ADOPTION OF COMPUTER PARTICLE SWARM OPTIMIZATION ALGORITHM UNDER THERMODYNAMIC MOTION MECHANISM

by

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In order to improve the stability and sensitivity of particle swarm optimization (PSO) algorithm and to solve the problem of premature convergence, in this research, a computer PSO algorithm based on thermodynamic motion mechanism is proposed based on the principle of thermodynamic motion mechanism. Firstly, the thermodynamic motion phenomenon, the diffusion law in kinematics and the standard PSO algorithm are introduced. Then, according to the basic idea of thermodynamic motion mechanism, the standardized PSO algorithm is optimized and its optimization process is introduced. Finally, the experimental results are analysed by setting the test function. The results show that among the five test functions, the computer PSO algorithm based on thermodynamic motion mechanism has a higher probability of jumping out of the local optimal solution. Its robustness and stability are much better than standard PSO algorithms. The evolution ability of the computer PSO algorithm based on thermodynamic motion mechanism is better than that of the standard PSO algorithm. The standard PSO algorithm is superior because it is based on thermodynamic motion mechanism.

The research in this paper can provide good guidance for improving the performance of PSO algorithm.

Key words: thermodynamic motion mechanism, convergence, PSO algorithm, test functions

Introduction

With the development of science and technology, in the late 20\textsuperscript{th} century, many scholars in the field of process put forward the artificial intelligence in order to achieve the computer simulation of natural intelligence. After decades of development, artificial intelligence has developed rapidly, and a lot of computer artificial intelligence algorithms have formed, such as ant colony algorithm, PSO algorithm, neural network algorithm, etc.. However, with the development of intelligent algorithms, artificial intelligence technology has encountered a development bottleneck, so it is necessary to find a new development approach to solve the bottleneck in the development of artificial intelligence [1, 2].

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The particle population algorithm is based on the study of animal behaviour and social psychology. It is a simulation of simple biological social system. The process is simple and easy to implement programmatically. It not only has the background of traditional algorithm, but also shows superior performance, which makes it widely applied after proposed [3-5]. However, although PSO has many excellent performances, it still has some problems in convergence and stability. To solve this problem, many researchers have tried to link PSO algorithms to the basic theories of mathematics, physics and biology. It is intended to improve the convergence and stability of the algorithm through the basic discipline theory [6]. Diffusion is a universal phenomenon in nature, a fundamental phenomenon in thermodynamic motion. It means that gases (solids, liquids) and other substances are in perpetual motion. In the same way that diffusion is related to the forces between molecules, PSO is essentially an intelligent algorithm that scientists use to inspire nature. The automatic dispersal of organisms in nature to seek new habitats to reproduce is essentially the same as the diffusion of heat, therefore, applying the idea of thermodynamic motion mechanism to PSO algorithm is of great significance to improve the performance of PSO algorithm.

To sum up, in order to improve the convergence and stability of the particle population algorithm, the theory of thermodynamic motion mechanism is used to introduce the phenomena and laws of thermal motion mechanism and the standard PSO algorithm. The particle population algorithm is optimized by means of thermodynamics. It is hoped that the research in this paper can provide a good idea for the performance of PSO algorithm.

Methodology

The thermodynamic motion phenomenon

Diffusion is a universal phenomenon in nature. For example, there is a saying that good wine needs no bush. It means that if the wine is very good, even in the deep alley, people will smell the aroma of the wine. In addition, if a person sprayed perfume, not long later, the whole room is perfume smell, if cooking at home, the smell of the food will come. All of these examples show that the molecules are in perpetual motion. There are many such examples in life [7]. In the concept of physics, this phenomenon is called diffusion motion. Diffusion motion can occur in solids, liquids, and gases. According to the knowledge of physics, the diffusion coefficient of the diffusion motion in the gas is the largest, followed by the liquid, and finally the solid. Because the forces between molecules are greater in a solid than in a gas or liquid, the resistance to molecular motion is greater. The diffusion coefficient of molecular motion is an inherent property, which is influenced by temperature, crystal structure, solid solution type, solid solution concentration, crystal defects and other factors, and is not immutable [8].

The thermodynamic diffusion laws

The law of diffusion was proposed by the German physicist Fick in the mid-19th century, and contains Fick's First law and the Second law. Fick's first law is steady-state diffusion. The law must satisfy that the concentration and concentration gradient in the diffusion process will not change because of time; Fick's Second law is the law of unsteady diffusion, in which the concentration and concentration gradient can change with time [9, 10].

Fick's First law can be expressed:

$$J = -D \frac{dc}{dx}$$  (1)
where the minus sign is the direction in which the concentration is going down, \( J \) represents the diffusion flow, and \( D \) is the diffusion coefficient. The diffusion coefficient is the material flow rate per unit area under the unit concentration gradient, and is an important physical quantity of the material diffusion speed. Equation (1) refers to the material flow rate that does not change with time. And it happens in a space where the material is not evenly distributed.

Fick’s Second law can be expressed:

\[
\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}
\]

(2)

where \( c \) is the concentration of the substance and \( t \) represents the diffusion time. The equation for the Second law of diffusion is the spatial position and concentration. For the Second law of thermodynamic motion mechanism, in the specific application, it needs to clearly know the initial conditions and boundary conditions before carrying out the integral solution.

The diffusion coefficient is generally related to the temperature, concentration, composition and structure of the substance \[11\]. The diffusion coefficient \( D \) can be expressed:

\[
D = D_0 e^{\frac{Q}{RT}}
\]

(3)

where \( Q \) is the diffusion activation energy, \( R \) stands for gas (solid, liquid) constant, \( D_0 \) – the diffusion constant, and \( T \) – the thermodynamic temperature.

**Standard PSO algorithm**

In PSO, each bird is a particle, which is the potential solution of the problem to be optimized in the search space. Each particle has fitness, and is determined by the optimized function and the position of the particle itself. There is also a speed that determines the direction and distance they will move the next moment. The particles then search the solution space based on where they were best and where they were best in the population. In each iteration, the particle determines its acceleration (and thus its velocity) from two pieces of data. The first is the optimal solution found by the particle itself, which is called the individual optimal solution, \( P_i \). The other extreme value is the optimal solution found by the whole population. This extreme value is the global optimal solution, \( P_g \). The individual optimal solution and the global optimal solution are updated once every iteration \[12\].

The PSO is mathematically described as a population \( S = \{x_1, x_2, \ldots, x_n\} \) of \( n \) potential solutions (particles) in an \( m \)-dimensional space, where the \( i \)th particle represents a \( d \)-dimensional vector: \( X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \), namely, the position of the \( i \)th particle in the \( m \)-dimension. The fitness (such as distance) of each particle is calculated based on the objective function. The \( P' \) represents the optimal solution experienced by particle \( i \) iteration up to \( t \) times, \( V_i = (v_{i1}, v_{i2}, \ldots, v_{id}) \) is the velocity of particle \( i \). The basic iterative mode of PSO algorithm is:

\[
V_i^{t+1} = wV_i^t + c_1 \text{rand}(1)(P_i^t - X_i^t) + c_2 \text{rand}(2)(P_g^t - X_i^t)
\]

(4)

\[
X_i^{t+1} = X_i^t + V_i^{t+1}
\]

(5)

Among them, \( i = 1, 2, \ldots, n \) is the particle label, \( t \) is the number of iterations, \( w \) is the inertia weight (there was no such parameter when the algorithm was first proposed, the default value is \( 1 \)), \( c_1 \) and \( c_2 \) are the learning factors (taking positives), \( \text{rand}(1) \) and \( \text{rand}(2) \) are the random numbers between \([0,1]\).
For the velocity of particle \( i \) in \( D \)-dimensions at time \( K \), eq. (6) should be satisfied:

\[
\begin{align*}
\vec{v}_{id}^k &= v_{d,\text{max}} \text{ if } \vec{v}_{id}^k = v_{d,\text{max}} \\
\vec{v}_{id}^k &= -v_{d,\text{max}} \text{ if } \vec{v}_{id}^k < v_{d,\text{max}} \\
i &= 1, \ldots, m \\
d &= 1, \ldots, n
\end{align*}
\]  

(6)

where \( v_{\text{max}} = (v_{1,\text{max}}, \ldots, v_{d,\text{max}}, v_{n,\text{max}}) \) is the given maximum speed limit and \( \omega \) is the inertial weight factor. The \( \omega \) value of the standard PSO algorithm is determined by:

\[
\omega^k = (\omega_{\text{max}} - \omega_{\text{min}}) \frac{k}{k_{\text{max}}}, k = 1, \ldots, k_{\text{max}}
\]

(7)

where \( \omega_{\text{max}} \) is the maximum of \( \omega \). Generally, \( \omega_{\text{max}} = 0.9 \). The \( \omega_{\text{min}} \) is the minimum of \( \omega \). Supposing \( \omega_{\text{min}} = 0.4 \), \( k_{\text{max}} \) is the specified maximum number of iterations and \( k \) is the current number of iterations.

The flow of PSO algorithm is shown in fig. 1.

**Figure 1. The flow graph of PSO**

The idea and flow of calculating PSO algorithm based on thermodynamic motion mechanism

Based on the revelation of the multi-population thought of thermodynamic motion mechanism, the dual-population according to the thermodynamic motion mechanism is simulated, and a double PSO (DPSO) algorithm based on thermodynamic motion mechanism is proposed. According to the concepts of diffusion activation energy and temperature in thermodynamic motion, the diffusion energy, temperature, and diffusion probability of population particles are defined. According to the thermodynamic motion mechanism, the principle and flow of the algorithm are introduced.

The diffusion energy of a particle is the energy due to the motion of an object, called kinetic energy. In this research, without considering the mass of the particle, the diffusion energy of population particles is expressed:
\[ Q_i = \frac{1}{2} \sum_{j=1}^{\text{Dim}} v_{ij}^2 \]  \hspace{1cm} (8)

where \( i \) is the subscript of the particles in the population, \( \text{Dim} \) represents the searchable space of particles, \( v_{ij} \) is the \( j^{th} \) vector of the velocity vector of the \( i^{th} \) particle.

According to the concept of temperature in physics, the population temperature is defined, which can be expressed:

\[ T = \frac{\sum_{i=1}^{M} Q_i}{M} \]  \hspace{1cm} (9)

where \( M \) denotes the number of population particles.

The diffusion probability of particles can be expressed:

\[ P_i = 1 - \frac{D_{Q_i}}{D_0} = 1 - \frac{D_0 e^{-Q_i/RT}}{D_0} = 1 - e^{-Q_i/RT} \]  \hspace{1cm} (10)

where \( P_i \) is the diffusion energy of particle \( I \), and \( R \) is the gas (solid, liquid) constant. Therefore, it can be concluded from the equation that the diffusion probability of a particle is determined by the diffusion energy of the particle and the temperature of the population. The diffusion probability of particles is inversely proportional to the population temperature and directly proportional to the population diffusion energy.

In the PSO algorithm based on thermodynamic motion mechanism, double population is used for reference to thermodynamic motion mechanism. In this research, two populations \( A \) and \( B \) are installed, and the operation settings for the two populations are the same.

According to the ideas of thermodynamic motion, the specific process of the dual-population ion swarm optimization algorithm based on thermodynamic motion can be divided into the following steps: Step 1 – the velocities and positions of all the ions in population \( A(B) \) were initialized, Step 2 – the adaptive values of the ions in populations \( A \) and \( B \) were evaluated, Step 3 – the particle history of population \( A \) and population \( B \) and the global extremum of population \( A \) and \( B \) are updated, Step 4 – the diffusion energy, temperature and diffusion probability of the two populations were calculated, Step 5 – particle \( i \) in population \( A \) is selected to be put into diffusion pool \( A \). Similarly, particle \( j \) in population \( B \) is selected to be put into diffusion pool \( B \), Step 6 – when the number of particles in diffusion pool \( A \) is greater than 2, two particles \( M \) and \( N \) are selected in the diffusion pool, Step 7 – the best of population \( A \) and population \( B \) are output, and then the positions and velocities of population particles are updated through eqs. (4) and (5), and Step 8 – if the algorithm is satisfied, the conditional transformation will be stopped; if not, the algorithm proposed in this paper will be terminated.

Results

The settings of the test function

In order to verify the superiority of the computer particle population optimization algorithm based on thermodynamic motion mechanism proposed in this paper, the perfor-
mance of various algorithms is compared. According to the test needs, five test functions are set in this paper:

\[ f_1(x) = \sum_{i} x_i^2, -100 \leq x_i \leq 100 \]  
\[ f_2(x) = \frac{1}{4000} \sum_{i} x_i^2, -\prod_{i} \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1, -100 \leq x_i \leq 100 \] 
\[ f_3(x) = \sum_{i} x_i^2 - 10 \cos(2\pi x_i) + 10, -100 \leq x_i \leq 100 \] 
\[ f_4(x) = -20 \exp \left( -0.2 \sqrt{\frac{1}{N} \sum_{i} x_i^2} \right) - \exp \left[ \frac{1}{N} \sum_{i} \cos(2\pi x_i) \right] + 20 + e \] 
\[ -100 \leq x_i \leq 100 \] 
\[ f_5(x) = \frac{\sin^2 \sqrt{x_1^2 + x_2^2} - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]} - 0.5, -100 \leq x_i \leq 100 \]  

Of the five test functions, \( f_1(x) \) is the unimodal function. In the unimodal function, there is only one local optimal solution. Functions \( f_2(x), f_3(x), \) and \( f_4(x) \) are multi-peaked function. There are several local optimal solutions in the function. \( f_5(x) \) means that at the point \((0,0)\), a global minimum of \(-1\) can be gotten. The infinite global minimum value points of the test function are within the range of 3.14 from this point, and the sub-global minimum value is approximately \(-0.00027\). Given the nature of the test function \( f_5(x) \), for general algorithms, the search for the global optimal solution is very complex, which can better test the performance of the algorithm.

**Experimental results and analysis**

Table 1 shows the convergence statistics of the DPSO and PSO algorithms for the five test functions, the optimal solution, the worst solution, the mean value, the standard deviation and the convergence ratio. In this research, the difference between the best price and the worst value less than \(1 \times 10^{-10}\) is viewed as a successful convergence ratio. According to tab. 1, according to the results of the five test functions, PSO algorithm and DPOS algorithm have certain differences in the optimal solution, the worst solution, the mean value and the standard deviation, and DPSO performance will be better. It also shows that with the increase of dimension, DPSO shows obvious superiority when dimension increases from 10 to 20 and 30. The performances improvements of functions \( f_1(x) \) and \( f_2(x) \) are most noticeable. Though functions \( f_3(x) \) and \( f_4(x) \) do not show obvious superiority, but the performance of DPSO algorithm is better than that of PSO according to the increase of dimension. For function \( f_5(x) \), its robustness and stability are much better than PSO. According to the data in the table, the global optimal solution can be obtained in 50 runs. Therefore, it can be proved that the performance of DPSO algorithm is superior to that of PSO.

Figures 2-5 show the convergence curves of DPSO algorithm and PSO algorithm in test functions \( f_1(x), f_2(x), f_3(x), \) and \( f_5(x) \), respectively. From fig. 2-5, it can be concluded that with the gradual implementation of the algorithm, the iteration of DPSO algorithm keeps going on, while PSO stagnates, indicating that the evolutionary ability of DPSO is better than
that of PSO, and DPSO has a greater probability of jumping out of the local optimal solution.

DPSO has a good advantage because it is caused by thermodynamic motion mechanism.

Table 1 Convergent statistic for five test functions of DPSO and PSO algorithms

<table>
<thead>
<tr>
<th>Test functions</th>
<th>Dimension</th>
<th>Algorithm</th>
<th>Optimum solution</th>
<th>Worst solution</th>
<th>Average</th>
<th>Standard deviation</th>
<th>Convergence rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x)$</td>
<td>10</td>
<td>PSO</td>
<td>9.06e-53</td>
<td>3.57e-37</td>
<td>7.16e-39</td>
<td>5.04e-38</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>2.07e-44</td>
<td>3.84e-39</td>
<td>5.06e-40</td>
<td>1.12e-39</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>PSO</td>
<td>5.63e-35</td>
<td>1.56e-20</td>
<td>9.81e-22</td>
<td>3.78e-21</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>3.23e-37</td>
<td>1.19e-31</td>
<td>5.52e33</td>
<td>1.98e-32</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>PSO</td>
<td>5.46e-26</td>
<td>3.21e-17</td>
<td>6.36e18</td>
<td>1.42e-17</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>5.48e-31</td>
<td>1.47e-27</td>
<td>5.03e-28</td>
<td>1.47e-27</td>
<td>50/50</td>
</tr>
<tr>
<td>$f_2(x)$</td>
<td>10</td>
<td>PSO</td>
<td>3.71e-02</td>
<td>1.64e-01</td>
<td>8.06e-02</td>
<td>3.52e-02</td>
<td>0/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>0</td>
<td>1.37e-01</td>
<td>6.92e-02</td>
<td>3.43e-02</td>
<td>2/50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>PSO</td>
<td>8.61e-02</td>
<td>3.47e-02</td>
<td>1.51e-02</td>
<td>1.62e-02</td>
<td>12/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>0</td>
<td>5.14e-02</td>
<td>1.23e-02</td>
<td>1.46e-02</td>
<td>10/50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>PSO</td>
<td>0</td>
<td>2.18e-02</td>
<td>7.63e-03</td>
<td>7.73e-03</td>
<td>15/50</td>
</tr>
<tr>
<td>$f_3(x)$</td>
<td>10</td>
<td>PSO</td>
<td>1.989921</td>
<td>11.93843</td>
<td>4.815644</td>
<td>2.577000</td>
<td>0/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>0.9949601</td>
<td>8.954633</td>
<td>5.487021</td>
<td>2.836235</td>
<td>0/50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>PSO</td>
<td>7.959643</td>
<td>39.79832</td>
<td>23.62486</td>
<td>8.605334</td>
<td>0/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>31.83869</td>
<td>1.04e+03</td>
<td>1.05e+03</td>
<td>3.16e+03</td>
<td>0/50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>PSO</td>
<td>27.85882</td>
<td>67.65702</td>
<td>49.05140</td>
<td>11.58457</td>
<td>0/50</td>
</tr>
<tr>
<td>$f_4(x)$</td>
<td>10</td>
<td>PSO</td>
<td>5.87e-16</td>
<td>7.73e-15</td>
<td>4.07e-15</td>
<td>8.76e-16</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>4.15e-15</td>
<td>7.72e-15</td>
<td>4.21e15</td>
<td>5.02e16</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>PSO</td>
<td>7.71e-15</td>
<td>2.13e-12</td>
<td>6.42e-12</td>
<td>4.49e11</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>7.69e-15</td>
<td>1.13e-14</td>
<td>9.39e-15</td>
<td>2.18e15</td>
<td>50/50</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>PSO</td>
<td>4.58e-13</td>
<td>1.57e-08</td>
<td>1.66e-09</td>
<td>4.85e-09</td>
<td>45/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>1.83e-14</td>
<td>3.65e-14</td>
<td>2.51e-14</td>
<td>3.66e15</td>
<td>50/50</td>
</tr>
<tr>
<td>$f_5(x)$</td>
<td>2</td>
<td>PSO</td>
<td>–1.00000</td>
<td>–0.990284</td>
<td>–0.990027</td>
<td>2.94e3</td>
<td>44/50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DPSO</td>
<td>–1.00000</td>
<td>–1.00000</td>
<td>–1.00000</td>
<td>4.48e-16</td>
<td>50/50</td>
</tr>
</tbody>
</table>

Figure 2. The convergence curves of PSO and DPSO algorithms on the test function $f_1(x)$

Figure 3. The convergence curves of PSO and DPSO algorithms on the test function $f_2(x)$
Discussion

In order to solve the problem of premature convergence of particle population and improve the performance of particle population optimization algorithm, the ideas of physics, thermodynamics and motion are applied to the optimization of PSO algorithm. Through the test function, the improvement of the population particle optimization algorithm with this idea not only improves the robustness and stability of the algorithm, but also enlarges the convergence range and effectively improves the performance of the PSO algorithm.

The diffusion mechanism of nomads is integrated with PSO algorithm. Experimental diffusion plays an important role in improving PSO. Although the thermodynamic motion mechanism is very different from the diffusion of nomads in the macro sense, its essence is the same in the micro sense. The migration of nomads is considered as a physical movement driven by the hunger for food of the animals in the pasture. In thermodynamic motion, the migration of matter is the difference between concentration, temperature, and diffusion energy. In the study of intelligent algorithms, scholars have applied the ideas of different disciplines to the design and improvement of particle population, and obtained certain results. Through the experiment of testing function, it is found that the particle population optimization algorithm based on thermodynamic motion mechanism proposed in this paper has better convergence, robustness and stability in function processing.

Conclusions

The premature convergence of the standard PSO algorithm will affect the stability and robustness of the algorithm. In order to improve the performance of PSO algorithm, the idea of thermodynamic motion mechanism is applied to PSO algorithm. The thermodynamic motion phenomenon, the law of thermodynamics diffusion, and the standard PSO algorithm are introduced, the idea of thermodynamics mechanism is adopted, and three new definitions for the computer PSO algorithm based on thermodynamics motion mechanism are obtained, which are respectively the particle diffusion energy, population temperature and particle diffusion probability. The algorithm flow is optimized according to three new definitions. Finally, the performance of the proposed computer PSO algorithm based on thermodynamic motion mechanism is tested by setting five different test functions. Compared with the standard PSO algorithm, the results show that the new algorithm proposed in this paper has a larger
iteration range, better robustness and stability. Therefore, it can be proved that the new algorithm proposed in this paper has better performance than PSO algorithm.

The research results of this paper apply the ideas in physics to the computer intelligent algorithm, providing a better idea for the development of intelligent algorithm. However, other strategies for the design of thermodynamic motion are not designed in this paper. It is hoped that the next research can improve the strategies of thermodynamic mechanism and make the research more profound and extensive.

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