SELF-EQUILIBRATED STRESSES IN THE SYSTEM COMPOSED OF A COVERING LAYER + SPATIALLY LOCALLY CURVED REINFORCING LAYER + HALF-SPACE

by

Ramazan TEKERCIOGLU *

Department of Mathematical Engineering, Yildiz Technical University, Istanbul, Turkey

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The system composed of a face covering layer + spatially locally curved substrate reinforcing layer + half-space is taken into consideration. It is presumed that this framework is compressed at infinity by uniformly distributed normal forces and it is required to establish the self-equilibrated normal stresses in that, caused by locally curved of the substrate reinforcing layer. The matching boundary and contact value problem is defined within the scope of 3-D geometrically non-linear exact equations. Formulated problem's solution is introduced with the series form of small parameter which represents the degree of the aforesaid locally curving. These series' zeroth and first approximation are ascertained with the utilization of double Fourier transform. The original of values that are searching is ascertained numerically. Corresponding numerical outcomes about the self-equilibrated normal stress caused by this spatially local curving are presented and discussed.

Key words: *self-equilibrated stresses, spatially local curving, double Fourier transform*

Introduction

Two field of studies have risen in the mechanics of fractures of composite materials which is under compression owing to the bulging of strengthening elements. The first field of study includes various approximate design models (the distribution of the compressive load between the filler and the binder is exemplified). The studies in [1-3] are a few of the first ones in this area, with recent investigations carried out in [4-7].

Such cases can occur where the material of the reinforcing layer is nanomaterial, for instance nanocarbon structures or graphene films [8-10].

The second field of study uses the 3-D geometrically non-linear exact equations of elasticity (or viscoelasticity) theory for studying fracture mechanisms for composite materials. Coherent account of these investigations has been made in [11, 12] and a review of the related works is given in [13]. State that the matching stability loss problems are considered in [14, 15]. From these references, it is concluded that the researches related to the second field of study have been worked within the scope of the piecewise homogeneous body model and continuum approach.

The foregoing investigations' analysis shows that the study of the self-equilibrated stresses in the layered and unidirected fibrous composites caused by the bulging of the strengthening elements has been made mainly for the cases where it is presumed that the material involves the infinite region.

* Author's e-mail: tramazan@yildiz.edu.tr

Consequently, up to now the influence of the boundary of the element of construction made of such materials on the aforementioned self-equilibrated stresses has not been taken into account.

The present paper attempts to fill this gap and studies the self-equilibrated stresses caused by the spatially locally curved substrate reinforcing layer.

Problem formulation and solution method

We take into account a semi-infinite half-space combined to a stack consists of two layers (covering layer and spatially curved substrate reinforcing layer) as shown in fig. 1. We associate the Cartesian co-ordinate system $Ox_1x_2x_3$ with the reinforcing layer as shown in fig. 1. we ascertain the position of the points of the constituents by using the Lagrangian co-ordinates in this co-ordinate system.

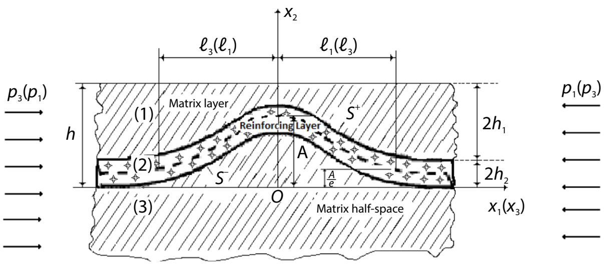

Figure 1. Geometry of the half-space covered by a stack consisting of two layers

Each reinforcing layer's thickness will be presumed as constant. Also, it will be presumed that the semi-infinite half-space and stack materials are homogeneous and isotropic. Now let us determine the stress deformation state in the aforementioned system under loading at *infinity* by uniformly distributed normal forces with intensity, $p_1(p_3)$, acting in layers' lying direction, $Ox_1(Ox_3)$.

Let us write equilibrium equations, constitutive and geometrical relations for each layer and half-space:

$$\frac{\partial}{\partial x_j^{(k)}}\left[\sigma_{jn}^{(k)}\left(\delta_i^n+\frac{\partial u_i^{(k)}}{\partial x_n^{(k)}}\right)\right]=0\ ,\quad \sigma_{ji}^{(k)}=\lambda^{(k)}\theta^{(k)}\delta_i^j+2\mu^{(k)}\varepsilon_{ji}^{(k)}$$
$$2\varepsilon_{ij}^{(k)}=\frac{\partial u_i^{(k)}}{\partial x_j^{(k)}}+\frac{\partial u_j^{(k)}}{\partial x_i^{(k)}}+\frac{\partial u_n^{(k)}}{\partial x_i^{(k)}}\frac{\partial u_n^{(k)}}{\partial x_j^{(k)}}\ ,\quad i;j;n=1,2,3,\ k=1,2 \tag{1}$$

In eq. (1), the conventional notation is applied and the upper indices (1), (2), and (3) indicate the binder layer, reinforcing layer and half-space, respectively.

The substrate layer's initial insignificant imperfection is given by the middle surface equation:

$$x_2^{(2)}=F(x_1^{(2)},x_3^{(2)})=\varepsilon f(x_1,x_3)=A\mathrm{e}^{-\left(\frac{x_1}{\ell_1}\right)^2-\gamma^2\left(\frac{x_3}{\ell_1}\right)^2}=\varepsilon\ell_1\mathrm{e}^{-\left(\frac{x_1}{\ell_1}\right)^2-\gamma^2\left(\frac{x_3}{\ell_1}\right)^2} \tag{2}$$

where ε is a dimensionless small parameter ($0\le\varepsilon\ll1$), $\varepsilon=A/\ell_1$, $\gamma=\ell_1/\ell_3$. The geometrical meaning of the parameters ℓ_1 and ℓ_3 is illustrated in fig. 1.

It is asumed that on the free-surface of the system, the traction-free condition and the components between complete contact conditions on the interfaces are satisfied.

The arbitrary components of the considered systems' quantities which characterize the stress-strain state are denoted as series in the parameter ε:

$$\left\{\sigma_{ij}^{(k)};\varepsilon_{ij}^{(k)};u_i^{(k)}\right\}=\sum_{q=0}^{\infty}\varepsilon^q\left\{\sigma_{ij}^{(k),q},\varepsilon_{ij}^{(k),q},u_i^{(k),q}\right\} \tag{3}$$

The method of solution developed in [12, 14, 15] is employed and the zeroth approximation's values are determined:

$$\sigma_{11}^{(k),0}=\frac{E^{(k)}[p_1+\nu^{(k)}p_3]}{E^{(3)}[1-(\nu^{(k)})^2]}-\frac{E^{(k)}\nu^{(3)}[p_3+\nu^{(k)}p_1]}{E^{(3)}[1-(\nu^{(k)})^2]} \tag{4}$$

$$\sigma_{33}^{(k),0}=\frac{E^{(k)}[p_3+\nu^{(k)}p_1]}{E^{(3)}[1-(\nu^{(k)})^2]}-\frac{E^{(k)}\nu^{(3)}[p_1+\nu^{(k)}p_3]}{E^{(3)}[1-(\nu^{(k)})^2]},\ \sigma_{ij}^{(k),0}=0 \text{ for } ij\neq 11;33 \tag{5}$$

Let us determine the first approximation's values. Following equations and relations are obtained from eqs. (1), (3)-(5).

The governing field equations:

$$\frac{\partial\sigma_{ji}^{(k),1}}{\partial x_j^{(k)}}+\sigma_{11}^{(k),0}\frac{\partial^2 u_i^{(k),1}}{\partial[x_1^{(k)}]^2}+\sigma_{33}^{(k),0}\frac{\partial^2 u_i^{(k),1}}{\partial[x_3^{(k)}]^2}=0 \tag{6}$$

The mechanical and geometrical relations:

$$\sigma_{ji}^{(k),1}=\lambda^{(k)}\theta^{(k),1}\delta_i^j+2\mu^{(k)}\varepsilon_{ji}^{(k),1},\quad \theta^{(k),1}=\varepsilon_{11}^{(k),1}+\varepsilon_{22}^{(k),1}+\varepsilon_{33}^{(k),1}$$

$$\varepsilon_{ji}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_i^{(k),1}}{\partial x_j^{(k)}}+\frac{\partial u_j^{(k),1}}{\partial x_i^{(k)}}\right) \tag{7}$$

The boundary conditions:

$$\sigma_{21}^{(1),1}\Big|_{x_2^{(1)}=+h_1}=0,\ \sigma_{23}^{(1),1}\Big|_{x_2^{(1)}=+h_1}=0,\ \sigma_{22}^{(1),1}\Big|_{x_2^{(1)}=+h_1}=0,\ \sigma_{ij}^{(1),1},\ u_i^{(1),1}\to 0 \text{ as } x_2^{(1)}\to+\infty \tag{8}$$

The contact conditions:

$$\sigma_{2i}^{(1),1}\Big|_{x_2^{(1)}=-h_1}-\sigma_{2i}^{(2),1}\Big|_{x_2^{(2)}=+h_2}=\left(\sigma_{11}^{(1),0}-\sigma_{11}^{(2),0}\right)\frac{\mathrm{d}f}{\mathrm{d}x_1^{(1)}}\delta_i^1+\left(\sigma_{33}^{(1),0}-\sigma_{33}^{(2),0}\right)\frac{\mathrm{d}f}{\mathrm{d}x_3^{(1)}}\delta_i^3$$

$$\sigma_{2i}^{(2),1}\Big|_{x_2^{(1)}=-h_2}-\sigma_{2i}^{(3),1}\Big|_{x_2^{(1)}=-h_2}=\left(\sigma_{11}^{(2),0}-p_1\right)\frac{\mathrm{d}f}{\mathrm{d}x_1^{(1)}}\delta_i^1+\left(\sigma_{33}^{(2),0}-p_3\right)\frac{\mathrm{d}f}{\mathrm{d}x_3^{(1)}}\delta_i^3 \tag{9}$$

$$u_i^{(1),1}\Big|_{x_2^{(1)}=-h_1}-u_i^{(2),1}\Big|_{x_2^{(2)}=+h_2}=0\ ,\quad u_i^{(2),1}\Big|_{x_2^{(1)}=-h_2}-u_i^{(3),1}\Big|_{x_2^{(1)}=-h_2}=0\ ,\ i=1,2,3$$

$$\sigma_{11}^{(3),1}\to p_1,\ \sigma_{33}^{(3),1}\to p_3,\ \sigma_{ij}^{(3),1}\to 0 \text{ for } j\neq 11;33 \text{ as } x_2^{(1)}\to-\infty \tag{10}$$

For the given problem's solution in eqs. (4)-(10), the exponential double Fourier transform with respect to x_1 and x_3 is applied:

$$\varphi(x_1,x_2,x_3)=\frac{1}{\pi^2}\int_0^{+\infty}\int_0^{+\infty}\varphi_{13F}(s_1,x_2,s_3)\cos(s_1x_1)\cos(s_3x_3)\mathrm{d}s_1\mathrm{d}s_3 \tag{11}$$

The solution procedure is not detailed here because it is similar to that detailed in [12, 14, 15].

Numerical results and discussion

We take into account the distribution of self-equilibrated normal stress σ_{nn}/p_1 effecting on the interface surface between the binder layer and reinforcing layer. Let us denote the Young's moduli through $E^{(1)}$, $E^{(2)}$, $E^{(3)}$, and the Poisson ratios through $\nu^{(1)}$, $\nu^{(2)}$, $\nu^{(3)}$. We assume that $E^{(2)}/E^{(1)} = E^{(2)}/E^{(3)} = 100$, $E^{(2)} > E^{(1)} = E^{(3)}$, $p_3 = p_1$, $\gamma = 1$, $\nu^{(1)} = \nu^{(2)} = \nu^{(3)} = 0.3$, $x_2/h_2 = 1.0$, and $x_3/h_2 = 0$. The geometrical non-linearity's influence on the aforementioned distribution will be characterized as $\sigma_{11}^{(2),0}/\mu^{(2)}$ $[\mu^{(2)} = E^{(2)}/2(1 + \nu^{(2)})]$. Thus, taking the previously-stated consideration into account we analyse the numerical outcomes. Let us begin the analysis related to the dependence between σ_{nn}/p_1 and x_1/h_2. This dependencies' graphs established for different values of $\sigma_{11}^{(2),0}/\mu^{(2)}$ are given in fig. 2:

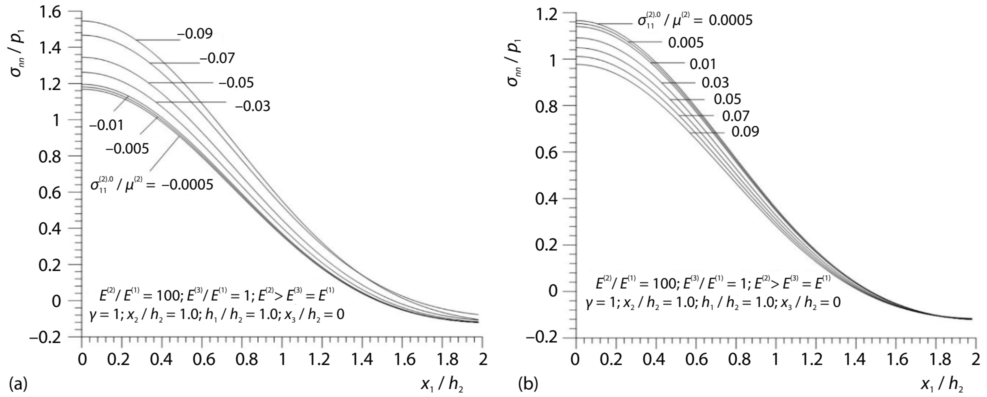

Figure 2. Graphs of dependencies; (a) between σ_{nn}/p_1 and x_1/h_2 (compressing), (b) the dependencies between σ_{nn}/p_1 and x_1/h_2 (stretching)

With respect to the mechanical consideration that is well-known, the $|\sigma_{nn}/p_1|$ values must approach zero with x_1/h_2. Proof of this prediction is seen by the graphs given in fig. 2. Nevertheless, these graphs show that because of the geometrical non-linearity, the absolute of σ_{nn}/p_1 values decrease with $\sigma_{11}^{(2),0}/\mu^{(2)}$ under tension and also increase with $\sigma_{11}^{(2),0}/\mu^{(2)}$ under compression of regarded material. Qualitatively these results are in agreement with the matching ones given in monograph [12]. Hereby, the outcomes exemplify the reliability and effectiveness of utilized algorithm and programs.

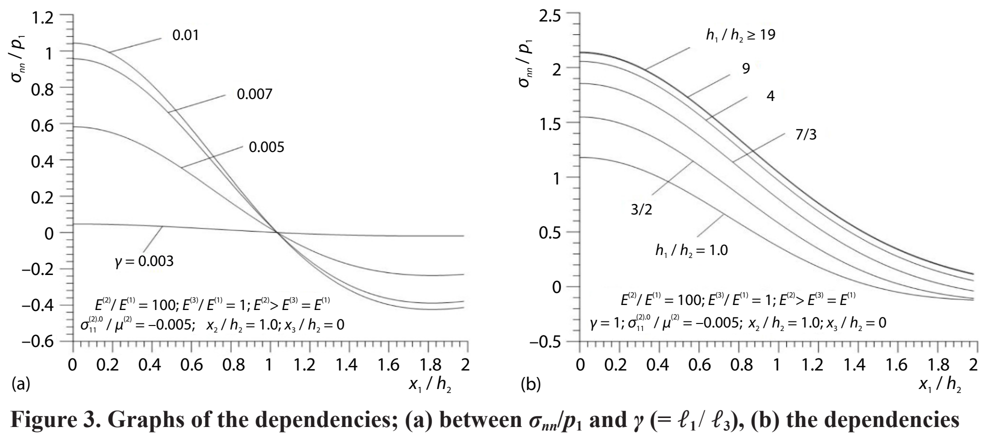

Figure 3. Graphs of the dependencies; (a) between σ_{nn}/p_1 and γ (= ℓ_1/ℓ_3), (b) the dependencies between σ_{nn}/p_1 and h_1/h_2

From fig. 3(a) it can easily be seen that as the values of γ is decrease, $|\sigma_{nn}/p_1|$ approaches a limit in which γ is equal to zero, that is to say, approaches the case where the curving of the reinforcing layer is a plane one. With respect to the mechanical consideration, σ_{nn}/p_1 must approach the certain limit value with the ratio h_1/h_2 and this limit value is σ_{nn}/p_1 which is obtained for the case where the spatially curved layer is in an infinite body. This estimation is approved with the graph seen in fig. 3(b).

Conclusion

Thus, in the present paper, within the framework of the piecewise homogeneous body model with the use of the exact equations of the geometrical non-linear theory, the system consisting of a face covering layer + spatially curved reinforcing layer + half-space has been studied. A method for the solution of the problem by applying the double Fourier transformation was developed. Numerical outcomes on the self-equilibrated stresses caused by the locally curved reinforcing layer under stretching and also under compressing of the mentioned system have been presented and analyzed.

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