol^{-1}]	45.023	45.023	44.947	0.17
$L_0 \cdot 10^{-3}$[m]	2.1018	2.1018	2.0968	0.24

Figure 3 shows the convergence history of the estimated values of the unknown parameters to the final values during the iterative process for the realized experiment E17 and root mean squared (RMS) error changes, [10].

In fig. 4 the transient variation of the determinant of the information matrix are presents, if simultaneously a_{m0}, E_0, $\rho_{\beta 0}$, β', L_0 are considered as unknown parameter. Elements on determinant of the information matrix were defined for a large [2, 11], but fixed number of transient temperature measurements (501 in this case). The maximum determinant of the information matrix value corresponds to the drying time when nearly equilibrium moisture content and temperature profiles were reached.

The estimated moisture diffusivity values for the potato slices obtained from this study are within the range from $5.14 \cdot 10^{-8}$ to $5.01 \cdot 10^{-9}$ m^2/s. These values are comparable with values for other dried food materials, which values reported in literature generally are in range from 10^{-13} to 10^{-6} m^2/s [12]. Also, these values are comparable with the published values for potato by the other authors that used diverse methods and different mathematical models for calculation of moisture diffusivity [13-16].

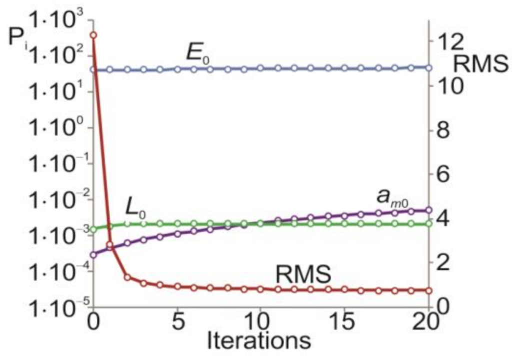

Figure 3. The convergence history of the estimated parametres and RMS changes

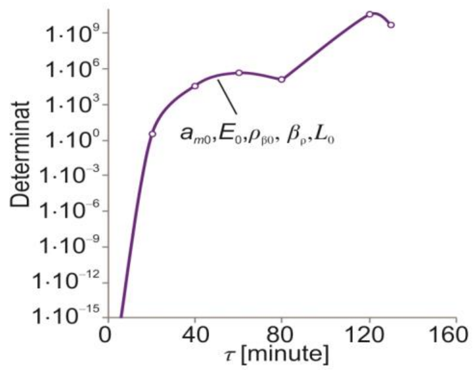

Figure 4. Determinat of the information matrix

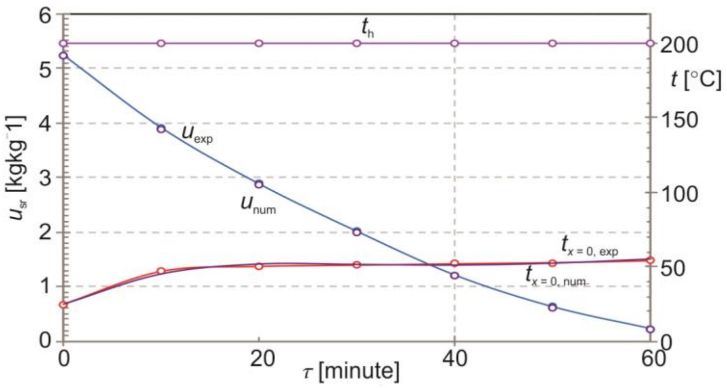

Figure 5. The mid-plane temperature, *t*, the temperature of the heaters, t_h, and the volume-averaged moisture content, *u*, changes during the drying of a potato slice

In fig. 5 the experimental transient temperature reading, $t_{x=0}$, and the experimental volume-averaged moisture content, u, change during the far-infrared vacuum drying of a potato slices (experiment E17), are compared with numerical solutions for the estimated parameters. From fig. 5 is obviously that have very good agreement between the experimental and numerical calculated temperature and moisture content changes, during the far-infrared vacuum drying of the potato slices.

Conclusions

The simultaneously estimation of two moisture diffusivity parameters, and thicknesses of the drying sample with other thermophysical properties based on a single micro-thermocouple temperature response of far infrared drying of potato slices was analyzed. The obtained results show good agreement between the estimated and exact values of parameters. An analysis of the influence of the temperature measurements errors on the accuracy of the estimated parameters was also presented. The estimated values of moisture diffusivity of potato obtained from this study are comparable with values published by other authors that used diverse methods and different mathematical models for calculation. The very good agreement between the experimental and numerical temperature and volume-averaged moisture content changes confirm that the mathematical model it would be a useful tool for calculations and optimization of far-infra red vacuum drying processes.

Nomenclature

a_m – moisture diffusivity, [m^2s^{-1}]
a_{m0} – Arrhenius factor, [m^2s^{-1}]
c – specific heat capacity (dry basis), [$JK^{-1}kg^{-1}$ db]
E_0 – activation energy, [Jkg^{-1}]
j_m – mass flux, [$kgm^{-2}s^{-1}$]
j_q – heat flux, [Wm^{-2}]
L – flat plate thickness, [m]
m – mass, [kg]
$\mathbf{P}$ – vector of unknown parameter
R – absolute gas constant, [$JK^{-1}mol^{-1}$]
p – pressure, [Pa]
r – specific latent heat of vaporization, [Jkg^{-1}]
t – temperature, [^oC]
T_k – temperature, [K]
$\mathbf{T}$ – vector of estimated temperature, [^oC]
v – specific volume, [m^3kg^{-1}]
V – volume, [m^3]
x – distance from the mid-plane, [m]
$\mathbf{Y}$ – vector of measured temperature, [^oC]
u – moisture content, [$kgkg^{-1}$ db]

Greek symbols

β – shrinkage coefficient, [–]
δ – thermo-gradient coefficient, [K^{-1}]
σ – Stefan-Boltzmann constant, [$Wm^{-2} K^{-4}$]
ε – phase conversion factor, [–]
ε_r – emissivity of the material, [–]
λ – thermal conductivity, [$Wm^{-1}K^{-1}$]
ρ – density, [kgm^{-3}]
τ – time, [s]
ψ – dimensionless coordinate, [–]

Subscripts

β0 – fully dried body
eq – equilibrium
f – potato thickness
h – heater
0 – initial
w – water
s – dry solid

References

[1] Kanevce, G., *et al*., An Inverse Approaches to Drying of Bodies with Significant Shrinkage Effects, *Proceedings*, 5th International Conference on Inverse Problems in Engineering: Theory and Practice, Cambridge, UK, 2005, K02, pp. 1-10
[2] Kanevce, G., *et al*., Inverse Approaches to Drying of Thin Bodies with Significant Shrinkage Effects, *Int. J. Heat Mass Transf.*, *129* (2007), 3, pp. 379-386
[3] Mitrevski, B., Investigation of the Drying Processes by Inverse Methods (in Macedonian), Ph. D. thesis, University St. Kliment Ohridski, Bitola, Macedonia, 2005
[4] Kanevce, G., *et al*., Application of Inverse Concepts to Drying, *Thermal Science*, *9* (2005) 2, pp. 31-44
[5] Kanevce, G., *et al*., Inverse Approaches to Drying with and Without Shrinkage, *Proceedings*, 15th International Drying Symposium, Budapest, Hungary, 2006, pp. 576-583
[6] Kanevce, G., *et al*., Estimation of Drying Parameters Including Moisture Diffusivity by Using Temperature Measurements, *WIT Trans. on Modelling and Sim.*, *51* (2011), 1, pp. 111-119
[7] Mitrevski, V., *et al*., Mathematical Modelling of Far Infrared Vacuum Drying of Apple Slices, *Thermal Science*, *23* (2019), 1, pp. 393-400
[8] Panagiotou, N. M. M., *et al*., Moisture Diffusivity: Literature Data Compilation for Foodstuffs, *In. J. Food Prop.*, *7* (2004), 2, pp. 273-279
[9] Mitrevski, V., *et al*., Experimental Investigation of Far Infrared Vacuum Drying of Apple Slices, *Appl. Eng. Let.*, *1* (2016), 2, pp. 35-39
[10] Bundalevski, S., Modelling of Far-Infrared Vacuum Drying Processes by Applying Inverse Approach (in Macedonian), Ph. D. thesis, University St. Kliment Ohridski, Bitola, Macedonia, 2015
[11] Ozisik M. N., Orlande, H. R. B., *Inverse Heat Transfer: Fundamentals and Applications*, Taylor and Francis, New York, U.S.A., 2000
[12] Saravacos, G. D., Maroulis, Z. B., *Transport Properties of Foods*, Marcel Dekker Inc., New York& Basel, USA, 2001
[13] Zogzas, N. P., Maroulis, Z. B., Effective Moisture Diffusivity Estimation from Drying Data. A Comparison Between Various Methods of Analysis, *Dry. Technol.*, *14* (1996), 7-8, pp. 1543-1573

[14] Chayjan, R. A., Modeling Some Drying Characteristics of High Moisture Potato Slices in Fixed, Semi Fluidized and Fluidized Bed Conditions, *J. Agri. Sci. Tech.*, *14* (2012), 1, pp. 1229-1241
[15] Kiranoudis, C. T., *et al*., Heat and Mass Transfer Model Building in Drying with Multiresponse Data, *Dry. Technol.*, *11* (1995), 6, pp. 463-480
[16] Kiranoudis, C. T., *et al*., Drying Kinetics of Some Fruits, *Dry. Technol.*, *15* (1997), 5, pp. 1399-1418

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