SIMPLIFIED MATHEMATICAL MODEL OF THE HYDROTHERMAL REGIME OF THE KRASNOYARSK RESERVOIR

by

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The scheme of the reservoir bed is applied to numerical study of the hydrothermal regime of the Krasnoyarsk Reservoir in the form of regions with continuous depth variation and abrupt width change. A numerical algorithm for the research of 2-D stratified currents in vertical plane in flowing reservoirs is considered taking into account Coriolis force. The numerical model determines the vertical distributions of water temperature in various areas of a reservoir real meteodata. There are given examples of water currents and temperature calculations in the Krasnoyarsk Reservoir. The results of the calculations are consistent with the observational data.

Keywords: *numerical modelling, hydrothermics, Krasnoyarsk Reservoir, stratified flows, Coriolis parameter*

Introduction

The construction of hydrosystem dams introduces significant changes in natural conditions of the adjacent areas. The temperature and flow regimes of the river vary both above and below the hydraulic system. Hydro-ice-thermal regime of the Yenisei River has changed markedly both above and below the dam after the construction of the Krasnoyarsk hydroelectric power station. The water temperature fell 10º-12º below the dam in summer and increased 1.5º-3.0º in winter.

Mathematical models of different levels of complexity are used for the numerical simulation of processes in both upstream and downstream areas. Mathematical models of hydrothermal processes in water reservoirs are based on the equations of fluid mechanics and heat transfer, taking into account the specifics of the problems. There is an extensive literature on mathematical modeling of temperature-stratified flows (see for example [1-11]), in [12] a simplified 2-D in the vertical plane model was used to study the hydrothermal regime of the Krasnoyarsk Reservoir. In this paper, we propose a method of accounting for Coriolis forces.

Bathymetry of the Krasnoyarsk Reservoir

For mathematical modeling of the reservoir hydrothermal regime, the natural channel is replaced by a fictitious one in the form of separate areas with continuous depth change and abrupt width change at the boundaries of the areas, tab. 1. The simplest schematization of the Krasnoyarsk Reservoir bed was used in the form of eight areas with constant width and depth [12, 13].

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Table 1. Calculated cross-sections

Number of region	1			2	3		4		5	
Distance to the cross-sections, [km]	0	25.9	31	33	36.3	44	83.2	110	122.3	138
Depth, [m]	4.6	5.6	9.2	10.9	14.2	16.6	19	22.5	26	28.75
Width without bays, [km]	6.9	6.9	6.9 (5.1)	5.1 (3.3)	3.3	3.3 (5.2)	5.2	5.2 (3.0)	3.0	3.0 (3.7)
Number of region		6		7		8		9	10	
Distance to the cross-sections, [km]		201.8	231	253.4	273	306.3	317	319	339.6	386
Depth, [m]		31.5	37.5	43.5	50.35	57.2	74.9	77.5	96	105
Width without bays, [km]		3.7	3.7 (6.2)	6.2	6.2 (5.5)	5.5	5.5 (3.4)	3.4 (1.3)	1.3	1.3

Two values are specified in the areas in which width changes abruptly. Two docking sections are added to reduce the sharp (large) changes in the width at the boundaries of areas in the scheme of work [12].

The simplified mathematical model of currents in reservoirs of big extent

Simplification of mathematical model is based on approximations: the Boussinesq approximation, the approximation of the slow movements, the approximation of hydrostatics, and the approximation of the boundary layer [12]. Hydro-thermodynamic equations are written in the standard Cartesian co-ordinate system (x, y, z), the axis x is directed to the east, the axis y – to the north, the axis z is directed downward. Because of the extension of the Krasnoyarsk Reservoir in the meridional direction, in 2-D approach the axis y is directed to the north. Then the simplified hydro-thermodynamic equations have the form in the standard Cartesian co-ordinate system in the variables stream function ψ – vortex ω:

$$\frac{\partial \omega}{\partial t} = \frac{\partial^2}{\partial z^2}(K_z \omega) + \mathrm{g}\frac{\partial \hat{\rho}}{\partial y} \tag{1}$$

$$\frac{\partial^2 \psi}{\partial z^2} = -\omega \tag{2}$$

$$V = \frac{\partial \psi}{\partial z}, \qquad W = -\frac{\partial \psi}{\partial y}, \qquad \frac{\partial T}{\partial t} + V\frac{\partial T}{\partial y} + W\frac{\partial T}{\partial z} = \frac{\partial}{\partial z} K_{zT} \frac{\partial T}{\partial z},$$
$$\rho = \rho_0 \left[1 + \hat{\rho}(T)\right], \qquad \hat{\rho} = -0.68 \cdot 10^{-5} (T-4)^2 \tag{3}$$

where V and W [ms^{-1}] are the components of water flow velocity in y- and z-directions correspondingly, t – the time, ρ – the water density, ρ_0 (= 10^3 [kgm^{-3}]), g [ms^{-2}] – the gravity acceleration, T [°C] – the water temperature, K_z, and K_{zT} [m^{2}s^{-1}] are the coefficients of turbulent exchange.

The flow problem for the m^{th} region is considered $0 \le y \le L_{\text{m}}$. A grid is entered

$$\Delta y_i = y_{i+1} - y_i, \quad \Delta z_i = \frac{H_i}{jj-1}, \quad i = 1, 2, 3, \cdots, ii, \quad j = 1, 2, 3, \ldots, jj$$

For this grid:

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y} - \frac{z}{H}\left(\frac{\mathrm{d}H}{\mathrm{d}y}\right)\frac{\partial}{\partial z}$$

Initial conditions are: $\psi = \psi^0(y,z), \quad \omega = \omega^0(y,z)$

Boundary conditions are: on the water surface (z = 0):

$$\psi = 0, \quad \omega = \frac{\tau_y}{\rho_0 K_z}, \quad K_{zT}\frac{\partial T}{\partial z} = -\frac{F_n}{c_p \rho_0}$$

on the bottom, $z = H(y_i) = H_i$:

$$\psi = q_m, \quad \omega = 2\frac{q_m - \psi_{jj-1}}{(\Delta z_i)^2}$$

Thomas's condition,

$$K_{zT}\frac{\partial T}{\partial z} = \frac{F_{\mathrm{bt}}}{c_p \rho_0}$$

where q_{m} is specific water discharge (per unit width), F_{n} – the total heat flux through the free surface, the components of the heat flux on the water surface are determined by the known formulas [1, 2, 14], F_{bt} – the heat exchange with the reservoir bed. The wind shear stress, $\vec{\tau}$, is defining by Saimons formula $\vec{\tau} = 1.5\cdot 10^{-3}\left|\vec{W}_a\right|\vec{W}_a$, where $\vec{\tau} = (\tau_x, \tau_y)$, $\vec{W}_a = (W_{ax}, W_{ay})$ is wind speed vector in [ms^{-1}] and wind stress in [kgm^{-1}s^{-2}].

The $\psi = \psi_{in}$, $\omega = \omega_{\mathrm{in}}$ are given in starting range of the first region (y = 0). The width and specific flow change abruptly at the boundaries of adjacent areas m^{th} and $(m+1)^{\mathrm{th}}$. The pairing of solutions is performed from the condition of water discharge balance by the ratios of:

$$\psi_{m+1} = \frac{B_m}{B_{m+1}}\psi_m, \qquad \omega_{m+1} = \frac{B_m}{B_{m+1}}\omega_m$$

The eqs. (1) and (3) are solved by an explicit scheme [12]. The difference approximation of the eq. (2) has the form:

$$\frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta z_i)^2} = -\omega_{i,j}^{n+1}$$

Difference equations for the stream function are solved by the sweep method. Flow velocity components are found by the formulas:

– on the water surface:

$$V_{i,1} = \frac{\psi_{i,2} - \psi_{i,1}}{\Delta z_i}, \quad W_{i,1} = 0$$

– on the bottom:

$$V_{i,1} = 0, \quad W_{i,1} = 0$$

– in the inner grid nodes:

$$V_{i,j} = \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta z_i}, \quad W_{i,j} = -\frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta y_m} + \frac{z_j}{H_i}\left(\frac{\mathrm{d}H}{\mathrm{d}y}\right)V_{i,j}$$

– at the border of regions:

$$V_{m+1,j} = \frac{B_m}{B_{m+1}}V_{m,j}$$

The water temperature in the internal nodes is found with the explicit scheme:

$$T_{i,j}^{n+1} = T_{i,j}^{n} - \Delta t A T_{i,j}^{n} + \Delta t \frac{K_{zT\,i,j+1/2}(T_{i,j+1}^{n} - T_{i,j}^{n}) - K_{zT\,i,j-1/2}(T_{i,j}^{n} - T_{i,j-1}^{n})}{(\Delta z_i)^2} \tag{4}$$

on the water surface ($j = 1,\ 2 \le i \le ii$) taking into account the boundary condition, we obtain:

$$T_{i,1}^{n+1} = T_{i,1}^{n} - \Delta t A T_{i,1}^{n} + 2\Delta t \frac{K_{zT\,i,3/2}(T_{i,2}^{n} - T_{i,1}^{n}) + \Delta z_i \dfrac{F_n}{c_p \rho_0}}{(\Delta z_i)^2} \tag{5}$$

on the bottom ($2 \le i \le ii, \quad j = jj$):

$$T_{i,jj}^{n+1} = T_{i,jj}^{n} - 2\Delta t \frac{K_{zT\,i,jj-1/2}(T_{i,jj}^{n} - T_{i,jj-1}^{n})}{(\Delta z_i)^2} \tag{6}$$

at the border of the adjacent regions $T_{m+1}^{n+1}(y_1, z_j) = T_m^{n+1}(y_{ii}, z_j)$.

Here A denotes a scheme with differences against the flow:

$$A f_{i,j} = V_{i,j}^{n} f_{i,j,y0} + 0.5\left|V_{i,j}^{n}\right|(f_{i,j,y1} - f_{i,j,y}) + W_{i,j}^{n} f_{i,j,z0} + 0.5\left|W_{i,j}^{n}\right|(f_{i,j,z1} - f_{i,j,z})$$

where

$$f_{i,j,y} = \frac{f_{i+1,j} - f_{i,j}}{\Delta y_i}, \quad f_{i,j,y1} = \frac{f_{i,j} - f_{i-1,j}}{\Delta y_{i-1}}, \quad f_{i,j,y0} = 0.5(f_{i,j,y} + f_{i,j,y1})$$

$$f_{i,j,z} = \frac{f_{i,j+1} - f_{i,j}}{\Delta z}, \quad f_{i,j,z1} = \frac{f_{i,j} - f_{i,j-1}}{\Delta z}, \quad f_{i,j,z0} = 0.5\frac{f_{i,j+1} - f_{i,j-1}}{\Delta z}$$

$$W_{i,j}^{n} = W_{i,j}^{n} - \frac{z_j}{H_i}\left(\frac{\mathrm{d}H}{\mathrm{d}y}\right) V_{i,j}^{n}$$

Parameterization of the vertical turbulent exchange coefficient

The averaged flow has both molecular and turbulent viscosities. There are wind mixing and dynamic mixing, determined by the flow rate of the reservoir. The total coefficient of vertical turbulent exchange of two terms $K_z = K_z^e + K_z^m$, where K_z^e corresponds to wind mixing and K_z^m – to dynamic one. The coefficient K_z^e is determined by the Prandtl-Obukhov formula with allowance for the Eckman approximation [12]:

$$K_z^e = \begin{cases} (0.05 h_1)^2 \sqrt{\left(\dfrac{\tau}{\rho_0 K_0}\right)^2 \mathrm{e}^{-2\alpha z} - \dfrac{\mathrm{g}}{\rho_0}\left(\dfrac{\partial \rho}{\partial z}\right)} + K_{\min} & \text{for} \quad B \ge 0 \\ K_{\min} & \text{for} \quad B < 0 \end{cases}$$

where

$$B = \left(\frac{\tau}{\rho_0 K_0}\right)^2 \exp(-2\alpha\,\mathrm{z}) - \frac{\mathrm{g}}{\rho_0}\left(\frac{\partial \rho}{\partial \mathrm{z}}\right)$$

$\tau = (\tau_x^2 + \tau_y^2)^{0.5}$ is the wind stress, $K_{\min} = 0.2 \cdot 10^{-5}$ [m^{2}s^{-1}] – the minimum value of the vertical turbulent exchange coefficient (corresponding to the molecular viscosity),

$$K_0 = (0.05\pi)^2 \tau \left(\rho_0 \sqrt{4 f^2 + (0.05\pi)^4 \mathrm{g} \frac{\Delta\rho}{\rho_0 H}} \right)^{-1}$$

$\alpha = (0.5 f K_0^{-1})^{0.5}$, $h_1 = \pi(0.5 K_0 f^{-1})^{0.5}$, $f = 2\Omega \sin\varphi$ – the Coriolis parameter, Ω – the angular speed of the Earth rotation, φ – the latitude. The coefficient K_z^m is determined by the formula of Makkaveyev [15]:

$$K_z^m = \frac{ngq}{37 H^{1/3}}$$

where n is the surface roughness of the bottom (n = 0.02), q [m^{2}s^{-1}] – the specific water discharge, g [m^{2}s^{-1}] – the acceleration of gravity, and H [m] – the depth, $K_{zT} = 0.1 \cdot K_z$.

Coriolis force accounting method

Drift currents are formed under the influence of wind in the top layer of a reservoir. Ekman [16] built the first solution of the problem of steady drift flow for homogeneous deep sea:

$$U^e = V_0 \mathrm{e}^{-\alpha z} \cos\left(\frac{1}{4}\pi - \alpha z \right), \quad V^e = V_0 \mathrm{e}^{-\alpha z} \sin\left(\frac{1}{4}\pi - \alpha z \right) \tag{7}$$

where U^e, V^e are the horizontal components of the velocity vector of the water flow, $\alpha = (0.5 f K_z^{-1})^{0.5}$, $V_0 = \tau_y [(2)^{1/2} \rho_0 K_z \alpha]^{-1}$, the wind is directed along the y-co-ordinate $(\tau_x = 0,\ \tau_y \neq 0)$. The wind flow velocity decreases exponentially with depth. The flow velocity is small below the horizon $z = D$, $D = \pi(2 K_z f^{-1})^{0.5}$ is the friction depth. The main part of the kinetic energy of the drift flow is concentrated in the friction layer between 0 and D. The influence of parameter, f, can be neglected for $H < D$ (H is reservoir depth). A method of accounting for the Coriolis force is proposed for deep-water bodies ($H > D$). The computational cycle consists of four stages.

Stage 1. Intermediate values of speeds V_1, W_1 at $\tau_y = 0$ are found by known field of speed in initial (or previous) time point from the eqs. (1) and (2).

Stage 2. The horizontal components of the velocities of the wind currents are calculated by the Ekman approximation (7) U^e, V^e, ($W^e = 0$).

Stage 3. The final speed values are calculated V, W ($V = V_1 + V^e$, $W = W_1 + W^e$).

Stage 4. According to the ratios (4)-(6) the temperature regime of the reservoir is calculated.

Results of calculations

A series of calculations of flow velocities in the Krasnoyarsk Reservoir was performed using the 2-D in the vertical plane model (variant A) and the proposed four-stage method (variant B). The values of the horizontal component of the flow velocity on the water surface calculated according to variants A and B are shown in tabs. 2 and 3. Accounting the Coriolis forces makes it possible to refine the surface velocities of the water flow.

Numerical experiments were performed to define the temperature regime of the Krasnoyarsk Reservoir during the summer period using the simplified 2-D in the vertical plane model taking into account the Coriolis force. Detailed weather data for the corresponding period were used. Figure 1 shows the calculated temperature profiles (solid lines) in the Krasnoyarsk Reservoir (July 4, 1977). Initial conditions are the measured data (June 2, 1977). The calculation results are consistent with the measured data (points). The data were taken from [17].

Table 2. Horizontal flow velocities on the water surface, W_a= 5 m/s

Depth, [m]	30	40	50	70	90
Velocity V, variant A, [m/s]	0.1	0.25	0.27	0.34	0.32
Velocity V, variant B, [m/s]	0.07	0.11	0.12	0.15	0.25

Table 3. Horizontal flow velocities on the water surface, W_a= 10 m/s

Depth, [m]	30	40	50	70	90
Velocity V, variant A, [m/s]	0.27	0.61	0.65	0.79	0.50
Velocity V, variant B, [m/s]	0.21	0.32	0.32	0.31	0.36

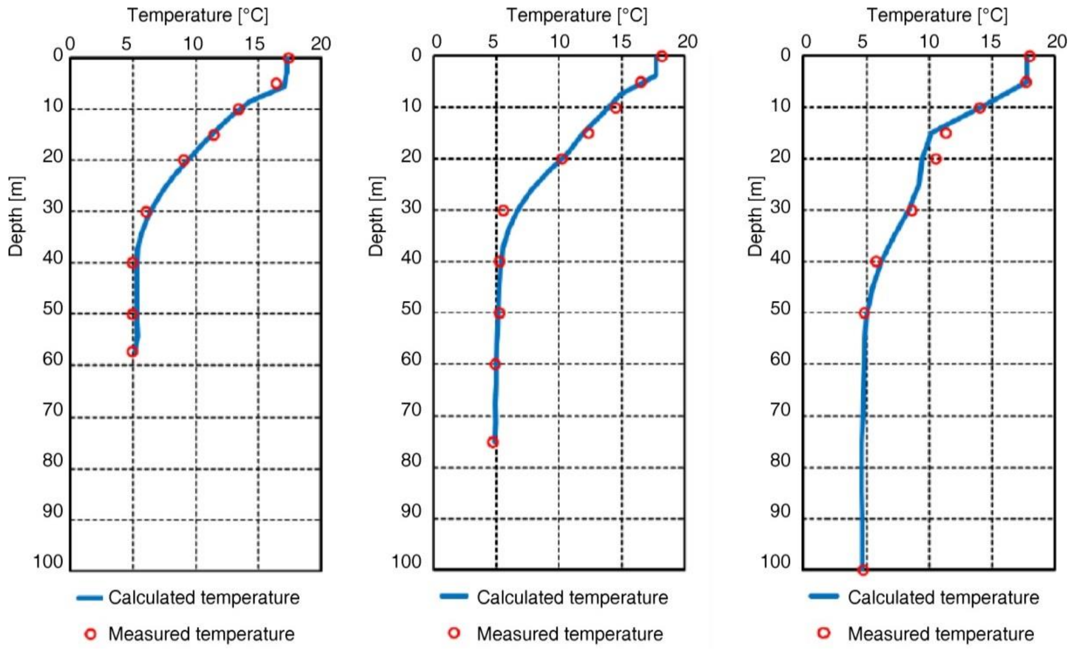

Figure 1. Vertical distribution of water temperature at some sections of the Krasnoyarsk Reservoir

Figure 2 shows the vertical distributions of the horizontal flow velocity in various sections of the reservoir. Calculations on the two-dimensional in a vertical plane model (variant A) significantly overestimate the speed of wind currents for deep sections of the reservoir. Offered method (variant B) allows to specify influence of Coriolis forces on the magnitude of the speeds of wind currents.

The developed computer model allows the prediction of the hydrothermal regime of a flowing reservoir for various weather conditions scenarios.

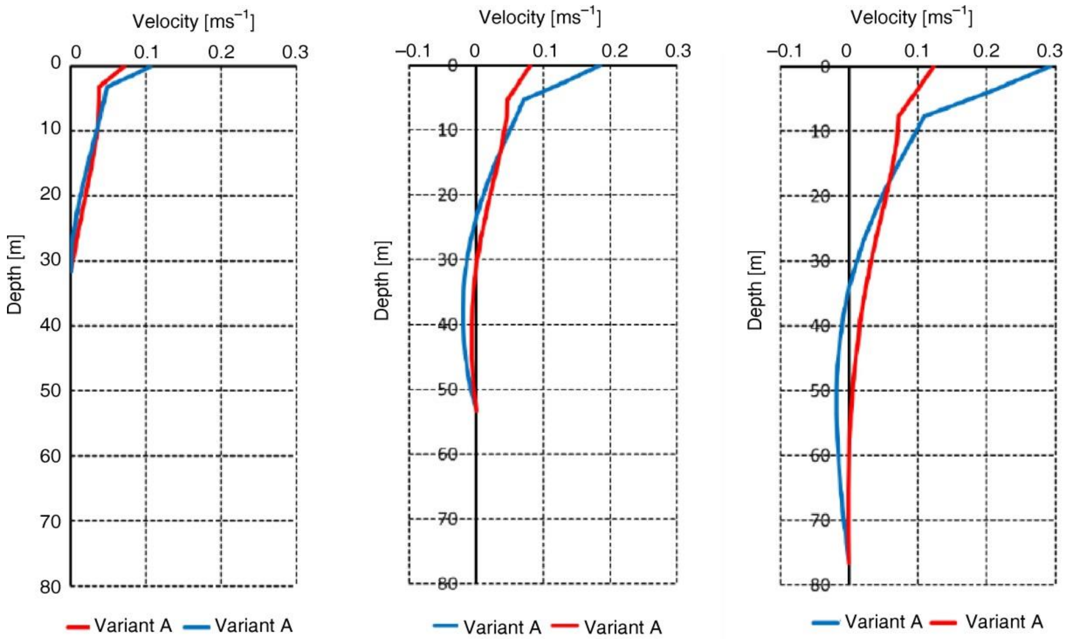

Figure 2. Vertical distribution of velocity at some sections of the Krasnoyarsk Reservoir

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