THE EFFECT OF UNIFORM MAGNETIC FIELD ON SPATIAL-TEMPORAL EVOLUTION OF THERMOCAPILLARY CONVECTION WITH THE SILICON OIL BASED FERROFLUID

by

Shuo YANG^{*}, Rui MA, Qiaosheng DENG, Guofeng WANG, Yu GAO, and Shanshan MA

Key Laboratory of Liaoning Province for Clean Combustion Power Generation and Heating Technology, Shenyang Institute of Engineering, Shenyang, China

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A uniform axial or transverse magnetic field is applied on the silicon oil based ferrofluid of high Prandtl number fluid (Pr ≈ 111.67), and the effect of magnetic field on the thermocapillary convection is investigated. It is shown that the location of vortex core of thermocapillary convection is mainly near the free surface of liquid bridge due to the inhibition of the axial magnetic field. A velocity stagnation region is formed inside the liquid bridge under the axial magnetic field (B = 0.3-0.5 T). The disturbance of bulk reflux and surface flow is suppressed by the increasing axial magnetic field. There is a dynamic response of free surface deformation to the axial magnetic field, and then the contact angle variation of the free surface at the hot corner is as following, $\phi_{hot, B = 0.5 T} = 83.34^{\circ} > \phi_{hot, B = 0.3 T} = 72.16^{\circ} >$ $> \phi_{hot, B = 0.1 T} = 54.21^{\circ} > \phi_{hot, B = 0.5 T} = 43.33^{\circ}$. The results show that temperature distribution near the free surface is less and less affected by thermocapillary convection with the increasing magnetic field, and it presents a characteristic of heat-conduction. In addition, the transverse magnetic field does not realize the fundamental inhibition for thermocapillary convection, but it transfers the influence of thermocapillary convection to the free surface.

Key words: uniform magnetic field, liquid bridge, level-set method, thermocapillary convection

Introduction

The researchers proposed the floating zone method for crystal growth in order to grow high-quality silicon crystals, and the liquid bridge is an ideal geometric model based on the single-crystal growth mechanism by the floating zone method. It is found that thermocapillary convection is the main cause of the impurity striations in a single silicon crystal at the micrometer scale under the deep space environment. Then, various means of inhibition are proposed, such as external magnetic field, vibration, surface coating, air-flow shear and other means used to suppress thermocapillary convection. Due to the effect of Lorenz force on the conductive fluid in the magnetic field and the non-contact advantage of the magnetic field, the applied magnetic field is an effective mean for controlling the thermocapillary convection in the crystal growth of floating zone. Therefore, a number of studies on the flow structure [1], temperature and concentration field inside liquid bridge under the magnetic field has been

^{*} Corresponding author, e-mail: ys_yang_shuo@163.com

conducted. Chedzey et al. [2] and Utech et al. [3] first carried out the experimental research on crystal growth under the static magnetic field. Kimura et al. [4] experimentally investigated the doping single crystal growth ($\phi = 20$ mm) of floating zone under the transverse magnetic field. The research showed that the crystal growth interface tends to be flat under the magnetic field of B = 0.18 T. When the magnetic field increases to the B = 0.55 T, the diffusion layer of melt interface is thickened, which is little significance to inhibit impurity stripes growth. Dold et al. [5] studied the silicon crystal growth ($\phi = 8$ mm) under the axial static magnetic field. It is found that the silicon crystal without stripe can be obtained under the imposition of magnetic field above 240 mT. By applied a axial static magnetic field above B = 5 T, Croll *et al.* [6] found that the melting zone is divided into a central stationary region and a strong convection region near the free surface. With the development of computer technology, a large number of numerical methods is used to analyze the effect of static magnetic field on crystal growth in the recent year [7]. Lan et al. [8] and Hermann [9] have carried out the experimental and numerical simulation on the single silicon crystal growth with floating zone method under the various static magnetic fields. Kaiser and Benz [10] calculated the temperature and flow field, and the impurity concentration distribution by the axial magnetic field for a 2-D model, which confirms the radial segregation under the axial magnetic field found by Croll's experiment. Yao et al. [11] studied the effect of rotating magnetic field on thermocapillary convection (Pr = 0.01) under microgravity in crystal growth of floating zone method. When applied magnetic field is B = 1.6 mT and 50 Hz, 3-D oscillatory flow appears. The oscillation frequency decreases correspondingly as soon as the increasing magnetic field, and the rotating magnetic field can effectively suppress the oscillatory flow of thermocapillary convective. Pang and Gong [12] discussed the effect of geometric parameters and axial magnetic field on buoyancy-thermocapillarity convection during the segregation and crystallization process. When the Marangoni number exceeds the critical Marangoni number, Ma_{cr}, the flow transitions to the instable phase. The Ma_{cr} increases with the increasing of melt height. The effect of different axial magnetic fields on cylindrical and hemispherical single crystal melt is simulated in the Czochralski silicon crystals growth process by Mokhtari et al. [13]. It is found that the pressure at the threephase point is an important parameter for controlling crystal growth. The pressure is more sensitive to the velocity variation of the flow field relative to the temperature. The axial magnetic field can effectively control the thermal disturbance inside silicon melt. Peng et al. [14] used the finite difference method to simulate the thermocapillary convection in the Cd-Zn-Te separation and crystallization process under the cusp magnetic field. The results showed that the inhibiting effect strengthen and the flow inside the melt weakens with the increasing magnetic field.

Therefore, it is necessary to study the control to thermocapillary convection by magnetic field under the real-time dynamic deformation of interface. It is found that the axial magnetic field is more effective in suppressing thermocapillary convection, comparing with the transverse magnetic field.

Mathematical and physical models

Experimental system and physical properties of material

Figure 1 is shown a schematic of the experimental device. The liquid bridge is suspended between the two coaxial disks. The upper disk, T_t , is heated and the liquid bridge is surrounded by the initial static ambient air-flow. The experimental device includes the high-speed camera equipped with micro-focus, liquid bridge generator, narrow-band filter, magnifying lens, backlight, laser light source, lifting device, flow-meter, thermocouple, digital display heating diaphragm, and cooling system. Since the medium of liquid bridge is silicon oil based ferrofluid,

the ferromagnetic fluid exhibits clusters phenomenon in silicone oil base with the high Prandtl number during the configuration process and the tracer effect is obvious (see the tab. 3). In the flow field, the image of silicon oil based ferrofluid sweep by the laser of 532 nm is better. In addition, the Stokes number (St = $= \rho_p D_p^2 U_{max}^l / 18 \mu^l H$) is chosen to evaluate the following performance of tracer particle, where ρ_p is the density of aluminum powder, D_p – the diameter of alu-



Figure 1. Experimental device of liquid bridge

minum powder, U_{max}^{l} – the maximum flow velocity in the liquid bridge, μ^{l} – the dynamic viscosity of silicone oil, and H – the height of liquid bridge. The Stokes number of the powder with the diameter of 10 µm is 8.03×10^{-7} (St ≤ 1).

In this research, the height of liquid bridge is h = 2.5 mm, the radius of coaxial support disk is R = 2.5 mm, the $\Gamma = h/R = 1$ represents the aspect ratio of liquid bridge. The physical parameters of fluid medium are shown in the tabs. 1 and 2.

Density [kgm ⁻³]	Initial susceptibility [-]	Nominal particle diameter [nm]	Magnetization coefficient [-]	Ferro fluid/ air interfacial tension at 25°C [Nm ⁻¹]	Saturation magnetization [Am ⁻¹]	Particles per unit volume [m ⁻³]
1020.0	2.2	10	0.8	25.8×10 ⁻³	1.75×10^{4}	1.51×10 ²³

Table 1. Physical properties of silicon oil-based ferromagnetic fluid

Table 2 Physical properties of the basic solution (10 cSt silicone o	Table 2	Physical	properties	of the bas	ic solution	(10)	cSt silicone	oil
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			Silicone oil
Density, ρ	[25 °C]	[kgm ⁻³]	935
Dynamic viscosity, μ	[25 °C]	[Pa·s]	9.35×10 ⁻³
Kinematic viscosity, v	[25 °C]	$[m^2s^{-1}]$	10 ⁻⁵
Thermal diffusivity, α	[25 °C]	$[m^2s^{-1}]$	8.96×10 ⁻⁸
Thermal conductivity, k	[25 °C]	$[Wm^{-1}K^{-1}]$	0.14
Coefficient of thermal expansion, b	[25-150 °C]	$[K^{-1}]$	1.08×10^{-3}
Surface tension, s	[25 °C]	$[Nm^{-1}]$	20.1×10 ⁻³
Coefficient of surface tension with the temperature, s_T		$[\mathrm{Nm}^{-1}\mathrm{K}^{-1}]$	-6.83×10 ⁻⁵
Coefficient of viscosity with the temperature, v_T		[-]	0.55
Specific heat, C_p		$[Jkg^{-1}K^{-1}]$	1672
Prandt number, Pr		[-]	111.67
Capillary number, Ca		[-]	0.08
Gravity acceleration, g		[ms ⁻²]	9.81

The liquid bridges under the different intensity magnetic fields, B, are chosen as the research object, as shown in tab. 3. In order to verify the accuracy of this experimental method and operation process, the experimental results are compared with theoretical values. The size

able 5. Images of hquid bridge with the different uniform magnetic fields							
	Transverse magnetic field Case 1: $B = 0$ T	Transverse magnetic field Case 2: $B = 0.1$ T	Transverse magnetic field Case 3: $B = 0.3$ T	Transverse magnetic field Case 4: $B = 0.5$ T			
Images							
	Axial magnetic field Case 5: $B = 0$ T	Axial magnetic field Case 6: $B = 0.1$ T	Axial magnetic field Case 7: $B = 0.3$ T	Axial magnetic field Case 8: $B = 0.5$ T			
Images			A No				

Table 3. Images of liquid bridge with the different uniform magnetic fields



Figure 2. Contrast of measurements and theoretical values (V = 0.951, $\Gamma = h/R = 1$)

and medium of liquid bridge in fig. 2 are different from the research object in this paper, the geometric and fluid physical parameters of the liquid bridge selected in the verification experiment are shown: V = 0.951, $\Gamma = h/R = 1$, h = 3 mm, Pr = 111.67, and the liquid bridge medium is 10 cSt silicone oil in fig. 2. The theoretical interface configuration obtained by the Young-Laplace equation is marked as the solid line in fig. 2 (F_0 represents the horizontal co-ordinates of the right free surface position of liquid bridge at steady-state, $g = 9.8 \text{ m/s}^2$). The free surface captured via the experiment is marked as the circular symbols. The maximum deviation $(\Delta h_{\max(x)} = |h_{yl} - h_{exp}|)$ of lateral position doesn't exceed 5.70 × 10⁻³ mm, and the average difference is $\Delta h \approx 2.56 \times 10^{-3}$ mm (h_{vl} is transverse position of free surface by solving Young-Laplace equation. The h_{exp} is the transverse position of free surface via the experiment). From fig. 2, it proves that this experiment is feasible for the precise capturing physical characteristic of liquid bridge.

The governing equations and boundary conditions

In the experiment, the radius and height of liquid bridge is R and H, respectively. The temperature of the upper and bottom disks is respectively T_t and T_b , and the temperature difference between the coaxial upper and bottom disks is $\Delta T_0 = T_t - T_b = 10$ K. The liquid bridge is surrounded by the initial static air-flow, the ambient temperature is controlled at $T_0 = 20$ °C. The outer diameter of the gas phase region is 2R. The uniform transverse or axial magnetic field is applied to the liquid bridge, and the direction of magnetic field is shown in fig. 3, respectively. The static magnetic force acting on the liquid bridge can be expressed as:

$$\mathbf{f} = \mathbf{j} \times \mathbf{B} - \frac{1}{2} \nabla \left(B^2 \frac{\partial \left(\mu_p\right)^{-1}}{\partial \rho} \rho \right) + \frac{1}{2} B^2 \nabla \frac{1}{\mu_p}$$
(1)

where **j** is a current density vector, **B** is an external magnetic field vector, μ_p is a magnetic permeability of medium, and **j**×**B** is the Lorentz force. The second and third terms on the right side



Figure 3. Schematic diagram of a half-zone liquid bridge under the different magnetic fields

of eq. (1) is the force caused by the magnetic permeability with the change of medium density and space, respectively. In addition, according to the principle of Joule, a current is produced when a magnetic field interacts with a fluid, furtherly, the joule heat is generated in magnetic fluid, it is expressed:

$$Q_{\mathbf{J}} = \frac{1}{\sigma_e} \mathbf{j} \cdot \mathbf{j}$$
(2)

The relative magnitude of the induced magnetic field produced by conducting ferro fluid in current is measured by magnetic Reynolds number, Re_{m} , $\text{Re}_{m} = \mu_{p}\sigma_{e}UL$, where μ_{p} is a magnetic permeability, σ_{e} – the electrical conductivity, and U – the characteristic velocity. The **j** is the current density vector defined by $\mathbf{j} = \sigma_{e}(\nabla \boldsymbol{\omega} + \mathbf{u} \times \mathbf{B})$, where σ_{e} is the electric conductivity of the liquid, $\boldsymbol{\omega}$ – the induced electric potential, \mathbf{u} – the velocity vector. The guiding potential in the current density is negligible in this paper, therefore, the expression of static magnetic force on the liquid bridge is reduced:

$$\mathbf{f} = \mathbf{j} \times \mathbf{B} = \sigma_e \left(\mathbf{u} \times \mathbf{B} \times \mathbf{B} \right) \tag{3}$$

The dimensionless control equations are shown in the following:

$$\Delta \mathbf{u} = 0 \tag{4}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} = \frac{1}{\rho} \left[-\nabla P + \frac{1}{\text{Re}}\nabla(2\mu\mathbf{D}) + \left(\frac{1}{\text{We}} - \frac{\text{Ca}}{\text{We}}\Theta\right)\kappa\delta(d)\mathbf{n} \right] + \frac{\text{Ha}^2}{\text{Re}}(\mathbf{u}\times\mathbf{B}\times\mathbf{B})$$
(5)

$$\frac{\partial \Theta}{\partial t} + \nabla \left(\mathbf{u} \, \boldsymbol{\Theta} \right) = \frac{1}{\mathrm{Ma}} \nabla^2 \boldsymbol{\Theta} \tag{6}$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \nabla \varphi = 0 \tag{7}$$

where $\mathbf{u} = (u, v)$ is the fluid velocity, $u(u = u_s/U)$, and $v(v = v_s/U)$ is the dimensionless transverse and longitudinal velocity, respectively. The $U(|\sigma_T|\Delta T/\mu)$ is the characteristic velocity. The $\rho = \rho(\mathbf{x}, t)$ is the fluid density, and $\mu = \mu(\mathbf{x}, t)$ is the fluid viscosity. The **D** is the viscous stress tensor, κ – the curvature of the interface, d – the normal distance to the interface, δ – the Dirac delta function, \mathbf{n} – the unit normal vector on the interface. The *t* is the dimensionless time, we denote $t = t_s U/L$, where t_s is dimensional time, L – the characteristic length and we take L = 2R. In addition, *x* and *y* are dimensionless co-ordinates. We denote x = X/L and y = Y/L. The *P* is the pressure. The surface tension coefficient is considered to be a linearly function of temperature and defined as $\sigma = \sigma_c - \sigma_T (T - T_b)$, where σ_c is a reference value of surface tension and σ_T is the temperature coefficient of surface tension and we denote $\sigma_T = \partial \sigma/\partial T$, and *T* is the temperature. The key parameters are dimensionless density ratio ρ_g/ρ_l and dimensionless viscosity ratio μ_g/μ_l inside the ambient air region. The ρ_l and μ_l are the dimensional density and viscosity of the liquid bridge, respectively, while ρ_g and μ_g are the dimensional density and viscosity of the ambient air, respectively. The dimensionless density (ρ_l/ρ_l) and viscosity (μ_l/μ_l) inside the liquid bridge are both equal to 1. Reynolds number, $\text{Re} = \rho_l UL/\mu_l$, Weber number, $\text{Me} = \sigma_T \Delta TL/\mu_l a = \text{RePr}$. The excess temperature is $\Theta = (T - T_b)/\Delta T$. The *a* is the thermal diffusivity. In the formula, Ha is the Hartmann number, $\text{Ha} = BL(\sigma_g/\mu_l)^{1/2}$.

The velocity boundary conditions and isothermal boundary conditions are given at both ends of the liquid bridge. Temperature boundary is $\Theta = 0$ (y = 0), $\Theta = 1.0$ (y = H). Upper disk is solid boundary, $\mathbf{u} = 0$, $\Theta = T_t$, including electrical insulation boundary $\mathbf{j} \times \mathbf{n} = 0$. Bottom disk is solid boundary, $\mathbf{u} = 0$, $\Theta = T_b$, including electrical insulation boundary $\mathbf{j} \times \mathbf{n} = 0$. Freesurface is also electrical insulation boundary $\mathbf{j} \times \mathbf{n}$. Initial condition is $\mathbf{u}(t = 0) = 0$, and the electric potential is 0.

Level set function

The level set method was originally introduced by Osher and Sethian [15] to numerically predict the moving interface, $\Gamma(t)$, between two fluids. Instead of explicitly tracking the interface, the level set method implicitly captures the interface by introducing a smooth signed distance from the interface in the entire computational domain. The level set function, $\phi(\mathbf{x}, t)$, is taken to be positive outside the liquid bridge, zero at the interface and negative inside the liquid bridge. The interface motion is predicted by solving the following convection equation for the level set function of $\phi(\mathbf{x}, t)$ given by:

$$\phi_t + \mathbf{u}\nabla\phi = 0 \tag{8}$$

Sussman *et al.* [16] adopted a second-order ENO method (Mulder *et al.* [17]) for the approximation of the convective terms as follows:

$$\mathbf{u}\nabla\phi = (u\phi)_x + (v\phi)_y \tag{9}$$

$$(u\phi)_{x} + (v\phi)_{y} = \frac{(u_{i+1/2,j} + u_{i-1/2,j})(\phi_{i+1/2,j} - \phi_{i-1/2,j})}{2h} + \frac{(\phi_{i+1/2,j} + \phi_{i-1/2,j})(u_{i+1/2,j} - u_{i-1/2,j})}{2h} + \frac{(\psi_{i,j+1/2} + \psi_{i,j-1/2})(\psi_{i,j+1/2} - \psi_{i,j-1/2})}{2h} + \frac{(\phi_{i,j+1/2} + \phi_{i,j-1/2})(v_{i,j+1/2} - v_{i,j-1/2})}{2h}$$
(10)

For smooth data, we have $(\phi_{i+1/2,j} + \phi_{i-1/2,j}) \approx (\phi_{i,j+1/2} + \phi_{i,j-1/2})$. In addition, we have $(u_{i+1/2,j} - u_{i-1/2,j}) \approx (v_{i,j+1/2} - v_{i,j-1/2})$ because **u** is numerically divergence free. Thus:

$$(u\phi)_{x} + (v\phi)_{y} \approx \frac{(u_{i+1/2,j} + u_{i-1/2,j})(\phi_{i+1/2,j} - \phi_{i-1/2,j})}{2h} + \frac{(v_{i,j+1/2} + v_{i,j-1/2})(\phi_{i,j+1/2} - \phi_{i,j-1/2})}{2h}$$
(11)

For computing $u_{i+1/2,j}$ (similarly for $u_{i,j+1/2}$, $\phi_{i+1/2,j}$,...), a second-order ENO scheme is used as follows, Define:

$$m(a,b) = \begin{cases} a & \text{if } |a| \le |b| \\ b & \text{otherwise} \end{cases}$$
(12)

Let:

$$u_{L} \equiv u_{i,j} + \frac{1}{2}m(u_{i+1,j} - u_{i,j}, u_{i,j} - u_{i-1,j})$$
(13a)

$$u_R \equiv u_{i+1,j} - \frac{1}{2}m(u_{i+2,j} - u_{i+1,j}, u_{i+1,j} - u_{i,j})$$
(13b)

$$u_M \equiv \frac{1}{2}(u_L + u_R) \tag{13c}$$

Thus:

$$u_{i+1/2,j} = \begin{cases} u_M & \text{if } u_L \le 0 \text{ and } u_R \ge 0\\ u_R & \text{if } u_M \le 0 \text{ and } u_R \le 0\\ u_L & \text{if } u_M \ge 0 \text{ and } u_L \ge 0 \end{cases}$$
(14)

Sussman *et al.* [16] proposed the following procedure shown in eqs. (15) and (16) to reinitialize the level set function after solving the convection equation for the level set function:

$$\phi_{t} = \frac{\phi_{0}}{\sqrt{\phi_{0}^{2} + (\Delta x)^{2}}} \left(1 - |\nabla \phi|\right)$$
(15)

$$\phi(\mathbf{x},0) = \phi_0(\mathbf{x}) \tag{16}$$

where $\phi_0(\mathbf{x})$ has the same zero level set as $\phi(\mathbf{x})$. However, even with the re-initialization procedure, it has been found that the total mass does not completely satisfy the mass conservation in time. Actually, the problem results from the numerical discretization of the level set formulation, which does not guarantee the mass conservation. To overcome this difficulty, a mass conserving procedure must be carried out by solving the following area compensation equation to a steady-state:

$$\frac{\partial \phi}{\partial t} + \left[1 - \frac{A(t)}{A_0}\right] F(c) \left|\nabla \phi\right| = 0$$
(17)

where A(t) is the liquid bridge area at time t corresponding to the level set function $\phi(t)$, A_0 – the initial area of the liquid bridge for the initial condition, and F(c) is defined:

$$F(c) = 1.0 + hk^n$$
(18)

The F(c) is an area constraint function and can be considered as a function of the local curvature. The F(c) varies with h and n. We denote h = 0 and n = 0, respectively, which leads to a faster convergence of the numerical procedure. The convergence criteria can be written:

$$|A(t) - A_0| < 1.0 \times 10^{-4} \tag{19}$$

In this way the total mass is conserved within 0.01%.

Conclusion and analysis

Influence of uniform transverse magnetic field on the flow characteristic of thermocapillary convection in the liquid bridge

In order to comparative analysis, the co-ordinates of numerical results and experimental results are dimensionless. The flow field image of right liquid bridge is at the left side of the fig. 4. The numerical result of velocity vector is at the right side of the fig. 4 including the fluid region and air region. In the velocity vector field of air region, there is no effect on it by uniform transverse magnetic field. However, the velocity vector field of fluid region is changed with the application of uniform transverse magnetic field ($B_0 = 0.5$ T). It is clearly observed that the velocity vector of cellular flow is constrained by the transverse magnetic field in the fig. 4(b).



Figure 4. Flow field with or without the transverse magnetic field (Ma = 47860, Ca = 0.104, D = 5.0 mm, $\Gamma = h/R = 1.0$); (a) B = 0 T and (b) B = 0.5 T

In addition, the longitudinal flow does not change dramatically along the interface. For the internal flow field near the symmetry axis (x = 1), the velocity vector doesn't present significantly change under the transverse magnetic field of B = 0.5 T. The cellular flow deviated towards the free interface under the action of transverse magnetic field, $\Delta x = 1.37 \rightarrow \Delta x = 1.44$, Δx presents the distance from the vortex center of a cellular flow to the central axis (x = 1).

From the fig. 5(a), the transverse velocity on the radius of R = 0.5 (or x = 1.5) is less than that on the radius of R = 0.4 (x = 1.4) without the application of transverse magnetic field. Under the effect of transverse magnetic field (B = 0.1 T, B = 0.2 T, and B = 0.3 T), the cellular flow moves toward the free surface, and the transverse velocity changes with the migration of thermalcapillary convection. The position of maximum transverse velocity horizon-

4166

tally migrates along the radial direction with increasing transverse magnetic field (as shown in figs. 5(a), 5(b), and 5(d), the spacing between the positive and negative peaks gradually increases. The $\Delta U_{(\text{max-min})}$ variation of transverse velocity is shown: $|\Delta U_{\text{R} = 0.1,\text{B} = 0.1}| = 0.45 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.1,\text{B} = 0.2}| = 0.30 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.1,\text{B} = 0.3}| = 0.24 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.2,\text{B} = 0.1}| = 0.95 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.2,\text{B} = 0.2}| = 0.80 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.2,\text{B} = 0.3}| = 0.48 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.3,\text{B} = 0.2}| = 1.70 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.3,\text{B} = 0.3}| = 0.66 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.3,\text{B} = 0.1}| = 2.50 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.4,\text{B} = 0.2}| = 1.55 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.4,\text{B} = 0.3}| = 1.00 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.4,\text{B} = 0.1}| = 2.5 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.5,\text{B} = 0.3}| = 1.25 \times 10^{-3}$, $|\Delta U_{\text{R} = 0.5,\text{B} = 0.3}| = 1.25 \times 10^{-3}$. By further observation, when the transverse magnetic field is $B_0 = 0.1$ T, fig. 5(b), the peak of transverse velocity has a fluctuation at the radius of R = 0.4 and R = 0.5, $U(\text{Max-}_{R = 0.4, B = 0.1}) = 1.25 \times 10^{-3}$, $U(\text{Max-}_{R = 0.4, B = 0.1}) = -1.2 \times 10^{-3}$. In figs. 5(c) and 5(d), the transverse magnetic field will influence the transverse velocity magnitude by changing the position of cellular flow.



Figure 5. Transverse velocity distribution at the different radius with or without the transverse magnetic fields (Ma = 47860, Ca = 0.104, D = 5.0 mm, $\Gamma = h/R = 1.0$); (a) B = 0 T, (b) B = 0.1 T, (c) B = 0.2 T, and (d) B = 0.3 T

Due to the circulation reflux, the direction of longitudinal velocity within the liquid bridge is opposite to that of surface flow, therefore, there are positive and negative longitudinal velocities, fig. 6(a). When there is no transverse magnetic field, the longitudinal velocities at the radius of R = 0.4 (x = 1.4) and R = 0.5 (x = 1.5) is negative, that is, the flow direction is from the hot disk (upper disk) to the cold disk (bottom disk). The longitudinal velocity near the

free surface is larger than that inside liquid bridge. The center of the thermocapillary convection gradually moves toward the free surface with the increasing transverse magnetic field (from B = 0.05 T). Therefore, it can be seen that the longitudinal velocity is negative near the free surface (R = 0.5 or x = 1.5) in fig. 6(b), while the direction of longitudinal velocity at R = 0.4 (x = 1.4) changes with the movement of cellular flow by thermalcapillary convection. At the same time, the overall level of longitudinal velocity is weakened due to the vertical resistance generated by the transverse magnetic field. Therefore, the longitudinal velocity decreases at the different radius as soon as the center of thermocapillary convection moves toward the free surface.



Figure 6. Distribution of longitudinal velocity at the different radius positions with or without the transverse magnetic fields (Ma = 47860, Ca = 0.104, D = 5.0 mm, $\Gamma = h/R = 1.0$); (a) B = 0 T and (b) B = 0.5 T

Figure 7 shows the temperature contour of liquid bridge and air region (t = 300) under the different magnetic fields. In fig. 7(a), when there is no transverse magnetic field, the convection heat transfer is severe near the gas-liquid free surface under the effect of thermalcapillary convection. Under the transverse magnetic field of B = 0.5 T, the convection heat transfer is weakened due to the direct inhibition of transverse magnetic field on the longitudinal velocity in the fig. 7(b), and the temperature distribution near the gas-liquid free surface is uniform.



Figure 7. Isothermals distribution with or without the transverse magnetic fields (Ma = 47860, Ca = 0.104, D = 5.0 mm, $\Gamma = h/R = 1.0$); (a) B = 0 T and (b) B = 0.5 T

Influence of uniform axial magnetic field on the flow characteristic of thermocapillary convection in the liquid bridge

The flow pattern in the right side of liquid bridge obviously changed with the increasing axial magnetic field. There is a larger cellular flow structure in the region of liquid bridge without applied axial magnetic field in the fig. 8(a). The influence sphere of cellular flow is suppressed and gradually withdrew to near the free interface by the axial magnetic field. The width variation of cellular flow is $\Delta x = 0.46-0.13$. Finally, the thermocapillary convection is mainly controlled near the free surface and a stagnant region is formed in the center of liquid bridge, which indicates that the axial magnetic field has a strong inhibitory effect on the thermocapillary convection. Pablo and Rivas [1] investigated the inhibitory effect of axial magnetic field on the thermocapillary convection for crystal growth process with the magnetic field of $B \approx 0.5$ T by floating zone method. It was found that there is also a stagnation region in the core of the liquid bridge in this paper. That is, the uniform axial magnetic field can effectively inhibit the thermocapillary convection in the central region.

The axial magnetic field has fundamentally the inhibitory effect on thermocapillary convection in liquid bridge. With the strength of the magnetic field (B = 0.0.5 T), the vortex core of cellular flow in the liquid bridge is close to the gas side, and the axial surface flow is



Figure 8. Streamlines under the different axial magnetic fields, (magnetic field strength B = 0, 0.1 T, 0.3 T, 0.5 T, corresponding Ha = 0, 16, 48, 80); (a) B = 0 T, (b) B = 0.1 T, (c) B = 0.3 T, and (d) B = 0.5 T

restrained from cold corner to hot corner. From fig. 9, the blue dotted line is used to represent the deformation amplitude of liquid bridge free surface under the action of magnetic field. The shape of free surface significantly changes under the effect of axial magnetic field, the free surface of liquid bridge has a tendency to be flat (see the slope change of blue dotted line in fig. 9). The contact angle, φ , variation of the free surface at the cold corner is as following, $\varphi_{\text{cold}, B=0.5 \text{ T}} =$ $= 118.19^\circ$, $\varphi_{\text{cold}, B=0.3 \text{ T}} = 129.53^\circ$, $\varphi_{\text{cold}, B=0.1 \text{ T}} = 98.36^\circ$, and $\varphi_{\text{cold}, B=0.7 \text{ T}} = 84.05^\circ$, meanwhile, the contact angle variation of the free surface at the hot corner is as following, $\varphi_{\text{hot}, B=0.5 \text{ T}} = 83.34^\circ$, $\varphi_{\text{hot}, B=0.3 \text{ T}} = 72.16^\circ$, $\varphi_{\text{hot}, B=0.1 \text{ T}} = 54.21^\circ$, and $\varphi_{\text{hot}, B=0.7 \text{ T}} = 43.33^\circ$. With the intensified axial magnetic field, the contact angle of free surface at the cold or hot corner is gradually increased.



Figure 9. Free surface deformation under the axial magnetic field

Conclusions

The influence of uniform transverse or axial magnetic field on the thermocapillary convection for a silicon oil based ferro fluid with the free surface deformation is numerically and experimental studied.

- Transverse magnetic field can generate vertical resistance to the internal flow of liquid bridge. Therefore, the uniform transverse magnetic field can directly suppress the longitudinal velocity of fluid in the liquid bridge. At the same time, the thermocapillary convection gradually approaches to the free surface with the increasing transverse magnetic field. The transverse magnetic field affects the lateral velocity of reflux by changing the space position of the cellular flow. The inhibition of transverse magnetic field on thermocapillary convection is realized by transferring influence to the free surface.
- Axial magnetic field can generate radial resistance to the internal flow of liquid bridge. Therefore, the uniform axial magnetic field can directly inhibit the lateral velocity of cellular flow, and the radial channel is impeded where the bulk reflux replenishes the surface flow. With the increasing axial magnetic field, the thermocapillary convection is gradually suppressed near the free surface, and the flow stagnant region is formed within the liquid bridge. The uniform axial magnetic field restrains the thermocapillary convec-

tion and reduces the convection heat transfer, which causes the heat transfer mode to be transformed into heat conduction. Further, the thermoisopleth is approximately uniform near the free surface.

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