RELIABILITY ANALYSIS OF GEAR TRANSMISSION SYSTEM OF AERONAUTICAL TURBINE STARTER UNDER MULTI-CONSTRAINT

by

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At present, there are some problems in the reliability analysis method for gear transmission system of aeronautical turbine starter, such as low accuracy of finite element model, low efficiency of analysis and low accuracy of analysis results. To this end, a reliability analysis method for the gear transmission system of aeronautical turbine starter under multi-constraint is presented. A 3-D model of the gear pair of aeronautical turbine starter is constructed in UG. The model is input into the finite element software for meshing. The gear transmission of aeronautical turbine starter under working conditions is simulated by defining boundary conditions and applying loads, which provides a basis for reliability analysis of gear transmission system of aeronautical turbine starter. The reliability of the gear transmission system of the aeronautical turbine starter is evaluated by the comprehensive evaluation method, and the reliability of the system is evaluated by the contact of the evaluation results. According to the evaluation results, the reliability analysis of the gear transmission system of the aeronautical turbine starter is realized under the condition of multi-constraint. The experimental results show that the proposed method has high analysis efficiency and high analysis accuracy.

Key words: *aeronautical turbine, starter, gear transmission system, reliability analysis*

Introduction

In recent years, the failure of rotating parts accounts for a high proportion of the major mechanical accidents of aircraft starters in China. The failure of rotating parts not only affects the normal operation of starter, but also poses a great threat to the safe flight of aircraft, and even leads to the destruction of aircraft [1, 2]. Therefore, it is very important to study the reliability of the gear transmission system of the aeronautical turbine starter [3, 4].

The coupling dynamic model of gears and bearings in the gear transmission system of aeronautical turbine starter is established based on the lumped mass method. The model considers the failure correlation and strength degradation factors of components, and realizes the reliability analysis of gear transmission system. The finite element model of the gear transmission system of the aeronautical turbine starter constructed by this method has a large error compared with the actual one, and the accuracy of the finite element model is low [5]. In the

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reliability analysis of gear transmission system based on Bayesian network, two main failure modes of gears, *i. e.* contact fatigue pitting and bending fatigue fracture of tooth root, are considered. Combining with fault tree analysis, the reliability index of gear transmission system is calculated by Bayesian network method, and the reliability analysis of aeronautical turbine starter system is realized by the bidirectional reasoning ability of Bayesian network [6]. This method takes a long time to analyze and has the problem of low analysis efficiency [7-9].

In order to solve the problems of low accuracy of finite element method, low efficiency of analysis and low accuracy of analysis results in the above methods, a reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint is proposed.

Finite element model for gear transmission system of starter

Establishment of geometric model of gear

Gear surface geometry and meshing gear pair structure are complex. The UG provides free modeling function and easy-to-operate editing and analysis tools. It provides convenience for establishing solid 3-D model of aeronautical turbine starter gear [10, 11].

(1) Determination of involute gear

According to the mechanical principle, the polar co-ordinate equation of the involute is:

$$r_k = \frac{r_b}{\cos\alpha_k}$$
$$\theta_k = inv\alpha_k = \tan\alpha_k - \alpha_k \tag{1}$$

where r_k is the directional diameter of any point k on the involute, α_k – the pressure angle of k, θ_k – the spreading angle of k, and r_b – the radius of the base circle.

The rectangular co-ordinate equation of the involute is obtained by the polar co-ordinate equation of the previous formula:

$$\begin{aligned} x_k &= r_b[\sin(\tan\alpha_k) - \cos(\tan\alpha_k)\tan\alpha_k] \\ y_k &= r_b[\cos(\tan\alpha_k) + \sin(\tan\alpha_k)\tan\alpha_k] \end{aligned} \tag{2}$$

where x_k and y_k are the abscissa and ordinate co-ordinates of the involute in the Cartesian co-ordinate system, and the origin of the co-ordinate is in the center of the gear [12].

In UG, the involute curve is generated according to the previous involute equation. Open the expression dialog box and add the following relationships in the expression edit box in turn:

$$\begin{gathered} t = 1 \\ u = \frac{\tan(\alpha_k t)180}{3.14} \\ r = \frac{mz\cos 20°}{2} \\ x_l = r\sin(u) - r\tan(\alpha_k t)\cos(u) \\ y_l = r\cos(u) + r\tan(\alpha_k t)\sin(u) \end{gathered} \tag{3}$$

where t is the variable between 0 and 1, r – the radius of the base circle, α – the pressure angle of the top circle of the gear, z_i – the number of each gear in the system.

Then, the function of *law curve* in UG is use to form the involute profile.

(2) Determination of radius of transition fillet of gear root

When the two gears are engaged, the point k of the gear 1 meshes with the gear tip of the wheel 2, and the radius of the point k is obtained:

$$r_k = \sqrt{r_{b1}^2 + (a \sin\alpha' - r_{a2}\sin\alpha_{a2})^2} \tag{4}$$

In formula, a, α', r_{b1}, r_{a2}, and α_{a2} are the center distance, radius of base circle of meshing angle gear 1, radius of top circle of gear 2 and pressure angle of top circle of gear 2. Among them:

$$r_{b1} = r_1 \cos\alpha = \frac{1}{2} m z_1 \cos\alpha \tag{5}$$

$$a = \frac{1}{2} m (z_1 + z_2) \tag{6}$$

$$\alpha_{a2} = \arccos\frac{r_{b2}}{r_{a2}} \tag{7}$$

$$\alpha' = \alpha \tag{8}$$

The pressure angle of the point k is as follows:

$$\cos\alpha_k = \frac{r_{b1}}{r_k} \tag{9}$$

The co-ordinate system is established. In the co-ordinate system, the X-axis is tangent to the root circle of gear 1, and the Y-axis is the symmetrical line of the alveolar. Then, the calculation formula of the center angle β of the tooth groove width e_k on the circle r_k is:

$$\beta = \frac{\pi}{z_1} 2[(\tan\alpha_k - \alpha_k) - (\tan\alpha - \alpha)] \tag{10}$$

The co-ordinates x_k and y_k of point k are:

$$x_k = r_k \sin\frac{\beta}{2} \tag{11}$$

$$y_k = r_k \cos\frac{\beta}{2} - \frac{1}{2} m (z_1 - 2c) \tag{12}$$

where c represents the damping coefficient.

The formula for calculating the angle between the tangent $t - t$ and the X-axis o the tooth profile of gear 1 at point k is:

$$\Delta = \frac{\pi}{2} - \alpha_k - \frac{\beta}{2} \tag{13}$$

There are two cases of the co-ordinates of point k: $x_k > y_k$ and $x_k < y_k$, which are discussed separately:

– $x_k > y_k$

$$\begin{gathered}\rho=\frac{y_k}{1-\cos\Delta}\\ x=x_k-\rho\sin\Delta\\ y=\rho\end{gathered}\tag{14}$$

– $x_k < y_k$

$$\begin{gathered}\rho=\frac{x_k-y_k\cot\Delta}{\tan\Delta}\\ x=0\\ y=\rho\end{gathered}\tag{15}$$

where ρ is the maximum radius of the transition fillet, x and y are the co-ordinates of the rounded center.

According to the aforementioned transition curve equation, the root's transition curve can be generated. After obtaining the tooth profile curve and the transition fillet of the gear root, the complete 3-D profile of the gear transmission system of aeronautical turbine starter is obtained by using the mirror and stretching functions provided in UG.

Mesh generation of gear pairs by finite element

Because of the irregular shape of the tooth shape of the ring gear, it is very difficult to divide the model into finite elements. Elements with singular angles often appear in areas such as fillet transition, which will cause singularity of stiffness matrix and make the calculation fail. This requires manual partitioning to avoid the generation of singular elements. In order to reduce the number of elements, a dense transition is needed. After partitioning the model, it needs to carefully check whether there are defects in the model. If the model contains unknown defects such as cell voids and overlapping nodes, the calculation results will be inaccurate. Seriously, it will make the calculation deviate from the expected direction, or even make the calculation unable to proceed. The reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint is based on free meshing. The meshing finite element model of gear pair in gear transmission system of aeronautical turbine starter is shown in fig. 1.

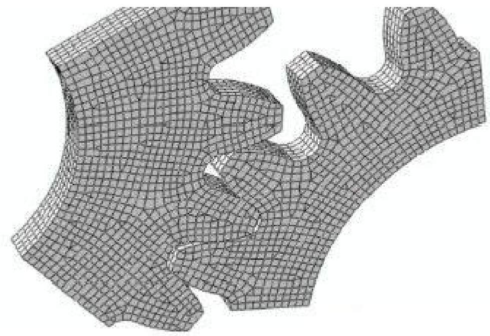

Figure 1. Finite element model for gear pair meshing

Reliability analysis of gear drive system of aeronautical turbine starter

Reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint condition realizes reliability analysis of gear transmission system of starter by constructing reliability rating model. The comments on the system safety assessment are:

$$V=\{v_1,v_2,v_3,v_4,v_5\}\tag{16}$$

where v_1 is very poor, v_2 is poor, v_3 is general, v_4 is good, v_5 is very good.

The factors set of the gear transmission system of the aeronautical turbine starter are:

$$U = \{U_1, U_2, U_3\} \tag{17}$$

where U_1 is the frequency of failure, U_2 is the severity of the impact, and U_3 is the degree of difficulty in testing. Each factor $U_i (i = 1,2,3)$ is determined by the basic factors. The factor set of U_i is:

$$U_i = \{u_1, u_2, u_3, u_4, u_5\} \tag{18}$$

In formula, the specific meanings of $u_i (i = 1,2,3,4,5)$ are: u_1 represents deformation, u_2 represents fatigue fracture of gear disc, u_3 represents stress corrosion cracking of birch groove bottom, u_4 represents birch tooth fracture, and u_5 represents crack of sealing castor tooth in outer edge of gear disc.

(1) Single factor evaluation

Firstly, the factor $u_i (i = 1,2,3,4,5)$ in factor set U is evaluated by single factor, and the membership degree d_{ij} of the comment $v_j (j = 1,2,\cdots,n)$ of factor u_i is determined. The single factor evaluation set of factor u_i is obtained:

$$d_i = (d_{i1}, d_{i2}, \cdots, d_{in}) \tag{19}$$

Fuzzy value mapping is as:

$$f : U \to F(V), u \to f(u_i) \tag{20}$$

In formula, $f(u_i) = d_i = (d_{i1}, d_{i2}, \cdots, d_{in})$ is the comment ambiguity vector of factor u_i, d_{ij} is the degree of a factor u_i which has comment v_j.

(2) Construction of a comprehensive evaluation matrix

With each single factor evaluation set as rows, the evaluation matrix of m rows can be obtained:

$$D = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix} \tag{21}$$

where D is a comprehensive evaluation matrix.

(3) Fuzzy set for determining the importance of factors

Because each factor has different influence on things, it is necessary to give the important degree of each factor, in the process of comprehensive evaluation, *i. e.* a fuzzy subset is given on factor set U.

$$A = (a_1, a_2, \cdots, a_m) \tag{22}$$

where a_i represents the importance degree of factor $u_i (i = 1,2,\cdots,m)$ in the overall evaluation. The A is called the fuzzy set of factor importance degree on U, and a_i is the important degree coefficient of factor u_i.

(4) Establishment of a comprehensive evaluation model

Comprehensive evaluation matrix D is carried out fuzzy linear transformation:

$$B = A * D = (b_1, b_2, \cdots, b_n) \tag{23}$$

where

$$b_j = (a_1 \hat{*} d_{1j}) \breve{*} (a_2 \hat{*} d_{2j}) \breve{*} \cdots a_n \hat{*} d_{nj}) \quad (j = 1, 2, \cdots, n) \tag{24}$$

where * denotes the generalized fuzzy synthesis operation, $\hat{*}$ is the generalized fuzzy *and* operation, "$\breve{*}$" is the generalized fuzzy "*or*" operation, B is the fuzzy comprehensive evaluation set on the commentary set V, $b_j (j = 1, 2, \cdots, n)$ is the membership degree of the rank v_j to the fuzzy comprehensive evaluation set B, and the former is the comprehensive evaluation model, which is recorded as $M(\hat{*}, \breve{*})$.

(5) Comprehensive evaluation

According to the principle of maximum membership degree, the grade v_j corresponding to the maximum b_j in the fuzzy comprehensive evaluation set $B = (b_1, b_2, \cdots, b_n)$ is selected as the result of the comprehensive evaluation.

When the evaluation system is complex or there are many influencing factors, the importance coefficient of each factor will become smaller, and the order of the factors is difficult to distinguish, so the accurate evaluation results cannot be obtained. In this regard, the elements in the factor set U are classified according to their attributes.

Experiments and discussions

In order to verify the overall validity of the reliability analysis method for the gear transmission system of an aeronautical turbine starter under multi-constraint, it is necessary to test it. The experimental platform of this experiment is NS2, operating system, Windows, simulation tool, MATLAB, processor is Intel (R) Celeron (R) 2.6 GHz, memory is 24 GB. The reliability of the gear transmission system can be analyzed by constructing the finite element model of the gear transmission system. The reliability analysis method for the gear transmission system of the aeronautical turbine starter under multi-constraint condition, represented by *a*, the reliability analysis method for the gear transmission system based on stress-strength interference theory, represented by *b*, the reliability analysis method for the gear transmission system based on Bayesian network, represented by *c* and the reliability analysis method for the gear transmission system based on ANSYS are adopted respectively, represented by *d*. Comparing the accuracy of the finite element models constructed by four different methods, the test results are shown in fig. 2.

Analysis fig. 2 shows that the accuracy of the finite element model constructed by the reliability analysis method for gear transmission system of aeronautical turbine starter under

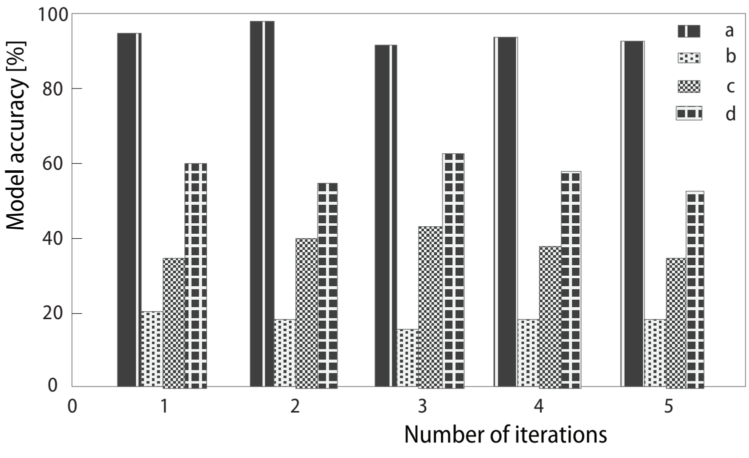

Figure 2. Model accuracy of four different methods

multi-constraint condition is higher than that constructed by the reliability analysis method for gear transmission system based on stress-strength interference theory, the reliability analysis method for gear transmission system based on Bayesian network and the reliability analysis method for gear transmission system based on ANSYS. Because this method establishes the solid 3-D model of gear pair in UG, and then imports the finite element software to mesh gear pair, which improves the accuracy of the finite element model.

Comparing the reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint condition, the reliability analysis method for gear transmission system based on stress-strength interference theory, the reliability analysis method for gear transmission system based on Bayesian network, and the reliability analysis method for gear transmission system based on ANSYS, the time used for reliability analysis of gear transmission system of starter can be analyzed. The testing results are shown in fig. 3.

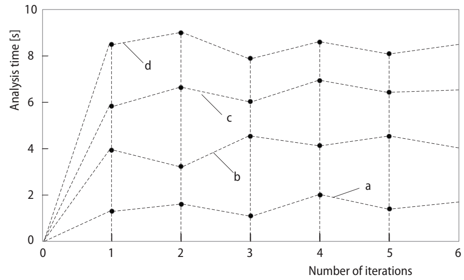

Figure 3. Analysis time of four different methods

Analysis fig. 3 shows that the reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint takes less time than the other two methods, because the finite element model of the system is established before the reliability analysis of the system is carried out. By applying load and defining boundary conditions, the finite element model of the system is established.

By analyzing the accuracy of reliability analysis, the validity of reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint condition is further verified. The accuracy of the reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint condition, the reliability analysis method for gear transmission system based on stress-strength interference theory, the reliability analysis method for gear transmission system based on Bayesian network are also verified, and the reliability analysis method for gear transmission system based on ANSYS is shown in fig. 4.

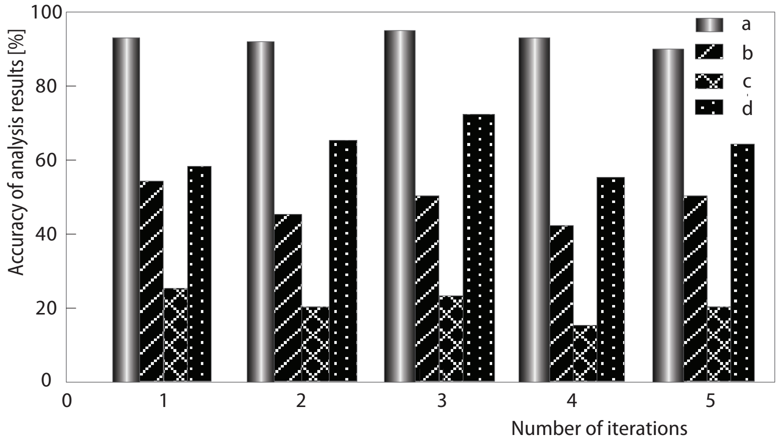

Figure 4. Analysis accuracy of four different methods

Conclusion

At present, the reliability analysis methods for gear transmission system of aeronautical turbine starter have some problems, such as low accuracy of finite element model, low efficiency of analysis and low accuracy of analysis results. To this end, a reliability analysis method for gear transmission system of aeronautical turbine starter under multi-constraint is proposed. Through high precision finite element model, the gear transmission of aeronautical turbine starter is analyzed accurately in a short time. The reliability of the system solves the problems existing in the reliability analysis method for the current transmission system, and provides a guarantee for the operation of the gear transmission system of the aeronautical turbine starter.

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