IMPACT OF SURFACE TEXTURE ON ENTROPY GENERATION IN NANOFLUID

by

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We consider a heat transfer augmentation problem to minimize the entropy generation by assuming boundary-layer flow of nanofluid over a moving wavy surface. The nanofluid demonstrates great potential in enhancing the heat transfer process due to its high thermal conductivity. The famous Tiwari and Das model has been used in the present article. Two types of water based nanofluids containing Cu and Fe_3O_4 nanoparticles are considered. Moreover, the surface texture is taken to be sinusoidal wavy to improve the thermal contact. The governing equations are transformed into a system of non-similar PDE by using suitable dimensionless variables and solved by the Keller-Box method. The effects of involved parameters like amplitude wavelength ratio, group parameters, and volume fraction on the total entropy number and the Bejan number are analyzed graphically. It is showed Fe_3O_4 base nanofluid is more effective to lessen the entropy production as compared to Cu base nanofluid.

Key words: *nanoparticles, wavy surface, entropy generation, Keller-Box method*

Introduction

For the last thirty years or so, much of the research was focused on how to enhance the efficiency of thermal and mechanical systems. If loss of useful energy is reduced, efficiency of the system can be increased. The situation becomes more uneconomical when useful energy is destroyed while reducing thermal and mechanical efficiency. This becomes the major reason in the disorder of a system which is called the entropy of the system. Entropy helps in studying the irreversibility effects in the system. The influence of entropy generation over flow/heat transfer phenomenon is of great interest due to its vast application in combustion engines, heat exchangers, turbo-machinery, and transport phenomena, *etc.* In fluid-flow there are many sources of entropy generation including convective transfer of heat, solar radiation, viscous dissipation, mass diffusion, *etc.*

Bejan [1] initiated the research in this field and explained various factors which are responsible for entropy production. Since then a spat of investigations has been done to examine the entropy production under various physical situations. San *et al.* [2] discussed

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the irreversibility process through an isothermal medium. Abu-Hijleh and Heilen [3] investigated the irreversibility analysis over an isothermal rotating cylinder. Yilbas [4] studied the entropy generation arises due to viscous dissipation through a cylindrical annulus. He found that when the value of Brinkmen number is large the entropy generation is negligible close to the rotating cylinder. Mahmud and Fraser [5] analyzed the entropy generation effects inside a channel. Tamayol *et al.* [6] discussed the irreversibility analysis of fluid motion and transfer of heat over the porous sheet. It was noted that irreversibility rate becomes greater when suction parameter increases. Makinde [7] discussed the variable viscosity using second law analysis in presence of thermal radiation. The entropy generation rate was shown to be increasing near the surface as radiation increases. Butt *et al.* [8] discussed the influence of hydrodynamic slip on the irreversibility phenomena along the vertical flat surface and found that entropy production reduces due to slip parameter. Effects of magnetic field on the entropy generation of mixed convection over a stretching surface were studied by Butt *et al.* [9]. Noghrehabadi *et al.* [10] considered the Boungiorno model of nanofluid over the stretching surface and examined the entropy generation phenomena. Recently, Munawar *et al.* [11] discussed entropy production in a multidimensional flow of viscous fluid over a swirling stretchable cylinder. Entropy generation in peristaltic flow of a variable viscosity fluid was studied by Munawar *et al.* [12].

The study of convective heat transport had been the subject of particular interest to the engineers and scientists working in the areas of convective heat transfer. To enhance the rate of heat transfer the conducting surfaces are deliberately made irregular and roughen so that the contact area of the surface is increased and heat exchange rate can be expedited. This is the reason why various parts of the heating systems are kept roughened. The heat generated by engines is transferred to air through radiators which have the rough surfaces. One way of increasing the surface area is to keep the sinusoidal wavy surface texture. Theoretically, convective heat transfer over an isothermal vertical wavy surface was analyzed by Yao [13]. Moulic and Yao [14, 15] discussed the impact of non-uniformities on natural-convection flow. Rees and Pop [16] studied the impact of rough surface over free convection in porous media. Hossain and Pop [17] inspected the influence of magnetic parameter and amplitude of the wavy surface on the local Nusselt number under small Prandtl number. Hossain *et al.* [18] discussed the free convection over a wavy cone with variable viscosity effects. Siddiqa *et al.* [19] analyzed thermally radiating fluid over the heated vertical wavy surface. They analyzed that the wavy amplitude decreases with increasing value of radiation parameter. Parveen *et al.* [20] investigated the influence of viscous dissipation in free convection flow over a wavy surface and showed that the thickness of boundary-layer becomes prominent with respect to viscous dissipation parameter.

Another way to enhance the heat transfer rate is the selection of nanofluid instead of pure fluid. Due to their higher thermal properties in comparison liquids the material particles present in the base fluid do increase the thermal conductivity and the density of the resulting nanofluid. Thus nanofluids can be used in industrial cooling for great energy savings. Due to high thermal conductivity nanofluids can be used as liquid coolant in computers and cell phones. Working on this idea Choi and Eastman [21] and Choi *et al.* [22] presented investigations on the thermal conductivity enhancement of fluids with the presence of nanoparticles. The nanofluids enhance the thermal conductivity, viscosity, thermal diffusivity, and convective heat transfer as compared to the ordinary fluids, see Nguyen *et al.* [23] and Akbarinia *et al.* [24]. Buongiorno [25] studied the comparison of conductivity of nanofluids with base fluids. The study of nanofluids is being applied in many fields like electronics, lasers, material processing and optimal devices, *etc.* Nowadays numerous scientists are considering convective heat transfer problems with nanofluids, see for instance [26-34].

The aim of present work is, to analyze the entropy generation of boundary-layer 2-D flow over a continuously moving wavy surface. By using non-similarity transformation, the governing equations are transformed into system of dimensionless non-similar PDE and solved numerically by finite difference scheme known as Keller-Box scheme. The influences of pertinent parameters on entropy generation are shown graphically.

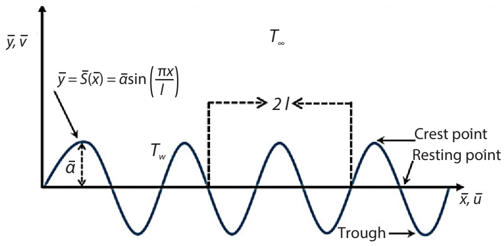

Figure 1. Schematic diagram with co-ordinate system

Mathematical formulation

Consider steady flow due to a horizontal wavy sheet moving uniformly in x-direction. The flow is 2-D in nature whose velocity varies continuously with the variable x which makes the flow non-similar. The wavy surface is assumed to be surrounded by the ambient nanofluid. The surface shape is described by the function $\bar{y} = \bar{S}(\bar{x}) = \bar{\alpha}\sin(\bar{x}/l)$, where $\bar{\alpha}$ represents the amplitude of the wavy boundary while l stands for the wavelength, fig. 1. The surface is held at a constant temperature T_w and the ambient temperature is $T_{\infty.}$ Any non-flat sheet surface can be considered in the analysis of such kind of problems. However, such a consideration must be accompanied by a proper justification in view of technological applications. In practice, a flat heat transferring surface of a heat exchanger can easily be replaced by a periodic wavy surface in order to improve the heat transferring capacity of the heat exchanger. Therefore, a sinusoidal wavy surface has been considered in this study. Moreover, a sinusoidal wavy surface is continuous, smooth, and has no sharp corners like the wavy surface of the triangular and square nature. Because of this reason, its mathematical handling is also simple. However, there are studies, see for instance Sadia *et al.* [35], where the authors have considered a triangular wavy surface as well.

In the literature, several theoretical and experimental models are available for describing the homogenous (single-phase) and non-homogenous (two-phase) nanofluids. To model nanofluid of single-phase, two important models namely the Das *et al*. [32] and Buongiorno [25] models are available in literature to discuss the convective transport phenomenon of nanofluid. Das *et al*. [32] model discusses the effective properties of the nanofluid whereas the Brownian motion and thermophoresis are considered in Buongiorno [25] model. Therefore, to capture the contribution of nanoparticles by considering the material properties of the particles in the flow and heat transfer analysis, Das *et al*. [32] model is utilized. The detailed derivation of the governing equations is given by Hossain and Pop [17] and for the nanofluid is given by Mehmood and Iqbal [29]. Following [17] and [29] the non-similar governing equations in the absence of body forces are therefore, directly written:

$$\frac{1}{d_1}f''' + \frac{1}{2}ff'' - \xi\frac{\Delta_\xi}{\Delta}\left(ff'' - f'^2\right) = \xi\left[f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right] \tag{1}$$

$$\frac{d}{d_3\mathrm{Pr}}\theta'' + \frac{1}{2}f\theta' + \xi\frac{\Delta_\xi}{\Delta}f\theta' = \xi\left[f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right] \tag{2}$$

In previous non-linear PDE, the dimensionless quantities are described:

$$\xi = x = \frac{\bar{x}}{l},\ y = \frac{\bar{y}}{l},\ u = \frac{\bar{u}}{U},\ v = \frac{\sqrt{\mathrm{Re}}}{U}\left(\bar{v} - S_\xi \bar{u}\right),\ \eta = \frac{\left[\bar{y} - \bar{S}(\bar{x})\right]\sqrt{\mathrm{Re}}}{l\Delta\sqrt{\xi}} y,\ p = \frac{\bar{p}}{\rho_f U^2}$$
$$\psi = \sqrt{\xi} f(\xi, \eta),\ u = \frac{\partial \psi}{\partial x},\ v = -\frac{\partial \psi}{\partial y},\ \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty},\ S = \frac{\bar{S}(\bar{x})}{l},\ \Delta = \sqrt{1 + S_\xi^2} \tag{3}$$

where Re and Pr are the Reynold's and Prandtl numbers, respectively, while Δ is the wavy parameter. The prime represents differentiation with respect to η. The material parameters d, d_1, d_2, d_3 and d_4 are given:

$$d = \frac{\kappa_{\mathrm{nf}}}{\kappa_{\mathrm{f}}},\quad d_1 = (1-\phi)^{2.5}\left[1 - \phi + \phi\left(\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{f}}}\right)\right],\ d_2 = \left[1 - \phi\phi\left(\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{f}}}\right)\right]$$
$$d_3 = \left[1 - \phi + \phi\left(\frac{(\rho c_p)_{\mathrm{p}}}{(\rho c_p)_{\mathrm{f}}}\right)\right],\ d_4 = 1 + \left[\frac{3\left(\dfrac{\sigma_{\mathrm{p}}}{\sigma_{\mathrm{f}}} - 1\right)\phi}{\left(\dfrac{\sigma_{\mathrm{p}}}{\sigma_{\mathrm{f}}} + 2\right) - \left(\dfrac{\sigma_{\mathrm{p}}}{\sigma_{\mathrm{f}}} - 1\right)\phi}\right] \tag{4}$$

where ϕ isthe concentration of nanoparticle, ρ – the density, σ – the electric conductivity, κ – the thermal conductivity*,* and c_p – the heat capacity. The subscripts p, f, and nf refer to the nanoparticle, base fluid, and the nanofluid, respectively. The thermal conductivity κ_{nf} of nanofluid given in eq. (4) was described by Maxwell [36]. The relations for μ_{nf}, ρ_{nf}, α^*_{nf}, and $(\rho c_p)_{\mathrm{nf}}$ are described:

$$\mu_{\mathrm{nf}} = \frac{\mu_{\mathrm{f}}}{(1-\phi)^{2.5}},\ (\rho c_p)_{\mathrm{nf}} = (1-\phi)(\rho c_p)_{\mathrm{f}} + \phi(\rho c_p)_{\mathrm{p}},\ \alpha^*_{\mathrm{nf}} = \frac{\kappa_{\mathrm{nf}}}{(\rho c_p)_{\mathrm{nf}}}$$
$$\rho_{\mathrm{nf}} = (1-\phi)\rho_{\mathrm{f}} + \phi\rho_{\mathrm{p}},\ \frac{\kappa_{\mathrm{nf}}}{\kappa_{\mathrm{f}}} = \frac{(\kappa_{\mathrm{p}} + 2\kappa_{\mathrm{f}}) - 2\phi(\kappa_{\mathrm{f}} - \kappa_{\mathrm{p}})}{(\kappa_{\mathrm{p}} + 2\kappa_{\mathrm{f}}) + \phi(\kappa_{\mathrm{f}} - \kappa_{\mathrm{p}})} \tag{5}$$

The appropriate boundary conditions in dimensionless form read:

$$f(\xi, 0) = 0,\ f'(\xi, 0) = \Delta,\ \theta(\xi, 0) = 1,\ f'(\xi, \infty) = 0,\ \theta(\xi, \infty) = 0 \tag{6}$$

The coefficient of skin friction and the Nusselt number:

$$C_{fx} = \frac{\tau_w}{\rho_{\mathrm{f}} U^2},\quad \mathrm{Nu}_x = \frac{\bar{x} q_{\mathrm{w}}}{\kappa_{\mathrm{f}}(T_{\mathrm{w}} - T_\infty)} \tag{7}$$

where τ_{w} and q_{w} are the shear stress and heat flux at the wall, respectively and are obtained:

$$\tau_{\mathrm{w}} = \mu_{\mathrm{nf}}(\nabla \bar{u} \hat{n})_{y=0},\quad q_{\mathrm{w}} = -\kappa_{\mathrm{nf}}(\nabla T \hat{n})_{y=0} \tag{8}$$

where $\hat{n} = (n_x, n_y) = (-S_\xi/\Delta, 1/\Delta)$, see [29], indicates unit normal vector of the sinusoidal boundary. Using eqs. (3) and (8) in eq. (7) the coefficient of skin friction and the Nusselt number:

$$C_f = C_{fx}\mathrm{Re}^{1/2} = \frac{x^{-3/2}}{\Delta^{3/2}(1-\phi)^{2.5}} f''(\xi, 0),\quad \mathrm{Nu} = \mathrm{Nu}_x \mathrm{Re}^{-1/2} = -\frac{\sqrt{x}}{\sqrt{\Delta}}\frac{\kappa_{\mathrm{nf}}}{\kappa_{\mathrm{f}}}\theta'(\xi, 0) \tag{9}$$

Entropy analysis

On the basis of Second law of thermodynamics the local entropy generation for nanofluid in rectangular co-ordinate system can be written:

$$S_G = \underbrace{\frac{\kappa_{\mathrm{nf}}}{T_\infty^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]}_{\text{Entropy generated by heat } (S_{G,H})} + \underbrace{\frac{\mu_{\mathrm{nf}}}{T_\infty}\left[2\left[\left(\frac{\partial \overline{u}}{\partial x}\right)^2 + \left(\frac{\partial \overline{v}}{\partial y}\right)^2\right] + \left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right)^2\right]}_{\text{Entropy generated by friction factor } (S_{G,F})} \tag{10}$$

where the thermal conductivity of nanofluid and the viscosity are two major causes of irreversibility. The first term represents the irreversibility due to heat transfer while the second term describes the fluid friction irreversibility. Entropy generation number is defined by Bejan [1], which is the ratio of volumetric entropy generation the characteristic entropy generation rate. Using the set of dimensional variables listed in eq. (3) the total entropy generation number is described:

$$\frac{N_S}{\mathrm{Re}} = \frac{S_G}{S_{G_0}} = \frac{d}{\xi}\theta'^2 + \frac{d_1\omega\Delta^2}{\xi} f''^2 = N_H + N_F \tag{11}$$

where d and d_1 are material parameters defined in eq. (4), $\omega = \mathrm{Br}/\Omega$ – the group parameter, $\mathrm{Br} = \mu_{\mathrm{f}} U_{\mathrm{w}}^2/\kappa_{\mathrm{f}}(T_{\mathrm{w}} - T_\infty)$ – the Brinkman number, $\Omega = (T_{\mathrm{w}} - T_\infty)/T_\infty$ – the dimensionless temperature difference, and $S_{G0} = \kappa_{\mathrm{f}}(\Omega/l)^2$ – the characteristic entropy. In eq. (11) N_H and N_F denote the entropy generation due to heat transfer and fluid friction, respectively. The Bejan number is defined to be the ratio of heat transfer irreversibility to the total irreversibility which is given by $\mathrm{Be} = 1/1 + \phi^*$ where $\phi^* = N_F/N_H$ is the irreversibility ratio. The heat transfer irreversibility dominates when $0 \leq \phi^* < 1$ and irreversibility due to fluid friction dominates for $\phi^* > 1$. However, entropy production rates due to both factors are equal if $\phi^* = 1$. To inspect the overall generation of entropy in the flow domain, the average values of entropy generation and Bejan number are computed:

$$\overline{N}_s = \frac{1}{\forall}\int_0^\xi N_s \mathrm{d}\xi, \quad \overline{\mathrm{Be}} = \frac{1}{\forall}\int_0^\xi \mathrm{Bed}\xi \tag{12}$$

where $\forall = S_0^\xi \Delta \mathrm{d}\xi$ is the surface area of the wavy sheet over a unit length.

Results and discussion

The governing non-linear PDE (1) and (2) along with boundary data (6) are solve numerically with the Keller-Box method. We suppress the detail of the method in order to keep the simplicity of the script. Readers are referred to the references [37, 38] for the detail of the method. However, a comparison with the data available in literature has been made. Table 1 gives the comparison of the current results of skin friction coefficient and Nusselt number with the result published by Hossain and Pop [17], Mehmood *et al*. [33] and Rees and Pop [39]. It shows an excellent agreement which authenticates the present numerical procedure.

Table 1. Comparison of numerical values of C_f and Nu at $\alpha = 0$, Pr = 0.7, $\xi = 0$

	Present	[39]	[17]	[33]
$-C_f$	0.4437	0.4438	0.4439	0.44375
–Nu	0.3492	0.3492	0.3509	0.34924

We use the solutions to make the further entropy analysis by inspecting the influence of pertinent parameters on the entropy generation with the help of graphs shown in figs. 2-16. In fig. 2 the entropy number is plotted against the variable ξ for variation in parameter α. It is observed that the entropy generation is high in the upstream region and decreases in the downstream. The reason behind this behavior of irreversibility is that the temperature and velocity gradients are high at upstream location. A careful look on the figure reveals that the entropy generation decreases at the resting points as the aspect ratio α increases. However, it increases at the crest and trough points of wavy surface. The Bejan number is plotted in fig. 3 to see the discriminating features of heat transfer and fluid friction irreversibilities against the variable ξ in presence of Cu- and Fe_3O_4-water nanoparticles. From the figure it is observed that overall heat transfer irreversibility dominates over fluid friction irreversibility at all resting points and fluid friction irreversibility increases at the crest/trough points of the surface. However, for nanofluid based on Fe_3O_4 nanoparticles the Bejan number is higher as compare to Cu-water base nanofluid. This indicates that Fe_3O_4-water based nanofluid are highly efficient in producing the heat transfer irreversibility and thus good for heat transfer augmentation problems. Moreover, the amplitude of oscillations in the Bejan number is high for the case of Cu-water base nanofluid in downstream which indicates that it is less efficient for the heat transfer augmentation process.

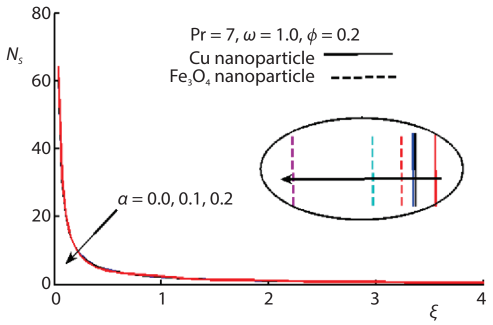

Figure 2. The entropy number N_S for different α

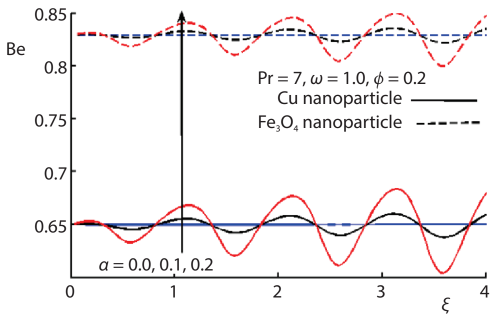

Figure 3. The Bejan number for different α

Figure 4 shows that the total entropy generation number increases as group parameter ω increases because of fluid friction enhancement. It is also noted that the amplitude of oscillation in the entropy number in presence of Cu-water based nanofluid is higher than Fe_3O_4-water base nanofluid as ω increases. Figure 5 illustrates that the Bejan number reduces as the group parameter ω increases which indicates the dominancy of fluid friction irreversibility for both types of nanofluids. Figure 6 describes the effects of nanoparticle volume fraction ϕ on the total entropy generation. The figure shows that the total entropy decreases as the concentration increases but not significantly. The graph of the Bejan number shown in fig. 7 indicates that fluid friction irreversibility increases as the volume concentration of nanoparticles increases. This happened because the increasing of volume concentration which causes an increase in the viscosity of the nanofluid leads to increase the friction factor.

Figures 8-13 show the graphs of the entropy number and the Bejan number against the variable η. Figure 8 demonstrates that the entropy decreases as the aspect ratio α increases for both types of nanoparticles. Moreover, the entropy is high near the wall and dies out when one moves away from the surface. However the entropy generation due to transfer of heat is dominant near the wall and irreversibility due to fluid friction is dominant in free stream region, see fig. 9. Figure 10 demonstrates that the total entropy generation number increases as

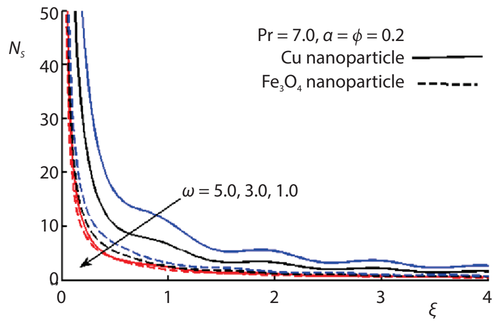

Figure 4. The entropy number N_S for different ω

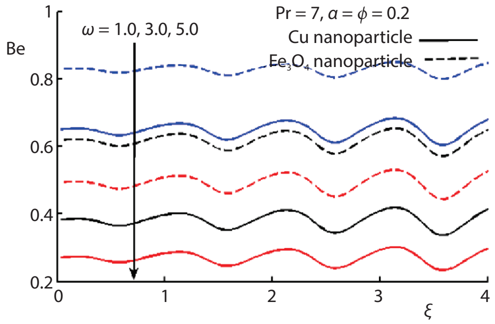

Figure 5. The Bejan number for different ω

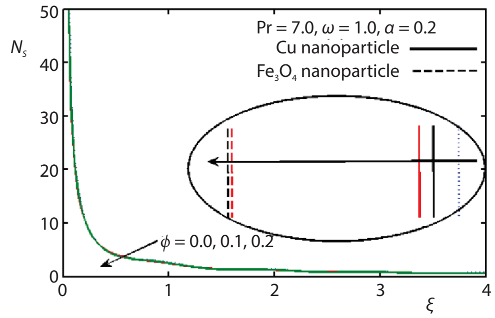

Figure 6. The entropy number N_S for different ϕ

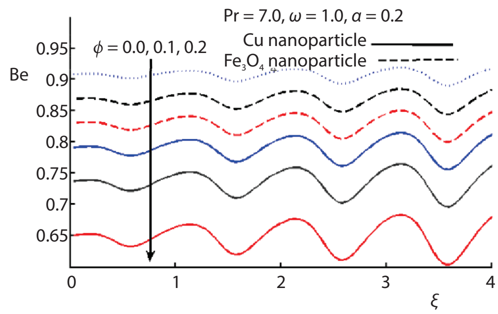

Figure 7. The Bejan number for variation in ϕ

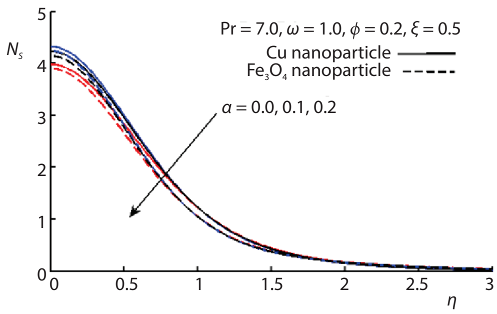

Figure 8. The entropy number N_S for different α

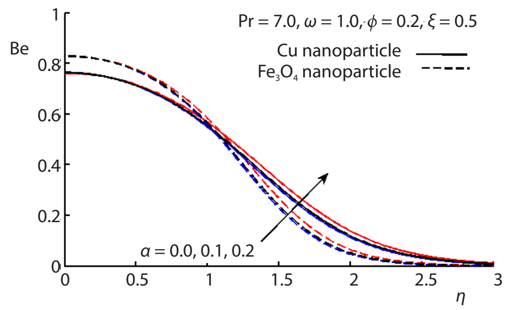

Figure 9. The Bejan number for different α

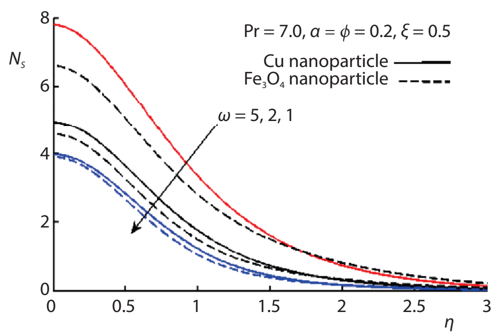

Figure 10. The entropy number N_S for different ω

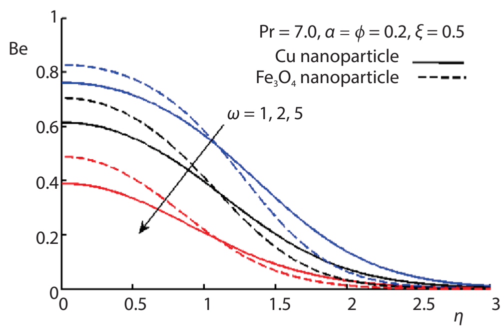

Figure 11. The Bejan number for different ω

the group parameter increases. Figure 11 shows that with increasing value of group parameter ω, the Bejan number decreases due to dominancy of fluid friction irreversibility. However, Fe_3O_4 nanoparticles display ability to persist heat transfer effect. Figure 12 reveals that the entropy generation number N_S increases as ϕ increases. This is because viscous effects due to the nanoparticles concentration increase and consequently enhance the overall irreversibility. From fig. 13 it is noted that as the value of ϕ increases the Bejan number reduces near the surface showing the strengthening of fluid friction irreversibility however an inverse behavior is seen in the main stream where heat transfer effects become stronger.

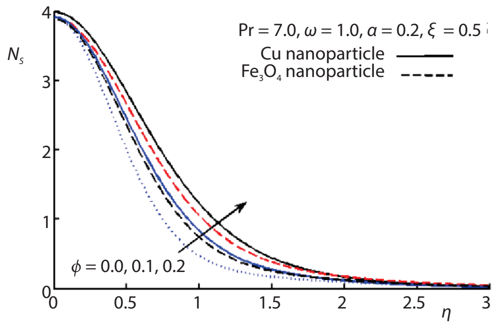

Figure 12. The entropy number N_S for different ϕ

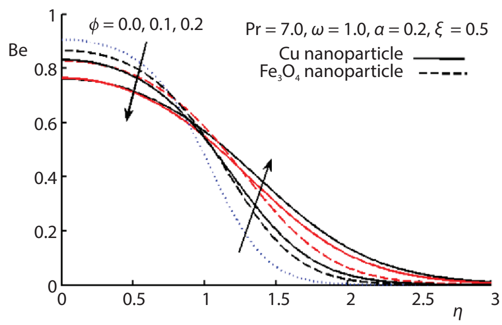

Figure 13. The Bejan number for different ϕ

The influence of wavy amplitude α on average entropy generation number, $\overline{N}_s$, and average Bejan number, $\overline{\text{Be}}$, is elaborated through tab. 2. It is seen that 0.8 times reduction in $\overline{N}_s$ occurs when wavy amplitude increases from 0 to 0.2 whereas $\overline{\text{Be}}$ reduces almost 0.9 times. This reduction in average value of Bejan number indicates that the viscous dissipation plays a vital role in the production of entropy in case of higher values of amplitude to wavelength ratio. Table 3 shows the behavior of $\overline{N}_s$ and $\overline{\text{Be}}$ with the variation of Prandtl number. It is observed that average entropy generation increases for the fluids having large Prandtl number. Also, from tabulated values of $\overline{\text{Be}}$ it is observed that thermal irreversibility dominates for the fluids having large Prandtl number whereas viscous irreversibility dominates for small Prandtl number.

Table 2. Average entropy generation number and average Bejan number for various values of wavy amplitude α at ϕ = 0.05, Pr = 0.7, ω = 1, ξ = 1

α	$\overline{Ns}$		$\overline{\text{Be}}$	
	Cu	Fe_3O_4	Cu	Fe_3O_4
0.0	1.9636	1.8138	0.2724	0.3180
0.1	1.8410	1.7006	0.2657	0.3102
0.2	1.5566	1.4377	0.2486	0.2903

Table 3. Average entropy generation number and average Bejan number for different base fluids at ϕ = 0.05, α = 0.1, ω = 1, ξ =1

Pr	$\overline{Ns}$		$\overline{\text{Be}}$	
	Cu	Fe_3O_4	Cu	Fe_3O_4
0.7	1.8410	1.7006	0.2657	0.3102
07	10.2879	10.2645	0.8451	0.8619

Conclusions

We examined the entropy generation of nanofluid over a continuously moving wavy surface. Numerical results are obtained for non-similar flow and heat transfer equations via Keller-Box scheme. The numerical results are plotted in different graphs to see the effect of surface texture and nanofluid on the entropy phenomenon. We concluded the following results.

- The total entropy generation reduces and rises at the resting points and crest/trough points of the surface, respectively, as the magnitude of oscillations increases.
- Nanofluid based on Fe_3O_4 nanoparticles reduces entropy more efficiently as compared to the nanofluid based on Cu nanoparticles. Since Fe_3O_4-water based nanofluid is more capable of producing heat transfer irreversibility and thus good for heat transfer augmentation problems.
- The volume concentration of nanoparticles should be kept under a certain range since it leads to increase the fluid friction irreversibility and thus results in increasing the total entropy of the system.
- Graphical results show that the entropy generation by the transfer of heat is dominant close to the boundary and fluid friction irreversibility is dominant in distant region.
- Enhancement of group parameter leads to decrease the Bejan number rapidly in case of Cu nanoparticles as compared to the nanoparticles of Fe_3O_4.

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