MATHEMATICAL MODELING AND NUMERICAL SIMULATION OF HEAT DISSIPATION IN LED BULBS

by

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We present a detailed study of heating and cooling processes in LED luminaires with passive heat sinks. Our analysis is supported by numerical simulations as well as experimental measurements, carried on commercial systems used for outdoor lighting. We have focused our analysis on the common case of a single LED source in thermal contact with an aluminum passive heat sink, obtaining an excellent agreement with experimental measurements and the numerical simulations performed. Our results can be easily expanded, without loss of generality, to similar systems.

Key words: *cooling law, high-power LED*

Introduction

The use of high-power light emitting diodes (HP-LED) for public illumination is an emerging research line, triggered by recent developments of different technologies including semiconductor materials [1-3], fluorescence techniques [4], driver electronics [5] or thermal control [6] among others [7, 8], and the quantification of its impact in construction [9].

One of the key aspects concerning the performance and durability of HP-LED lighting systems is the adequate control of the temperature of the LED chip [10]. As it has been pointed by recent studies [11], LEDs have a high energy efficiency and long lifespan. However, a large amount of heat is dissipated during operation due to the Joule effect; thus, cooling HP-LED is an important challenge in package designs, where a correct evacuation of the heat will substantially enlarge the lifetime of the device.

Apart from the previous constraint, other practical aspects like a compact configuration, low cost, mass production, or even esthetic considerations can play an essential role in market-oriented products. Thus, we face on the design and analysis of two types of efficient solutions for systems of lighting:
– one led bulb, for using in the retrofit of existing lighting systems, and
– one entire luminaire for implementing new lighting systems.

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In both cases, numerical analysis of the thermal stabilization of $50W$ LED chips attached to passive heat sinks are performed, which is a usual configuration for outdoor luminaires used in the street lighting market. We propose some luminaires that keeps the temperature of the LED chip well-below 70 °C under realistic conditions, yielding long-term operation of the device, with the corresponding savings in energy consumption and maintenance.

After the numerical calculations performed, we constructed a set of prototypes in a compact and ready-to-install configuration for comparing the results of the computational simulations with the experimental measurements. As we will show, there is an excellent agreement between the numerical simulations and the corresponding data obtained.

Theoretical model

Here, we detail the definition of the mathematical model that will be implemented to deal with this case.

The relationship between the linear wave equation $-\partial^2\psi/\partial t^2+\nabla^2\psi=0$, the Schroedinger wave equation $i\partial\psi/\partial t+\nabla^2\psi=0$, and the heat equation $-\partial\psi/\partial t+\nabla^2\psi=0$ is well known. Namely, the Schroedinger wave equation is the non-relativistic limit of the linear wave equation, and further rotating the time coordinate into the imaginary axis yields the heat equation. Here, we will focus on the dissipative equation of the above three, namely, the heat equation.

We need to guarantee that the HP-LED temperature stays below the maximum value specified by the producer. Specifically, the *heat sink* presented in [12] must not exceed 70 ℃. Some studies [13] show that LED units have a high life expectancy, despite Joule dissipation.

We model the heat sink as axially symmetric, gravity **g** (dimensionally an acceleration) acting vertically along the symmetry axis (taken as *Oz*). The velocity field **v** of the air in which the heat sink is immersedsatisfies the Navier-Stokes equation.

$$\rho\frac{\partial\mathbf{v}}{\partial t}+\rho\left(\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\nabla p+\rho\nu\nabla^2\mathbf{v}+\rho\mathbf{g} \tag{1}$$

where ρ is the volume density of the air, ν – the kinematic viscosity, and p – the pressure. The velocity and the density are related together by the continuity equation:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\mathbf{v}\right)=0 \tag{2}$$

Heat flow in a temperature gradient is governed by the equation:

$$\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T=\kappa\nabla^2 T+\frac{J}{\rho C_\rho} \tag{3}$$

where C_ρ denotes the heat capacity per unit volume of the air, κ – the thermal diffusivity, and J – the time rate of internal heat production per unit volume. Equations (1)-(3) apply to the air sorrounding the LED. The heat sink itself is assumed to be a black body at temperature T, thus radiating energy according to the Stefan-Boltzmann law:

$$\dot{q}=\varepsilon\sigma T^4 \tag{4}$$

where $\dot{q}$ (energy per unit time per unit area) is the exit flow of energy from the heat sink, ε its emissivity, and σ is the Stefan-Boltzmann constant.

We will numerically solve the coupled system of eqs. (1)-(4) subject to Dirichlet boundary conditions. Let T_0 be the (fixed) temperature of the air surrounding the LED, far enough from the heat source. Weimpose the Dirichlet condition $T = T_0$ on the boundary of the numerical simulation window.

Additionally, the to the Dirichlet boundary condition has been supplemented with:
- heat transfer by solid to solid conduction between the LED and the dissipator, with a distributed thermal resistance of 0.001 m^2K/W and
- heat transfer by solid to air convection.

The heat transfer coefficient h has been taken to equal 12 W/m^2K. Since the values for h found in the literature vary widely, this latter input was adjusted until the simulation results coincided with those from the experiment.

Once we have found the best fit, we take the corresponding value of h for the rest of the numerical simulations.

Whether or not our coupled system of eqs. (1)-(4) defines a well-posed problem is a very delicate issue. The question of well-posedness of the Navier-Stokes equation alone is an unsolved problem, at least globally (although partial results concerning local issues are known). Of course, coupling Navier-Stokes to the remaining equations does not simplify matters. However, we make the following observation: a velocity field $\mathbf{v} = \mathbf{v}_0$ that is constant, both in space and time, reduces the Navier-Stokes eq. (1) to Newton's law $\nabla p = \rho\mathbf{g}$. In this limit, our coupled system of equations (Newton's law + continuity equation + heat equation) is known to define a well-posed problem. This makes the following statement plausible: *perturbatively around a constant velocity field* $\mathbf{v}_0$, *our coupled system of equations defines a well-posed-problem under Dirichlet boundary conditions*.

The Navier-Stokes eq. (1) is quadratic in $\mathbf{v}$, so the phenomenon described by the above system of equations is genuinely non-linear. However, the fluid velocities involved are small, and we can linearize (1) by simply dropping the term $(\mathbf{v}\cdot\nabla)\mathbf{v}$. This also simplifies the numerical resolution of the above system of equations. A detailed analysis of the fully non-linear problem, without dropping the term $(\mathbf{v}\cdot\nabla)\mathbf{v}$, is deferred to a later publication. There is also the option of using other numerical methods such as (MDDiM), see [14].

Materials and methods

In order to solve the mathematical model proposed in section *Theoretical model*, we used the Solidworks program. This program is computer-aided design software that allows 2-D and 3-D modeling. In addition, the different modules allow simulating diverse physical phenomena. In our case, the geometry of the lamp was imported.

The program module used was that of heat transfer. As a requirement of this module, several parameters must be defined in order to start the simulation. The parameters are linked to the different forms that may exist in the exchange of heat between the components and with the environment.

In this work we have first considered 3 different types of sinks. All of them consist on 42 equidistant baldes distributed along an inner cylinder of 18 mm of radius and 100 mm of length. The LED is located on one base of the cylinder. The temperature of the LED is 64°. Their descriptions are graphically represented below and they can visualize in figs. 1 and 2.
- *Heat sink 1*: Rectangular fins with dimensions: Length: 26 mm, Width: 2 mm, Height: 100 mm (heat sink height).
- *Heat sink 2*: Fins are bifurcated at the end with dimensions: Length: 10 mm, bifurcation radius: 1.5 mm and bifurcation length: 13 mm.
- *Heat sink 3*: Fins are curved and bifurcated at the end, with similar dimensions as heat sink 2.

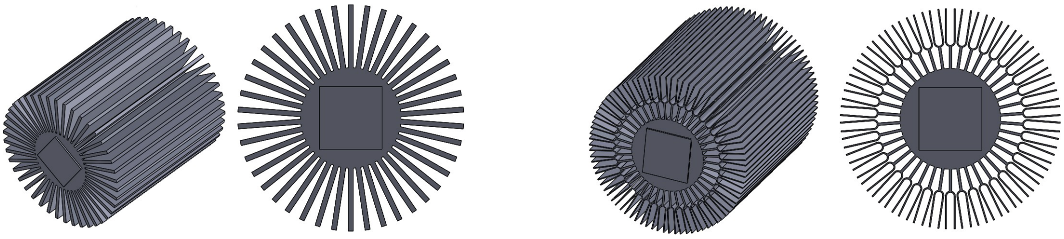

Figure 1. Geometry of the heat sinks 1 and 2

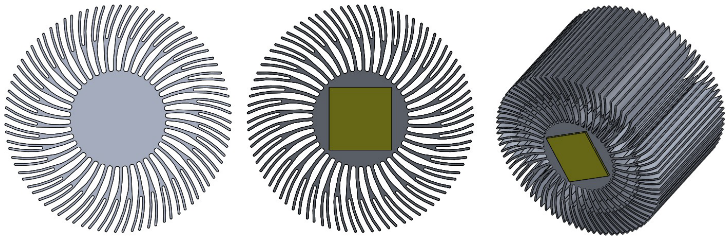

Figure 2. Geometry of the heat sink 3

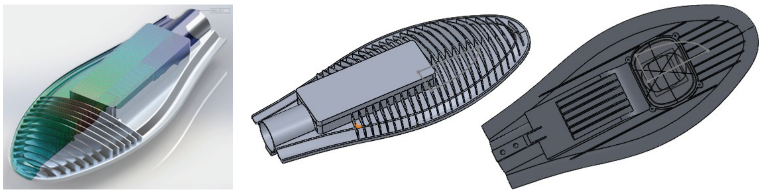

Figure 3: Geometry of the heat sink 4

As an alternate geometry, we have also considered the geometry shown in fig. 3, namely heat sink 4. This is a luminaire shaped public lighting structure to protect the LED chip of dimensions 480.25 mm (length) and 174.4 mm (width). In this case, we considered the LED temperature of 51.6 °C.

We describe the overall pattern for the air-flow as follows: the cooling air enters from the outer region of the heat sink and is heated while passing through the fins. The heated air rises upward from the inner regions of the heat sink. This is due to the lower density of the air there, respect to the density of the surrounding air.

Table 1. Material properties

Property	Value	Units
Elastic modulus	6.9e+10	N/m^2
Poisson coefficient	0.33	N/D
Shear modulus	2.7e+010	N/m^2
Mass density	2700	kg/m^3
Traction limit	68935600	N/m^2
Elastic limit	27574200	N/m^2
Thermal expansion coefficient	2.4e–005	K^{-1}
Thermal conductivity	200	W/(mK)
Specific heat	900	J/(kgK)

Besides, a thermal boundary-layer develops discontinuously after some delay. Thus, we expect a relatively high local heat transfer coefficient in the inner regions of the heat sink.

In tab. 1 we present the properties of the material used in all the simulations.

Once we have defined all the parameters that are used in the system, we proceed to define a mesh that best adapts to the geometry to be studied. Within the mesh tool, it is possible to define the different parameters to find the best mesh. First, we define the type of mesh to be implemented and, if necessary, modify the dimensions of the finite element. Later, the program presents the results directly in the studied geometry, which let us analyze the results. One of the advantages of this software is the variety of graphical and numerical ways to show the results.

Results

The mathematical model (heat sink geometry and governing equations) is applied for generating the simulations. All of them have been carried out for an input LED power of 50 W. In the heat sinks, the HP-LED is modeled as a square piece. Our aim is to determine the maximal temperature attained by the LED. In fig. 4, we exhibit the numerical results obtained for the geometries of sinks 1 and 2.

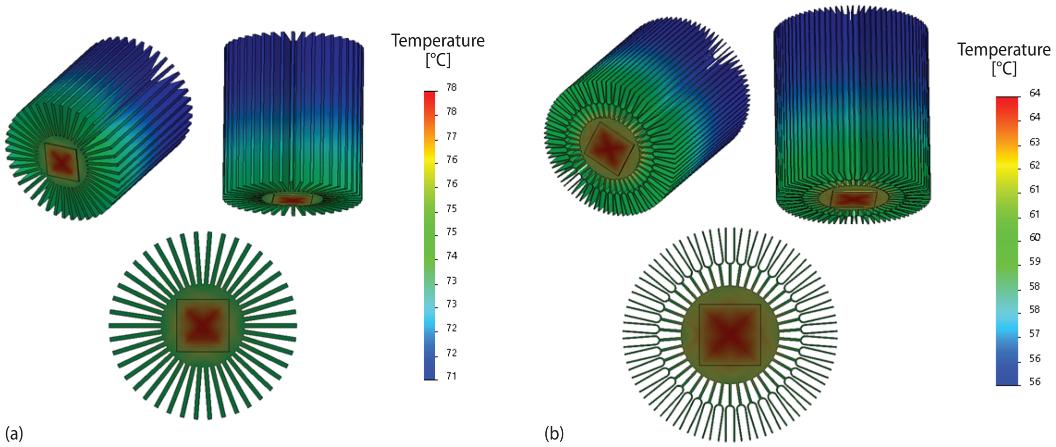

Figure 4. Numerical results of the simulation of heat sinks 1 (a) and 2 (b)

The best results are obtained for the heat sink 3, as it is shown in fig. 5. That sink was already introduced in [12]. The same general features already observed in the previous simulation continue to hold in this case. Only the specific numerical values change: the maximal temperature attained is 64 °C, the minimal is 56 °C. Observe the geometry of the flaps, which allows a higher flow of air through the heat sink. The latter surrounds a solid cylinder, on the base of which the square HP-LED is located; this cylinder distributes the heat across the whole device.

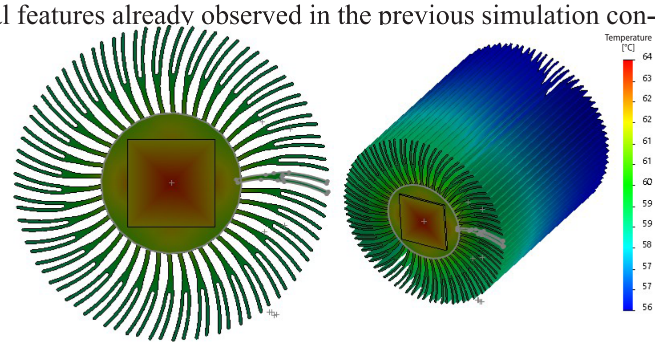

Figure 5. Numerical results of the simulation of heat sink 3

These results lead to the following conclusion. Heat sink no. 1 attains a maximal temperature of 78 °C. This is the highest value obtained in all our simulations. The absence of a bifurcation in the heat sink's fins reduces the contact area with the surrounding air. Clearly this reduces convection, thus leading to less heat dissipation. On the other hand, heat sink no. 2 attains a maximal temperature of 64 °C, similar to thatreached by the heat sink no. 3.

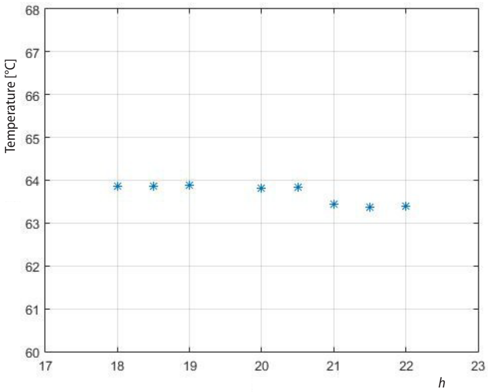

Figure 6. Relationship between finite element size and temperature for heat sink 3

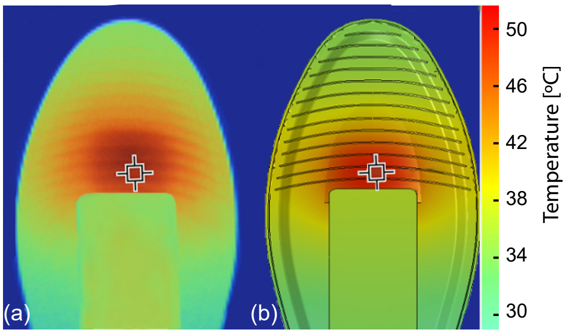

Figure 7. (a) Temperatures measured with a thermal camera (the maximum temperature indicated by the cross- shaped cursor is 51.6 °C), (b) results of the numerical simulations (the maximum temperature indicated by the cross-shaped cursor is 50.7 °C)

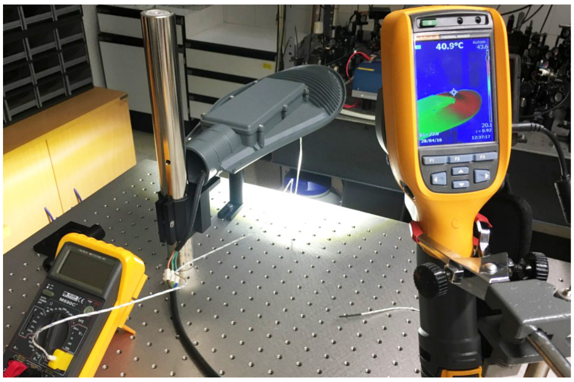

Figure 8. Experimental set-up used to perform the temperature measurements

Different simulations have been carried out to study the convergence of the program. Simulations have been carried out for the different defined values of the maximum size of the finite element. In the results, it can be seen that when the size is smaller, the temperature value practically remains constant, which confirms the convergence. Figure 6 shows the relationship between the variation of the size of the finite element and the maximum temperature reached in the simulation of heat sink 3.

In the case of the heat sink 4, it is the carcass itself of the LED (as designed for public lighting) that acts as a heat sink. Here, the chip is also located on the bottom of the heat sink. The results of the numerical calculations are shown in fig. 7 and are adjusted to what was expected.

As it can be appreciated, the maximum of the temperature distribution is obviously located at the source, and the values of the temperature, T, diminish depending on the distance between the point and the LED chip, showing a quasi-radially symmetric distribution around the perpendicular to the LED plate. The used ambient temperature was 24 °C, and the maximum temperature calculated, in steady-state distribution, is 52 °C. The temperature obtained at the critical point is below the maximum temperature recommended by the manufacturer, which helps to guarantee the useful life of the HP-LED. Additionally, this result approaches the experimental value obtained in a real system. The experimental setup used to perform the temperature measurements is shown in fig. 8.

Discussion

In order to check the validity of our numerical model, a series of experiments were made. Aluminum (Al6061) black anodized heat sinks were constructed with the same geometry as in figs. 1-3.

In all the cases, to minimize the thermal contact resistance between the LED chip and the heat sink a graphite film of high thermal conductivity (240 W/mK) was used. Finally, the luminaries were mounted in an optical table, reproducing usual operation conditions. The emissivity of the heat sink with the black-anodized surface treatment is 0.8. A temperature sensor with three different probe heads (DAQ-9172, NI9211), a power supply, a watt-meter, and a laptop were used in order to collect data. The results obtained are totally satisfactory, as the critical temperature is kept well below the control temperature.

Conclusions

We have presented numerical study of the steady-state temperature distribution of a realistic HP-LED bulb. The results obtained in the simulation have been compared with experimental values obtained through the use of prototypes following the geometrical characteristics of the simulated design. A nice agreement between the simulated results and the corresponding experimental data is demonstrated.

Finally, we have studied another case where the carcass itself of the LED (as designed for public lighting) is acting as a heat sink. The results of the numerical calculations fit the experimental data and are totally satisfactory, as the critical temperature is kept well below the control temperature. Taking into account that one of the key aspects concerning the performance and durability of HP-LED lighting systems is the adequate control of the temperature of the LED chip, we can conclude that we have presented two efficient solutions (one led bulb for the retrofit of existing lighting systems and one luminaire for implementing new lighting systems).

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