

MECHANICAL GRINDING METHOD OF MULTI-AXIS LINKAGE FOR ELLIPTIC CONTOUR BASED ON ALGEBRAIC GEOMETRY

by

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Aiming at the shortcomings of traditional elliptic contour grinding methods, such as large grinding roughness and poor contour accuracy, a multi-axis mechanical grinding method based on algebraic geometry is proposed. This method studies the first attempt of elliptic contour. The process includes selecting suitable multi-axis grinding tools and selecting appropriate grinding methods according to grinding process. Using specific workpieces, grinding path planning, motion attitude determination and error compensation based on algebraic geometry for multi-axis mechanical grinding are carried out. The results show that compared with the traditional grinding method, the device has the smallest surface roughness, the contour radian is closer to the expected results, and has higher performance, which solves the grinding defects.

Key words: *algebraic geometry, elliptical contour, multi-axis linkage, mechanical grinding*

Introduction

The rapid development of modern industry has improved the performance and quality requirements for electromechanical products. The manufacturing precision requirements of mechanical parts are getting higher and higher, and the shape is becoming more and more complicated. For example, the transverse section of the piston of automobile engine is designed as an ellipse, and the ellipse of the transverse section varies in the direction of piston skirt height, the mechanical shaft also has a rhombus which is not cylindrical but consists of several arcs. For the processing of non-circular parts with various kinds, high precision and high frequency servo control requirements, the traditional processing scheme cannot meet the requirements. With the continuous progress of spindle unit technology, feed unit technology, detection and control technology, the conditions for precise grinding of ellipse and other non-circular parts are becoming more and more mature, which has become a hot research topic in developed industrial countries all over the world. Therefore, it is of great practical value to focus on the high-speed and high-precision grinding of elliptical contour parts [1].

At present, for the processing of elliptical contour parts of medium and small calibers, foreign processing technology has been relatively mature and industrialized, and mass production can be carried out. However, domestic processing technology is still just starting and there are still many problems to be solved. Molding techniques or injection molding techniques can be selected to process elliptical contoured devices, typically with less stringent processing requirements for elliptical profiling devices. The biggest disadvantage of these two types of

grinding technology is the lack of precision, which makes the roughness of the device after grinding larger [2, 3]. For large aspheric surface processing, the main method is grinding before polishing, which increases the probability of error and takes a long time to polish, greatly reducing the efficiency of processing [4]. In order to improve the smoothness of elliptical contour parts, it is necessary to improve the surface accuracy of the workpiece and effectively control the surface damage layer in the grinding process.

In order to solve the above problems, the principle of algebraic geometry is applied, and a new multi-axis mechanical grinding method of elliptic contour based on algebraic geometry is proposed. In this method, while grinding workpiece with multi-axis linkage grinding tool, the error compensation method based on algebraic geometry is used to repair the error and improve the grinding accuracy [5, 6]. It has been proved that compared with the traditional grinding method, the elliptical contour grinding of the device has been carried out by this method. The surface roughness of the device is the smallest, and the contour curvature is closer to the expected result, which shows that the method has higher performance and solves the problems of the conventional method.

Mechanical grinding technology of multi-axis linkage for elliptical contour

Mechanical grinding of elliptical contours is a form of machining that simultaneously grinds and polishes the surface of a workpiece using a special form of flexible coating tool. In

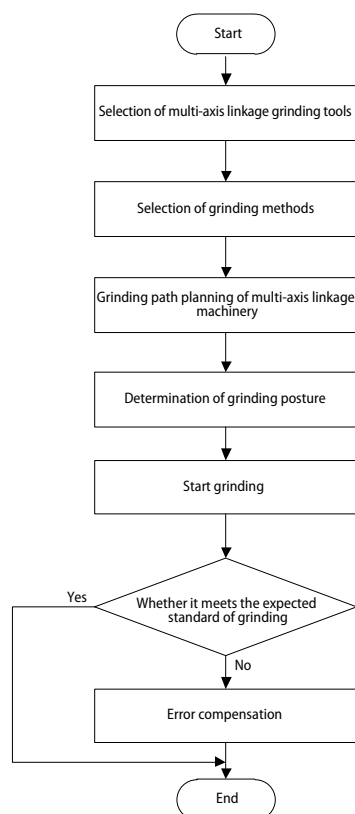


Figure 1. Flow chart of mechanical grinding process for multi-axis linkage

the process of grinding with abrasive belts, the flexible contact between the belt and the workpiece enables the belt to closely conform to the complex surface of the workpiece, so mechanical grinding is ideal for machining workpieces with complex surfaces such as blades and faucets [7].

Compared with other machining processes, multi-axis linkage grinding has a very superior processing performance in the finishing of complex surface workpiece. When abrasive belt is used as a tool for surface finishing, the traditional NC milling machine generally adopts five-axis series structure to realize the finishing movement between tool and workpiece [8, 9]. The grinding of elliptical contour devices by means of multi-axis linkage is of great significance to the research, development and application of its machining accuracy, efficiency and NC programming system. Figure 1 is the flowchart of the multi-axis linkage grinding process for elliptical contour.

Mechanical grinding tool of multi-axis linkage

According to the principle of curvature matching, the B-axis is rotated to match the curvature of the workpiece with the workpiece, and the x , y , z , and C axes are controlled. Through the five-axis linkage, the complex contour of the stone carving can be processed most efficiently [10].

The axis direction of the contact wheel in NC abrasive belt grinding is parallel to a tangent direction of the surface at the processing point. Therefore, the efficiency,

error and surface quality of abrasive belt grinding are directly related to the width of abrasive band and the diameter of contact wheel. At the same time, grinding linear speed, feed speed, floating pressure and cutting line spacing have different effects on grinding efficiency and quality [11].

The width and length of the belt are related to the blade grinding efficiency and the design of the belt movement mechanism of the machine tool, which should be the primary object of the belt selection technology. Under the conditions of machine power, the larger the belt width, the higher the grinding efficiency. However, nuclear power blades and steam turbine blades are generally twisted blades, and the shape of the curved surface is complex. The wider the abrasive belt, the greater the amount of grinding interference, which affects the grinding precision and surface quality [12]. The length of the belt is as long as the machine space allows, to reduce the frequent replacement time of the belt. Table 1 is a comparison of the performance of several commonly used belts.

Table 1. Performance comparison of common abrasive belts

Model	577F	XK850X	SK850X	KK712 J
Specifications	15 mm × 330 mm	10 mm × 450 mm	20 mm × 520 mm	25 mm × 550 mm
Condition	22000 rpm	25000 rpm	25000 rpm	22000 rpm
Characteristic	The grinding efficiency is high, the effect of the workpiece is good, the service life is long, and the interface is not easy to break.	The grinding efficiency is super fast, the cutting quantity is large, the service life of SK850X is 5 times that of 577F abrasive belt, the quality is stable and the price is moderate.	Grinding efficiency is more than 25% higher than that of general gravity sand planting belt, and the surface roughness is uniform and better.	Abrasive belt grinding accuracy is high, the highest accuracy has reached less than 0.1 mm; the quality of abrasive belt is light; the rigidity and strength requirements of machine tools are low.

In addition to the width and length of the abrasive belt, the abrasive material and particle size also have a great influence on the surface quality and grinding efficiency of the blade.

The choice of abrasive grain size should be based on different grinding requirements, considering machine tool performance and specific processing conditions (such as workpiece processing allowance, surface condition, material, heat treatment, accuracy, roughness, etc.) to select different grain size of abrasive belt.

According to the grinding mechanism, the choice of belt linear speed is greatly influenced by the material of the workpiece. When the same material is grinded at different linear speeds, the shape of debris, grinding heat and material removal rate are different. At the same time, it has a great influence on the surface quality of device grinding and the degree of abrasive belt wear. The linear velocity of abrasive belt is mostly determined by experience. When grinding elliptical contour device of stainless steel material, the linear velocity of abrasive belt is generally set to 20-30 m/s; when grinding elliptical contour device of Qin alloy material, the linear velocity of abrasive belt is generally set to 85-90 m/s.

Grinding mode selection

There are three main types of grinding used in mechanical grinding: longitudinal grinding, lateral grinding, and helical grinding, as shown. In the longitudinal grinding, the contact area between the contact wheel and the blade surface is small, so that the processing bandwidth of each grinding path is narrow. Therefore, in order to ensure the surface processing

quality, the grinding track density must be increased, resulting in low processing efficiency. However, since the calibration accuracy of the tool co-ordinate system of the contact wheel during longitudinal grinding has little effect on the machining quality, the longitudinal grinding has a good processing quality [13]. In the transverse grinding, the relative movement direction between the contact wheel and the blade is perpendicular to the width direction of the contact wheel, so the processing bandwidth of each path is equal to the width of the contact wheel, which can significantly reduce the track density and greatly improve the processing efficiency. However, due to the wide processing belt, it is easy to have different grinding depths on the same path, so the machining accuracy of this grinding method is relatively low [14]. When the spiral is ground, it also has a large processing bandwidth, and the processing method of the spiral can reduce the number of lifting and improve the processing efficiency. However, this method also has the problem of uneven grinding depth, and the curvature of the blade profile changes greatly when grinding the inlet and outlet edges, resulting in a sharp change in the joint angle of the robot and a large acceleration, which increases the elliptical contour device. The jitter further reduces its grinding accuracy.

Path planning

In the machining of CNC machine tools, the design of the running path of the model tool is the basis for the precise machining of the tool to the workpiece. The trajectory is closely related to the tool and the processing equipment. Therefore, the mechanical grinding path planning is of great significance. Before the mechanical grinding path planning, the following basic concepts related to the tool path need to be clarified [15]:

- *Grinding point curve*: Refers to the curve formed by the grinding point during the machining process. Grinding point curves are the basic elements of generating tool trajectories, either explicitly defined on the surface of the workpiece or implicitly defined to meet certain conditions. The grinding point curve can be a real curve on the curve or a *virtual* curve implied by the constraints on the grinding point.
- *Tool path curve*: Refers to the curve formed by the grinding point during the grinding process, that is, each point on the curve contains a tool axis vector. The grinding tool path curve can be calculated from the grinding point curve based on the tool offset [16-18].
- *Guide rule*: Refers to the method of generating the grinding point curve on the curved surface or the tool path curve on the equidistant surface and some related parameters of the machining precision, such as the step size and the line spacing.

Determination of motion attitude

In determining grinding attitude, it is necessary to ensure good fit between device surface and contact wheel, sufficient contact pressure, simplify algorithm and improve efficiency. The bonding between the device surface and the contact wheel will directly affect the final processing quality. Good bonding effect can evenly remove the surface allowance of the device and improve the processing quality. If the bonding effect is not good, it is not only difficult to guarantee the uniformity of grinding quantity, but also prone to interference leading to device scrap.

Considering the aforementioned factors, when determining the attitude of the target point co-ordinate system, the z -axis of the co-ordinate system coincides with the normal, n , of the surface at the target point, and the tangent direction, a , of the u parameter line at the target point coincides with the direction of the y -axis. According to the right hand, the direction of the x -axis can be determined. Namely:

$$\begin{aligned}x &= yz \\y &= a \\z &= n = ab\end{aligned}\tag{1}$$

where a and b represent the tangent vectors of u -parametric lines and v -parametric lines on NURBS surfaces, respectively.

The tool co-ordinate system of the grinding machine is built on the outer edge of the contact wheel. The direction of the supporting axis of the grinding wheel is taken as the z -axis direction of the tool co-ordinate system, and the y -axis direction is parallel to the axis direction of the contact wheel. The direction of the x -axis is determined according to the right hand.

According to the principle of establishing the target point co-ordinate system and tool co-ordinate system, and the requirement of overlapping the two co-ordinate systems in grinding process, the relationship between device characteristics and contact wheel in grinding process is discussed.

Therefore, the relationship between tool co-ordinate system, target point co-ordinate system, workpiece co-ordinate system and grinding tool can be obtained:

$$T1 = T2 \cdot T3 \cdot T4 \cdot T5\tag{2}$$

where $T1$ represents the transformation matrix of the grinding tool co-ordinate system, $T2$ – the transformation matrix of the track target co-ordinate system, $T3$ – the transformation matrix of the target co-ordinate system, $T4$ – the transformation matrix of the workpiece co-ordinate system, and $T5$ – the transformation matrix of the end of the grinding tool relative to the grinding tool co-ordinate system.

According to the known $T3$, $T4$, and $T5$, the position and posture information of the grinding tool in grinding the target point can be determined.

$$T0 = T3 \cdot T4^{-1} \cdot T5^{-1}\tag{3}$$

During the movement, the tool co-ordinate system of the contact wheel is sequentially coincident with the co-ordinate system of the target point, thereby realizing the control of the grinding motion of the grinding tool.

Error compensation based on algebraic geometry

In the mechanical grinding process of multi-axis linkage, the tool path of the machining can be any free curve in the space. The movement of the tool in the numerical control system is actually the combined motion of each motion axis and each component, and the movement of the tool relative to the workpiece is actually a segment. A small straight line, the error between these straight line segments and the original curve is the machining error, and the final result is the relative position error between the workpiece and the tool, collectively referred to as geometric error. Therefore, in order to reduce the lift error and improve the machining accuracy, the error compensation programming based on the algebraic geometry principle is needed to realize the automatic repair of the elliptical contour error.

If it is used in actual processing, because the overall quality of the grinding wheel rack will be relatively large, instantaneous speed and acceleration will usually produce a large jump, so that the response of the grinding machine cannot meet the requirements, resulting in lateral tracking error. The direct result is the cam profile error. If the situation is worse, it will form obvious corrugated or prismatic surface on the surface of the camshaft. The surface quality of the workpiece is very poor. Therefore, it is very important to ensure that the virtual lift curve

obtained by pre-compensation can be smooth, and that the transverse velocity and acceleration generated by the virtual lift curve can be guaranteed in the maximum response range of the grinding machine system.

Analysis of simulation experiment

Simulated user interface

Firstly, the simulated user interface is designed, and the GUI interface is created by using GUIDE of MATLAB. In the menu bar of MATLAB, click File-> New-> GUI, or click the shortcut icon GUIDE to create a new blank GUIDE template, set the interface attributes, and add the required control components to the template, such as co-ordinate axis, combo box, edit box, static text and button control.

Arrange the position of each control according to the need, right-click the control, select *Property Inspector*, set the properties of each component, beautify the simulation interface.

Testing equipment

Form Talysurf PGI 1240 Surface Roughness Profiler of Taylor Hopson Company is used to measure the surface roughness of workpiece, and the grinding accuracy is tested by MATLAB software.

Grinding process experiment

Firstly, grinding process for elliptical contour of the SiC ceramic was carried out on an electroformed diamond grinding wheel on the machine tool to study the surface roughness of SiC ceramics during end grinding. The process parameters used in the experiment are shown in tab. 2, where the bold parameter is the base value of the parameter when other parameters change.

Table 2. Process parameters

Parameter	Numerical value
Feed speed [$\text{mm} \cdot \text{min}^{-1}$]	100
Grinding depth [μm]	5
Spindle speed [rpm]	5500

Test results

The surface roughness measurement result (R_a value) measured by the profilometer after the workpiece is ground is shown in tab. 3.

It can be seen from tab. 3 that, under the application of the method, the surface roughness R_a is minimum after the device is ground, and thus the surface of the device is smoother, indicating that the method has higher performance.

Table 3. Surface roughness results of the device

Methods	This method	Literature [2] method	Literature [3] method	Literature [4] method
Surface roughness	0.5712	0.7241	0.7921	0.7121

Table 4. Goodness of fit results

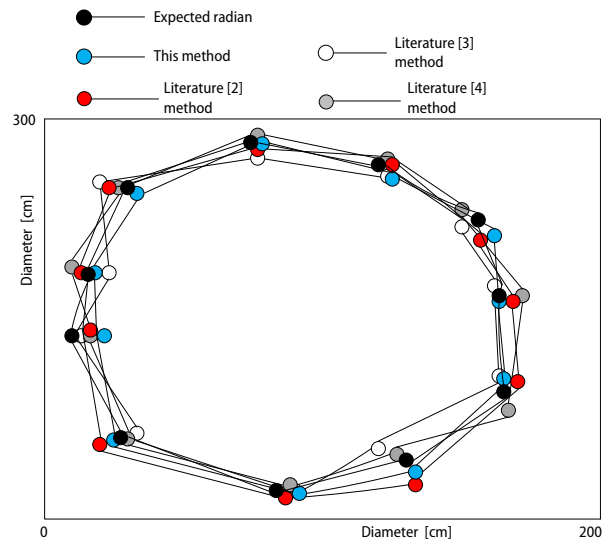
Methods	This method	Literature [2] method	Literature [3] method	Literature [4] method
Goodness of fit R^2	0.952	0.921	0.824	0.872

Note: The closer the value of R^2 is to 1, the better the degree of fit, otherwise, the worse the degree of fit.

The actual contour curvature after grinding the workpiece in fig. 2 is compared with the expected curvature, and the obtained goodness of fit results are shown in tab. 4.

It can be seen from tab. 4 that under the application of

Figure 2. Grinding workpiece curvature



this method, the goodness of fit is 0.95, the actual contour curvature after the workpiece is polished is closer to the expected curvature, and the expected effect is basically achieved; while the other three methods are applied, the fitting is excellent. The degrees are 0.921, 0.824, and 0.872, respectively. The actual contour curvature after the workpiece is polished is greatly different from the expected curvature. It can be seen that the grinding performance of the method is better.

Conclusion

Elliptical grinding, as an important branch of non-circular grinding, plays an important role in various fields of fine device processing. In view of the shortcomings of the three traditional methods, a new multi-axis mechanical grinding method based on algebraic geometry for elliptic contour is designed. This method applies the algebraic geometry principle to the error compensation in real time during the grinding process. After testing, the method solves the problems of the three conventional methods, and the grinding performance is higher.

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