FILTRATION EFFICIENCY OF A CIGARETTE FILTER WITH X- OR Y-SHAPED FIBERS

by

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Nicotine is harmful to the health of smokers and the inhalation of second-hand smoke is more harmful. A high efficiency of nicotine filtration for a cigarette filter is much needed. This paper suggests a formulation to predict filtration efficiency of a filter with X- or Y-shaped fibers. Main effects affecting the efficiency are elucidated using a fractal modification of the Darcy's law.

Key words: *fractal theory, fractal dimensions, shaped fiber, fractal calculus, cellulose acetate, trilobal-shaped fiber, Y-cross-sectional shape, X-cross-sectional shape, Darcy's law, hierarchical structure*

Introduction

According to the Science magazine (science.sciencemag.org), US teens' use of nicotine soars [1], other part of the world has a similar trend. Though nicotine in moderation is helpful for health, because it can prevent the formation of protein clumps which leads to Alzheimer's disease, and smokers appear to have a lower incidence of Alzheimer's disease [2]. Small amount of nicotine from smoking can greatly improve learning ability and prominently enhance memory, and it can also result in a rapid information processing with great arousal and attention [3]. However, nicotine has a high concentration in a tobacco, constituting up to 3.0%, too much nicotine from tobacco is harmful to the health of smokers and the inhalation of second-hand smoke is more harmful [4]. Tobacco factories have been trying to reduce the harm to an acceptable level by controlling fiber morphology, fiber number and tobacco's weight in a cigarette filter. The most used fibers in a cigarette filter is Y-shaped, which is often called as a trilobal-shaped fiber. Nantong Zhuhai Kunming Cellulose Fibers Co., Ltd. produces a new shaped fiber with X-cross-section, which has obvious advantages over its Y-shaped fibers in a low pressure drop and high filtration efficiency of nicotine. This paper will establish a theoretical model to predict nicotine's filtration efficiency.

Filter geometry

A cigarette filter [4] is used to prevent smokers from harm, it is generally made from cellulose acetate fibers [5-8]. The pressure drop and filtration efficiency are two main factors affecting consumer's feeling and attitude to the quality of cigarette. The most used fibers are either X-shaped or Y-shaped, see figs. 1 and 2. A filter consists of many fibers as illustrated in fig. 3.

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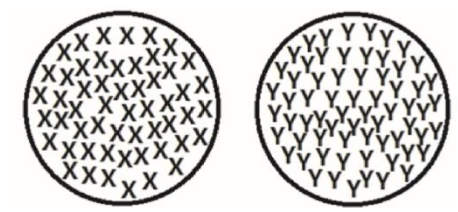

Figure 1. Cross-section of a cigarette filter with different shaped fibers

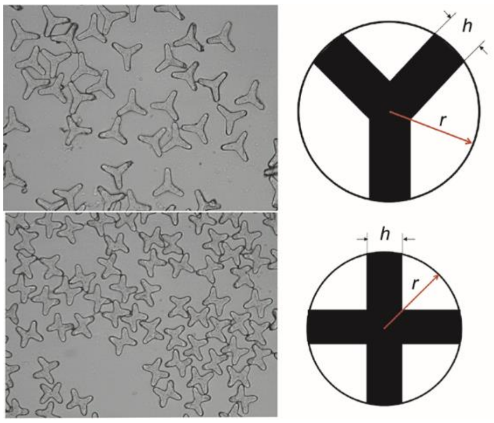

Figure 2. Different cross-sections of shaped fibers at 400× magnification

Figure 3. Fiber bundle in a cigarette filter

Fibers' cross-section, fiber number, and the filter's weight are three main factors affecting the filter's pressure drop and filtration efficiency.

Pressure drop through the filter

The pressure drop, Δp, through a porous medium can be predicted approximately by Darcy's law [9]:

$$\Delta p = \frac{\mu Q L}{\varepsilon A} \tag{1}$$

where A is the section area, ε – the constant, Q – the flow rate, μ – the permeability, and L – the tube length.

According to the fractal theory, a fractal modification of Darcy's law was proposed in [9], which reads:

$$\Delta p = k\frac{Q L^{\gamma}}{R^{\alpha}} \tag{2}$$

where R is the porous pipe's radius, Q – the volumetric flow rate, α and γ – the values of fractal dimensions for the filter section and the compacted fiber in the filter, respectively. When $\alpha = 2$ and $\gamma = 1$, eq. (2) turns out to be the Darcy's law.

Fractal approach is widely used to study the air permeability [10, 11], heat transfer [12-16] of discontinuous media including the natural fibers and nanofiber membranes [17-22]. Fractal theory is also a theoretical foundation for explanation of the Hall-Petch effect [23], lotus effect [24], and geometric potential [25, 26]. The fractional dimensions are relative to the order of the fractal order in the fractal calculus [27-29].

The fractal dimensions can be obtained by:

$$\alpha = \frac{\ln N}{\ln M} \tag{3}$$

where two scales are used, the larger one is L, and the smaller is l, $M = L/l$, on the larger scale, we have a unit, while on the smaller scale, the units are N.

To calculate the value of the fiber's fractal dimensions, we use the two scales, one is the filter's radius, R, and the other is the lobal half width, $h/2$, of a shaped fiber with radius of r, see fig. 2. For the scale of R, we have a unit area, and when we use the scale of $h/2$, we have the real area $\pi r^2 - 3rh$ for a trilobal-shaped fiber, *i. e.*, the number of new units is:

$$N = \frac{4n(\pi r^2 - 3rh)}{\pi h^2} \tag{4}$$

where n is the total number of fibers in a filter.

The scale ratio is:

$$M = \frac{2R}{h} \tag{5}$$

The fractal dimensions for the filter's section are:

$$\alpha = \frac{\ln N}{\ln M} = \frac{\ln \dfrac{4n(\pi r^2 - 3rh)}{\pi h^2}}{\ln \dfrac{2R}{h}} \tag{6}$$

For a multi-lobal shaped fiber, the fractal dimensions are:

$$\alpha = \frac{\ln \dfrac{4n(\pi r^2 - \lambda rh)}{\pi h^2}}{\ln \dfrac{2R}{h}} \tag{7}$$

where λ is the lobal number, for the X-shaped fiber as illustrated in fig. 2, $\lambda = 4$.

The value for γ can be calculated:

$$\gamma = \frac{\ln \dfrac{L_0}{R}}{\ln \dfrac{L}{R}} \tag{8}$$

where L_0 is the length of a fiber in a filter at a free condition, and it is compacted into a filter with length of L.

Filtration efficiency

Filtration efficiency of nicotine depends upon two factors, one is the fiber's surface area, and the other is the intake rate:

$$\eta \propto nCLQ \tag{9}$$

where η is the filtration efficiency, n – the fiber number in a filter, L – the filter's length, C – the perimeter of a shaped fiber, which can be written:

$$C = \lambda(h + 2r) \tag{10}$$

where λ is the lobal number, for the X-shaped fiber as illustrated in fig. 2, $\lambda = 4$ and $\lambda = 3$ for Y-shaped fiber.

The intake rate, Q, can be calculated from eq. (2), finally the filtration efficiency can be expressed:

$$\eta = Kn\lambda(h + 2r)\frac{\Delta p R^{\alpha}}{L^{\gamma-1}} \tag{11}$$

Filtration efficiency comparison between the X- and Y-shaped fibers

We assume all geometrical parameters for X- and Y-shaped filters are same, *i. e.*, the equivalent fiber's radius, r, and initial length, L_0, the filter's radius, R, and length, L, and the pressure drop is also same. The filtration efficiency comparison between the X- and Y-shaped fibers reads:

$$\frac{\eta_X}{\eta_Y} = \frac{4}{3} R^{\alpha_X - \alpha_Y} \tag{12}$$

where

$$\alpha_X - \alpha_Y = \frac{\ln \dfrac{4n(\pi r^2 - 4rh)}{\pi h^2}}{\ln \dfrac{2R}{h}} - \frac{\ln \dfrac{4n(\pi r^2 - 3rh)}{\pi h^2}}{\ln \dfrac{2R}{h}} = \frac{\ln \dfrac{\pi r^2 - 4rh}{\pi r^2 - 3rh}}{\ln \dfrac{2R}{h}} \tag{13}$$

It is obvious that the efficiency depends on filter's radius, fibers' radius and the lobal's length.

It is obvious that:

$$\frac{\pi r^2 - 4rh}{\pi r^2 - 3rh} < 1 \tag{14}$$

Equation (13) implies that:

$$\alpha_X - \alpha_Y < 0 \tag{15}$$

So there is a threshold of the filter's radius, R_{critical}, when $R < R_{\text{critical}}$ we have $\eta_X > \eta_Y$.

Experiment

Equation (11) reveals that $\eta \propto \Delta p.$. We compare the nicotine’s filtration efficiency between Y-shaped and X-shaped fibers. All cigarette parameters are kept same except the pressure drop. In our example we choose two samples 6.0Y17000 and 6.0X17000 for comparison. The experiment results are shown in tab. 1. For the samples of 6.0Y17000 and 6.0X17000, we have, respectively:

$$\eta = 3.384 \cdot 10^{-5} \Delta P \text{ for 6.0Y17000} \tag{16}$$

and

$$\eta = 3.439 \cdot 10^{-5} \Delta P \text{ for 6.0X17000} \tag{17}$$

It is shown that at the same pressure drop, the X-shaped filter has a higher nicotine’s filtration efficiency.

Table 1. Nicotine’s Filtration efficiency vs pressure drop

Samples	Nicotine’s filtration efficiency, [%]	Pressure drop, [Pa]
6.0Y17000 (Y-shaped fiber)	14.04	4100
	14.83	4300
	15.24	4500
	15.72	4800
6.0X17000 (X-shaped fiber)	13.54	3800
	14.27	4100
	14.77	4300
	15.34	4500
	15.87	4800

Conclusion

This paper gives a theoretical model for prediction of the filtration efficiency of a cigarette filter with X or Y-shaped fibers based on a fractal modification of the Darcy law. The filtration efficiency of filters with X-shaped fiber is higher when filter’s diameter is less than a threshold value.

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