NUMERICAL SIMULATION AND ANALYSIS OF PHASE CHANGE HEAT TRANSFER IN CRUDE OIL

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Accurately obtaining the temperature distribution of the medium in the shutdown pipeline of waxy crude oil has important guiding significance for making maintenance plan and restart plan. The phase transition process of waxy crude oil involves complex problems such as natural convection heat transfer, latent heat release, difficulty in tracing liquid-solid interface. In this paper, the concept and significance of breaking point were proposed. Taking the breaking point and the freezing point as dividing point, and a new zonal partition model was established based on the influence of phase change of crude oil wax crystal on heat transfer mode, with the corresponding governing equations being established for different regions. With the proposed model, the effects of natural convection on heat transfer, latent heat release, location change of condensate reservoir, heat transfer mechanism and other key issues in the process of oil phase transition were analyzed.

Key words: phase change; waxy crude oil; heat transfer; numerical simulation; condensate reservoir

1. Introduction

Waxy crude oil has a wide temperature range because of its complex wax composition. It is important to describe the heat transfer characteristics in the process of phase transition and to obtain the temperature distribution of crude oil at different stoppage times. The research of wax and other phase change materials has been relatively mature [1–5], but for the complex component of crude oil, the phase change heat transfer research needs to be further developed.

How to describe the natural convection heat transfer in the process of phase transition is one of the key problems in obtaining accurate temperature field [6-9]. In the early models
natural convection heat transfer was neglected, and the liquid-solid two-phase partition was adopted, and an equivalent thermal conductivity equation was established in the liquid phase region, the influence of natural convection on heat transfer process cannot be analyzed by this method. For the liquid-solid two-phase partition model, latent heat was considered to be completely released at an infinitely thin phase interface [14-15].

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With the development of DSC technology, latent heat was transformed into variable specific heat capacity, and equivalent specific heat capacity is used to describe the heat transfer process in the governing equation [16-20]. This treatment of latent heat of phase change was a reasonable and feasible way to accurately represent the latent heat release at different temperatures.

In fact, after the temperature of wax-containing crude oil drops to the wax point, wax precipitation and latent heat release begin. In the initial stage, a small amount of wax crystals are suspended in the liquid phase crude oil. Subsequently, when the amount of wax crystals increases continuously, a grid structure will be formed through gelling [10, 21-26]. Therefore, there is a liquid-solid mixed fuzzy zone in the process of phase transition in the pipeline, which can be regarded as porous medium. For this fuzzy region, most scholars used enthalpy-porous media model to describe heat transfer and flow [27-30]. In the above models, Darcy formula was used to describe the natural convection of liquid phase in the pores. And the release of crude oil latent heat of wax appearance was considered as occurring in a narrow temperature zone near the freezing point [29-30]. However, there was no explanation regarding determination of these temperature ranges.

When the temperature of crude oil is lower than the abnormal point, the crude oil has Non-Newtonian characteristics, and the viscosity of crude oil varies significantly with temperature and shear rate [31-33]. For the Non-Newtonian properties, most scholars used the viscosity change characterization [15, 20]. In Xu’s models, momentum equations were established respectively for Newtonian fluid and non-Newtonian fluid, with the influence of crude oil viscosity change on the shutdown process being analyzed, and the final results show that the influence of viscosity change on the shutdown pipeline can be ignored [34].

In this paper, combined with the influence of the phase change of wax crystal on the heat
transfer process in the process of wax evolution in crude oil, a new zonal mathematical model will be established to describe the heat transfer characteristics of crude oil phase transition more accurately. Based on the new model, the effects of natural convection on heat transfer, latent heat release, location change of condensate reservoir, heat transfer mechanism and other key issues in the process of oil phase transition will be analyzed in depth, so as to enrich the heat transfer mechanism of oil.

2. Physical model

The physical model of an overhead pipeline in shutdown was shown in the figure 1. From the inside to the outside, there were crude oil, steel pipe and insulation layer.

![Physical model for an overhead pipeline](image)

Fig. 1 Physical model for an overhead pipeline

2.1 Phase change and heat transfer of wax crystal

In the process of crude oil cooling, the phase state of crude oil can be divided into four stages: (1) when the oil temperature is higher than the wax point, the crude oil in the whole pipeline is liquid phase crude oil, which is in the stage of natural convection heat transfer. (2) When the temperature of crude oil is lower than the wax point, a small amount of wax crystals will be precipitated out and suspended in the liquid phase. At this time, due to the small amount of wax crystals, the presence of wax crystals will not affect the change of heat transfer mode of the main body in the pipeline. (3) When the wax crystal precipitates about 2%, it will form a cementitious structure [35-36]. At this time, the crude oil can be regarded as porous medium, and the heat transfer mode is natural convection heat transfer in the liquid phase crude oil in the void, and the wax crystal is mainly thermal conduction mode. (4) When the temperature of crude oil drops to the freezing point, it can be regarded as a pure solid phase, and its heat transfer mode is heat conduction.

2.2 Definition and significance of breaking point

It can be seen from the above analysis that the heat transfer modes in the first and second stages are natural convection heat transfer, which can be described by the same model.
Therefore, they are all regarded as “liquid phase regions”. When the amount of wax crystal increases to a grid structure, the corresponding temperature is “the breaking point temperature”, and it can be regarded as the temperature point at which the porous medium is formed.

2.3 Assumptions

In order to simplify the calculation, there were some assumptions as follows: (1) Neglecting viscous dissipation in the fluid; (2) The medium in the pipe is incompressible fluid; (3) At the beginning of shutdown, the temperature of crude oil is consistent throughout the pipeline.

3. Mathematical model

3.1 Liquid zone

The governing equations in the liquid phase region include the continuity equation (1), the momentum equation (2) and the energy equation (3).

\[ \frac{\partial \rho}{\partial t} + \text{div} (\rho \mathbf{u}) = 0 \]

\[ \frac{\partial (\rho \mathbf{u})}{\partial t} + (\rho \mathbf{u} \cdot \nabla) \mathbf{u} = - (\nabla P) + (\nabla \cdot \mathbf{\tau}) + \mathbf{F} \]

\[ \frac{\partial (\rho \mathbf{u})}{\partial t} + \text{div}(\rho \mathbf{u} \cdot \mathbf{u}) = - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \mathbf{F}, \]

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + \text{div}(\rho \mathbf{v} \cdot \mathbf{u}) = - \frac{\partial P}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \mathbf{F}, \]

\[ \tau_{xx} = 2\mu \frac{\partial \mathbf{u}}{\partial x} + \lambda \text{div}(\mathbf{u}) \]

\[ \tau_{yy} = 2\mu \frac{\partial \mathbf{v}}{\partial y} + \lambda \text{div}(\mathbf{u}) \]

\[ \tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial \mathbf{u}}{\partial y} + \frac{\partial \mathbf{v}}{\partial x} \right) \]
\[ \lambda = \frac{-2}{3} \mu \]  \hspace{2cm} (2-f)

In the momentum equation (2), \( F \) is the source term of the external force, which includes the components in the X and Y directions, \( F_x = 0 \), and \( F_y = -\rho g \).

\[ \frac{\partial (\rho \epsilon T)}{\partial t} + (\rho \epsilon u \cdot \nabla T) = \lambda \nabla^2 T \]  \hspace{2cm} (3)

When oil temperature is higher than the wax precipitation point, \( \lambda_e = \lambda_s \), and when the oil temperature is between breaking point and wax precipitation point, \( \lambda_e \) is determined by the following formula:

\[ \varepsilon = 1 - \frac{T_o - T}{T_i - T_o} \]  \hspace{2cm} (3-a)

\[ \phi = 1 - \varepsilon \]  \hspace{2cm} (3-b)

\[ \lambda_e = \lambda_s \cdot \frac{2 + \lambda_s / \lambda_i + 2\phi(\lambda_s / \lambda_i - 1)}{2 + \lambda_s / \lambda_i - \phi(\lambda_s / \lambda_i - 1)} \]  \hspace{2cm} (3-c)

3.2 Solid zone

The governing equation of the solid phase region is the energy equation, as shown in the formula (3), where \( \lambda_e = \lambda_s \).

3.3 Liquid-solid mixed porous media zone

In the porous media range, Brinkmann-Forchheimer-Darcy porous media seepage model is adopted, where, permeability (\( K \)), shape factor (\( F_s \)) and equivalent thermal conductivity (\( \lambda_e \)) are determined by liquid phase rate calculation.

\[ \frac{\partial (\rho \epsilon u)}{\partial t} + (\rho \epsilon u \cdot \nabla) u \frac{\epsilon}{\rho} = -\nabla(\rho P) + \mu \nabla^2 u + F \]  \hspace{2cm} (4)

\[ F = -\frac{\varepsilon u \rho}{K} - \frac{\varepsilon F}{\sqrt{K}} |u| + \varepsilon \rho g \beta (T - T_o) \]  \hspace{2cm} (4-a)
\[ F_e = \frac{1.75}{\sqrt{150\varepsilon}} \]  

(4-b)

\[ K = \frac{\varepsilon}{C(1-\varepsilon)^3} \]  

(4-c)

\[ \dot{\lambda}_i = (1-\varepsilon)\dot{\lambda}_x + \varepsilon \dot{\lambda}_i \]  

(4-d)

3.4. Piping and insulation

The heat transfer equations of steel pipe and insulation material are shown below,

\[ \rho c, \frac{\partial T}{\partial t} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad r_1 \leq \sqrt{x^2 + y^2} \leq r_2 \]  

(5)

\[ \rho c, \frac{\partial T}{\partial t} = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad r_2 \leq \sqrt{x^2 + y^2} \leq r_i \]  

(6)

3.5 Boundary and initial conditions

\[ -\dot{\lambda}_i \frac{dt}{dr} \bigg|_{r_i} = h(T - T_j) \quad \sqrt{x^2 + y^2} = r_i \]  

(7)

\[ T \bigg|_{r_i} = \text{const} \tan t \]  

(8)

4. Verification and analysis

The SIMPLE algorithm was adopted for simulation. The independence of the grid number and time step was verified. First, three models with grid numbers of 10100, 29765 and 59311 were simulated, and the curve of the average temperature of crude oil in the tube with time for different grid numbers was compared. The results were as followed in figure 2. It can be found that the temperature drop curve with the number of grids of 10100 was obviously different from other curves, and the temperature drop curves of crude oil with the number of grids of 29765 and 59311 almost overlap, so it can be determined that 29765 was the appropriate number of grids. The calculation accuracy of the model had reached the maximum, and increasing the number of grids basically had no effect.
Then, the model was verified with time step for the grid number of 29765, and the curves of the average temperature in the pipeline with time steps of 60s, 40s, 20s and 10s were compared, as shown in figure 3. The temperature drop curve showed a clear tendency to converge as the time step decreases, until the time step of 20s. Also, the temperature drop curve at the step of 20s and the step of 10s basically overlap, so it can be determined that 20s was the appropriate time step of the 29765 grid number model.

The pipe was bare without insulation layer, and the air temperature was held constant at 290 K. The experimental crude oil was from the oil field site in Daqing, with the wax precipitation temperature of 319 K, the breaking point of 313 K and the freezing point of 305 K. Physical parameters of crude oil steel pipe were listed in table 1. The experimental flow diagram was shown in the figure 3.
The crude oil was heated to 333 K in a heating tank and then entered a 0.5m long experimental tube with a 60mm insulation material at both ends. Automatically closed the tank valve when the level gauge showed full tube. Two monitoring positions were arranged on the pipeline, and five monitoring points were set at each position. The external environment of the pipeline was at a constant temperature of 290 K. Heating tank temperature control device, thermocouple, level gauge, temperature sensor and other instruments all had been calibrated, the test accuracy can reach ±2%. The experimental results were taken as the arithmetic mean value of the corresponding positions at the two monitoring points. Shutdown process under experimental conditions was calculated by proposed model. The experimental and simulated results of crude oil temperature at typical locations were shown in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil density $\rho$</td>
<td>kg/m$^3$</td>
<td>$\rho = 0.902 - 8.177 \times 10^{-4} (T - 273.15) + 1.54 \times 10^{-6} (T - 273.15)^2$</td>
</tr>
</tbody>
</table>
| Oil thermal conductivity $\lambda$ | W/(m·K)  | $\lambda_i = 0.15, \quad T > 315K$  
$\lambda_v = 0.25, \quad T < 310K$ |
Oil dynamic viscosity $\mu$ (Pa·s)

For Newtonian fluid:

$$\mu = 10^{5.06039 \pm 0.0195^\circ F} \cdot T \geq T_1$$

For non-Newtonian fluid:

$$\mu = \begin{cases} 10^{13.30785 \pm 0.1223^\circ F}, & T_1 \leq T < T_2 \\ 10^{14.44974 \pm 0.051^\circ F}, & T_2 \leq T < T_3 \\ 10^{20.81207 \pm 0.0679^\circ F}, & T_3 \leq T < T_4 \end{cases}$$

$$T_1 = 313.15 \text{ K}, \quad T_2 = 298.15 \text{ K}, \quad T_3 = 296.15 \text{ K}, \quad T_4 = 292.15 \text{ K}.$$ 

Specific heat of oil $c_p$ (J/(K·kg))

$$c_p = \begin{cases} 2140 & T \geq 322 \text{ K} \\ -35.8T + 13667.5 & 303.15 \text{ K} \leq T < 322 \text{ K} \\ 18.33T - 2737.75 & 273.15 \text{ K} \leq T < 303.15 \text{ K} \end{cases}$$

Density of steel pipe $\rho$ (kg/m³)

7850

Specific heat of steel pipe $c_p$ (J/(K·kg))

500

Conductivity of steel pipe $k$ (W/(m·K))

48

Dimensions/size of steel pipe

219×7

### Table 2 Comparison between the test data and calculated results

<table>
<thead>
<tr>
<th>time/min</th>
<th>5point Cal</th>
<th>5point test</th>
<th>5point relative error</th>
<th>3point Cal</th>
<th>3point test</th>
<th>3point relative error</th>
<th>2point Cal</th>
<th>2point test</th>
<th>2point relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>56.1</td>
<td>55.3</td>
<td>1.42%</td>
<td>58.3</td>
<td>58.8</td>
<td>-0.85%</td>
<td>59</td>
<td>59.5</td>
<td>0.84%</td>
</tr>
<tr>
<td>20</td>
<td>51.2</td>
<td>49.4</td>
<td>3.51%</td>
<td>55.3</td>
<td>54.4</td>
<td>1.62%</td>
<td>56.6</td>
<td>56.1</td>
<td>0.88%</td>
</tr>
<tr>
<td>40</td>
<td>47.4</td>
<td>46.9</td>
<td>1.05%</td>
<td>50.7</td>
<td>50.0</td>
<td>1.38%</td>
<td>52.2</td>
<td>51.3</td>
<td>1.72%</td>
</tr>
<tr>
<td>60</td>
<td>43.6</td>
<td>44.1</td>
<td>-1.14%</td>
<td>46.8</td>
<td>46.9</td>
<td>-0.21%</td>
<td>48.5</td>
<td>47.8</td>
<td>1.44%</td>
</tr>
</tbody>
</table>
Table 1 presented the comparison of the oil temperature profiles at positions 2, 3, and 5. It can be found that the deviation between the calculated results and the test results was satisfactory, with the max relative error of 3.51%. The figure 4 was the comparison between the test data and calculated results at the pipe center. According to the figure, the two curves were in good agreement. Next, the proposed model was used for numerical simulation to analyze the phase change heat transfer process of crude oil.

![Fig.4 Comparison of temperature drop curve of pipeline center point](image)

**4.1. Effect of natural convection on heat transfer under different pipe diameters**

The change of average Ra number in the shutdown process of overhead pipelines with different diameters was shown in the figure 5. \( \text{Ra} = \text{Gr} \cdot \text{Pr} \), which can indicate the degree of natural convection.
As can be seen from the figure 5, the trends of the two curves were consistent, in the initial stage of shutdown, Ra rapidly increased to the peak, and then gradually decreased. The larger the pipe diameter, the higher the peak value of Ra, this was due to the large diameter pipeline, its natural convection space was larger, more fully developed. The peaks of the two pipe diameters were between $10^8$ and $10^9$, it meant that at the initial stage of shutdown, the natural convection was in the transition stage of laminar flow turbulence.

When the pipeline was shutdown for 20 min, the distribution of temperature field and flow field in the pipeline were shown in the figure 6. For large-diameter pipelines, the internal crude oil content was large and the heat storage capacity was large, so the overall temperature distribution in the pipeline was higher. For $\Phi$ 800 mm pipe, the vortex area was larger, the natural convection was more pronounced, and the effect of natural convection lasted longer.
4.2 Change in heat flux

The figure 7 showed the heat flux at different positions of the pipe wall.

![Fig.7 Change in heat flux](image)

Because the temperature distribution in the pipe decreased gradually from top to bottom, the maximum heat flux was at the top of the pipe and the minimum was at the bottom wall. Due to the pipe flow field and temperature field on the Y axis approximate symmetrical distribution (as shown in the figure 5), and, so the heat flux density curves at 45° and 135° coincided, and the pattern was the same at 225° and 315°. Although the curve peaks at different positions were different, the overall change trend was consistent, which can be divided into three stages: (1) The period of severe fluctuation was also the initial stage of shutdown (the red part of the coil in the picture). At this time, the mixing of cold and hot fluids was severe, and the effect of natural convection was the most significant. (2) As the influence of natural convection was weakened, the heat flux dropped sharply and then rised slightly, because the latent heat release increased the heat transfer temperature difference. (3) Since then, with the appearance of condensate reservoir, the heat flux tended to be consistent at different locations.

4.3 temperature drop and heat transfer characteristic

The temperature distribution on the Y-axis of the pipeline at different shutdown times were shown in the figure 8.
As can be seen from the figure 8, the highest temperature point was in the positive Y-axis, while the lowest temperature point was in the lowest Y-axis at the same shutdown time, this was due to natural convection, with hot oil moving up and cold oil moving down. With the increase of shutdown time, condensate layer appeared in the inner wall of the pipeline, and the liquid crude oil gradually decreases, so the maximum temperature point was gradually close to the center of the pipe, but the final freezing point was above the center of the pipeline. When the influence of natural convection can be ignored, the temperature distribution in the pipe was approximately symmetric about the Y-axis.

The crude oil freezing point was 305K, and the above the dotted lines in the figure 8 were all liquid crude oil regions. The graph can be used to determine the position of condensate reservoir on the Y-axis at a typical shutdown time. For example, when the transmission was shutdown for 1h, the temperature of the positive half of the Y-axis was all higher than 305K, while the temperature of the crude oil from about 85mm of the negative half axis to the bottom of the pipe wall was all lower than the condensation point, and the thickness of the condensate reservoir was about 19mm. When the shutdown lasted for 5 hours, the temperature curve on the entire Y axis was below 305K, and the crude oil in the pipeline was all solidified. The above conclusions can be further proved by the figure 9, which was solidified cloud picture at different shutdown time, and the blue part represented solidified crude oil.

The curves were spaced 0.5 h apart, so the density can reflect the temperature drop rate at the corresponding stage. Obviously, the temperature drop rate was the highest at the initial moment of shutdown, this is because the temperature difference between inside and outside the pipeline was the largest at this stage, and there was natural convection. For the center of
the pipe, namely \( Y=0 \), the temperature drop rate went through a process from large to small and then increased. The curve circled by the ellipse showed a marked increase in density due to the release of oil latent heat.

Fig.9 Solidified cloud picture at different shutdown time

The figure 10 was the temperature drop curves at typical position on \( Y^+ \) axis.

Fig.10 Temperature drop curve at typical position on \( Y^+ \) axis

From the figure 10 we can see that the slope change of temperature drop curves can be divided into four sections. In the first stage, the curve had the highest slope and the fastest temperature drop rate. In the second stage, the rate of temperature drop was slowed down due to latent heat release, and the slope was obviously reduced. In the third stage, the influence of latent heat was exhausted and the rate of temperature drop increased. As the temperature difference decreased and the thickness of condensate reservoir increased, the temperature
drop rate was lower than that in the first stage. In the fourth stage, the crude oil in the whole pipeline had all solidified, and the temperature difference was getting lower and lower, and the thermal resistance was large, resulting in a very small slope of the temperature drop curve, which was finally 0.

According to the curves in the figure 9, for different positions, the latent heat influence duration in the second stage of the temperature drop curve were different. The thermal resistance at the pipe wall was the minimum, so the latent heat released by wax precipitation was much smaller than the heat lost to the outside of the pipe. Compared with other positions, the slope in the second stage was extremely high, and the temperature at this point dropped rapidly to the ambient temperature. According to the comparison of curves, the closer the center was, the more significant the influence of latent heat was.

5. Conclusions

A new partition model was established, with the latent heat being treated as an additional specific heat capacity. Compared with the experimental results, the maximum relative error was 3.12%, and the agreement was good.

1. For overhead pipelines with different diameters, the changes of flow field, temperature field and Ra number in the pipeline after shutdown were analyzed. The results showed that natural convection develops more fully and the influence of convection heat transfer lasted longer.

2. Due to the approximate Y-axis distribution of the flow field and the temperature field, the heat flux at the symmetric points on both sides of the Y-axis was approximately equal. Because of natural convection, the heat flux in the whole pipe decreased gradually from top to bottom. The trend of heat flux curve reflected the influence of natural convection and latent heat release.

3. The heat transfer process and characteristics of phase change were described by the temperature distribution curve of Y axis and the solidification cloud diagram. With the extension of shutdown time and the increase of condensate thickness, the highest temperature point in the pipeline was gradually close to the center of the pipeline, but the final freezing point was above the center of the pipeline.

4. The temperature drop curves at typical positions on Y axis were used to analyze the influence of latent heat release on temperature drop rate. The results showed that the closer to the pipe wall, the smaller the thermal resistance was, the faster the heat dissipation was, and the less obvious the influence of latent heat was.
Acknowledgment

This work was supported by the National Natural Science Foundation of China (no.51534004), and Youth Science Foundation of Northeast Petroleum University (2018GPZD-01 and 2018QNL-15).

Nomenclature

$\rho$  oil density, kg/m$^3$  
$t$  shutdown time, h  
$U$  kinematic velocity, m/s  
$P$  apparent stress, Pa  
$F$  force, N  
$\mu$  kinematic viscosity, kg/(m·s), (pa·s)  
$g$  gravitational acceleration, m/s$^2$  
$T$  oil temperature, K;  
$\lambda_l$  thermal conductivity of liquid crude oil, W/(m·K)  
$c_p$  specific heat capacity of crude oil, J/(kg·K)  
$c_s$  specific heat capacity of insulating layer, J/(kg·K)  
$\beta$  expansion coefficient, l/K  
$\phi$  solid fraction  
$\rho_i$  insulating layer density, kg/m$^3$  
$\rho_p$  pipe density, kg/m$^3$  
$c_{i,p}$  specific heat capacity of pipe, J/(kg·K)  
$r_i$  pipe radius, m  
$\lambda_i$  pipe thermal conductivity, W/(m·K)  
$\lambda_s$  thermal conductivity of insulation, W/(m·K)  
$\varepsilon$  liquid fraction  
$F_s$  shape factor  
$K$  permeability  
$C$  coefficient, C=10$^4$–10$^7$  
$T_{ref}$  reference temperature, K  
$\lambda_e$  effective thermal conductivity,  W/(m·K)  
$\rho_c$  specific heat capacity of crude oil, J/(kg·K)  
$\lambda_{i,p}$  thermal conductivity of insulation, W/(m·K)  
$\lambda_{t,\lambda}$  effective thermal conductivity,  W/(m·K)  
$\rho_{i,p}$  pipe density, kg/m$^3$  
$\phi_{i,p}$  pipe radius, m  
$\lambda_{i,p}$  effective thermal conductivity,  W/(m·K)  
$\lambda_{s,i}$  thermal conductivity of insulation, W/(m·K)
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