THERMO DIFFUSION ASPECTS IN JEFFREY NANOFLUID OVER PERIODICALLY MOVING SURFACE WITH TIME DEPENDENT THERMAL CONDUCTIVITY

by

Sami ULLAH KHAN^a, Sabir ALI SHEHZAD^{a*}, Fahad MUNIR ABBASI^b, and Shahid HUSSAIN ARSHAD^c

^a Department of Mathematics, COMSATS University Islamabad, Sahiwal, Pakistan ^b Department of Mathematics, COMSATS University Islamabad, Islamabad, Pakistan ^c Department of Applied Science, National Textile University (NTU), Faisalabad, Pakistan

> Original scientific paper https://doi.org/10.2298/TSCI190428312U

Double diffusion flow of Jeffrey fluid in presence of nanoparticles is studied theoretically under time dependent thermal conductivity. The considered nanoparticles are evaporated over convectively heated surface which moves periodically in its own plane. The appropriate dimensionless variables are employed to obtain the dimensionless forms of governing equations. We computed the analytical solution of non-linear differential equations by utilizing homotopy analysis method. The present investigation reveals the features of various emerging parameters like Deborah number, combined parameter, oscillation frequency to stretching rate ratio, Prandtl number, Lewis number, thermophoresis parameter, Brownian motion parameter, nano Lewis number, modified Dufour parameter, and Dufour solutal Lewis number. A useful enhancement in movement of nanoparticles is observed by utilizing the combined magnetic and porosity effects. Unlike traditional studies, present analysis is confined with the unsteady transportation phenomenon from periodically moving surfaces. Such computation may be attributable in flow results from tensional vibrations due to stretching and elastic surfaces. The simulation presented here can be attractable significance in the bioengineered nanoparticles manufacturing. It is observed that the heat transportation of nanoparticles may efficiently enhance through the utilization of variable thermal conductivity. The solutal concentration decreases with increasing Deborah number and Lewis number. It is further noted that the nano Lewis number causes reduction of nanoparticles concentration.

Key words: Jeffrey nanofluid, time dependent thermal conductivity, double diffusion flow, oscillatory stretching surface

Introduction

Substantial explorations pertaining to the flow of non-Newtonian fluids are proposed in recent century because of their convenient applications in many chemical industries. The general applications include polymer, biological solutions, manufacturing crude soft material, greases, glues, chemicals, paints, petroleum, and oil reservoir engineering. The exclusive aspect of these fluid models is the existence of non-linear relationship between shear stress and defor-

^{*} Corresponding author, e-mail: ali_qau70@yahoo.com

mation rate and consequently referred power law model. Unlike viscous fluid models, the rheology of non-Newtonian fluids is not easy to entertain. Therefore, in modern era, various models are enlarged by engineers to thrash out the diverse rheological properties. Jeffrey fluid is probably one of model which attained special attraction by researchers as it profitably forecast the relaxation and retardation time effects. Owing to such interesting rheological behavior, various attempts for flow of Jeffrey fluid have been made using various flow features. Jawad et al. [1] reported the Jeffrey fluid flow over convectively heated surface in presence of applied magnetic field. The analytical computation performed by Hayat et al. [2] for unsteady flow of Jeffrey fluid considered over stretching surface. The slip flow and melting heat feature in radiative flow of Jeffrey fluid has been examined by Das et al. [3]. Another investigation concentrated by Ahmad and Ishak [4] on stretching flow of Jeffrey fluid in presence of porous medium. Nam et al. [5] simulated numerical computations on free convection flow of Jeffrey fluid in presence of wall ramped temperature over vertical moving surface. In the work of Ramesh [6], results were suggested for stagnation point flow of Jeffrey fluid over vertical moving surface. The addition of heat source/sink in chemical reactive flow of Jeffrey fluid specified by vertical cone was analyzed by Saleem et al. [7]. The study of chemically reactive Jeffrey fluid in presence of heat and mass transport phenomenon has been numerically treated by Narayana and Babu [8]. Hussain et al. [9] studied peristaltic transport of Jeffrey fluid in presence of variable thermal conductivity. They performed the analytical computations for the dimensionless problem. Babu and Narayana [10] investigated the melting heat transfer effects in the radiative flow of Jeffrey fluid caused by a parallel moving surface. A numerical based continuation dealing with the flow of Jeffrey fluid over a porous stretched surface was performed by Narayana et al. [11].

Recent advancement in the field of nanotechnology has been impelled in the field of chemical, mechanical, biomedical products, industrial applications, and to improve the energy consumptions. The term *nanofluid* is compliance in the era of scientific research recently and attracted the interest of researchers. Nanofluids are frequently used enhance the heat and mass transportation, nuclear systems, cooling of objects, solar generation systems, biomedical applications, storage of energy, cooling of reactors etc. These are mixture of tiny particles immersed in the base fluid which improve the thermal properties due to interaction of nanoparticles. The concept of nanofluid was theoretical formulated in 1995 by Choi and Estman [12] in which he experimentally showed that the enhancement in poor thermal conductivity can be efficiently improved by addition of tiny sized particles. The Brownian and thermophoresis aspect in flow of nanofluid has been examined by Buongiorno [13]. Liu et al. [14] pointed out impact of heat absorption in flow of rate type nanofluid over finite thin film. The numerical treatment regarding flow of pseudo-plastic fluid in presence of nanoparticles was reported by Lin et al. [15]. Sheikholeslami et al. [16] theoretically justified the impact of magnetic field in presence of nanoparticles distribution over rotating system. In more continuation, Sheikholeslami [17] utilized the CuO-water nanoparticles in channel flow by means of using mesoscopic method. The numerical results for flow of Eyring Powell nanofluid has been accomplished by Malik et al. [18]. Hayat et al. [19] examined the stretching flow of nanofluid over convectively heated surface analytically. The enhancement of heat transfer for forced convection flow of nanofluid was originated by Sheikholeslami and Bhatti [20]. The peristaltic transport of nanofluid in presence of Hall current and entropy generation was explored by Abbasi et al. [21]. The electro-osmotic flow of Jeffrey nanofluid over rotating micro-channel has been directed by Reddy et al. [22]. Mahantesh et al. [23] performed numerical computations on the characteristics of Carreau nanofluid over stretching surface in presence of nanoparticles. Gireesha et al. [24] explained the non-linear thermal radiation flow of Maxwell nanofluid caused by stretching surface. Babu *et al.* [25] performed the numerical computations based on Runge-Kutta method for Jeffrey nanofluid over a moving stretched sheet. The bioconvection flow of viscous nanofluid with various slip features over a rotating cone has been numerically analyzed by Latif *et al.* [26]. Abro *et al.* [27] examined the characteristics of heat transfer in electrically conducting flow of single as well as multiple carbon nanotubes in flow of Casson fluid under the influence of magnetic field. Goyal and Bhargava [28] numerically investigated the thermodiffusion features in the nanofluid flow over a non-linear stretched surface.

The aim of current investigation is to report the transportation solutal concentration in flow of non-Newtonian nanoparticles over periodically moving convectively heated surface. The combined magnetic and porosity aspects are also implemented. These theoretical computations can be valuable in manufacturing processes, enhancement of energy transport and heat resources. An efficient analytical method, homotopy analysis method, is occupied to determine the solution of dimensionless equations. The characteristics of various parameters are evaluated graphically.

Flow problem

We assume 2-D and unsteady flow of Jeffrey nanofluid over in presence of variable thermal conductivity, fig. 1. The flow is engaged due to the periodically stretching surface which account the accillatory.

which governed the oscillatory velocity *i. e.* $u = b\overline{x} \sin \omega t$. The constant *b* denotes maximum rate of stretched while ω expresses the frequency. Further, the aspect of thermophoresis and Brownian motion are also utilized in aspect of thermo-diffusion effects. The effects of magnetic field are carried out in the transverse directions. Let T_w is the temperature of the sur-



Figure 1. Geometry of the problem

face, C_w – the solutal concentration, while ϕ_w – the nanoparticles concentration, while T_{∞} , C_{∞} , and ϕ_{∞} – the respective ambient values for related profiles.

The dimensional equations concerned the mass, momentum, heat and diffusion of concentrations for nanoparticles are examined as [2, 28]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial \overline{x}} + v \frac{\partial u}{\partial \overline{y}} = \frac{v}{1+\lambda} \left[\frac{\partial^2 u}{\partial \overline{y}^2} + \lambda_1 \left(\frac{\partial^3 u}{\partial \overline{y}^2 \partial t} + u \frac{\partial^3 u}{\partial \overline{x} \partial \overline{y}^2} - \frac{\partial u}{\partial \overline{x}} \frac{\partial^2 u}{\partial \overline{y}^2} + \frac{\partial u}{\partial \overline{y}} \frac{\partial^2 u}{\partial x \partial \overline{y}} + v \frac{\partial^3 u}{\partial \overline{y}^3} \right) \right] - \frac{\sigma_e B_0^2}{\rho_f} u - \frac{v \varphi^*}{k^*} u$$
(2)

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial \overline{x}} + v \frac{\partial T}{\partial \overline{y}} = \frac{1}{(\rho c)_p} \frac{\partial}{\partial \overline{y}} \left[k(T) \frac{\partial T}{\partial \overline{y}} \right] + \tau_1 \left[D_T \frac{\partial \phi}{\partial \overline{y}} \frac{\partial T}{\partial \overline{y}} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial \overline{y}} \right)^2 \right] + DK_{TC} \frac{\partial^2 C}{\partial \overline{y}^2} \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial \overline{x}} + v \frac{\partial C}{\partial \overline{y}} = D_{s} \frac{\partial^{2} C}{\partial \overline{y}^{2}} + DK_{CT} \frac{\partial^{2} T}{\partial \overline{y}^{2}}$$
(4)

$$\frac{\partial \varphi}{\partial t} + u \frac{\partial \varphi}{\partial \overline{x}} + v \frac{\partial \varphi}{\partial \overline{y}} = D_{\rm B} \frac{\partial^2 \varphi}{\partial \overline{y}^2} + \frac{D_{\rm T}}{T_{\infty}} \frac{\partial^2 T}{\partial \overline{y}^2}$$
(5)

where *u* is the horizontal velocity component, *v* – the vertical component, *v* – the kinematic viscosity, λ – the ratio of relaxation to retardation time, λ_1 – the retardation constant, σ_e – the electrical conductivity, ρ_f – the base fluid density, B_0 – governed with magnetic field strength, φ^* – the porous medium, k^* – imply permeability of porous medium, *T* – entails temperature, $\tau_1 = (\rho c)_p / (\rho c)_f$ – the ratio between heat capacity of nanoparticles material to heat capacity of fluid, *k* – the thermal conductivity, DK_{TC} – the Dufour diffusivity, DK_{CT} – the Soret diffusivity, *C* – the solutal concentration, φ – the nanoparticles volume fraction, D_s – the solutal diffusivity, and D_B – the Brownian diffusion coefficient. The initial and boundary conditions concerned to the present flow field are articulated:

$$u = b\overline{x}\sin\omega t, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad \varphi = \varphi_w \quad \text{at} \quad \overline{y} = 0, \quad t > 0$$
(6)

$$u \to 0, \quad \frac{\partial u}{\partial \overline{y}} \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty}, \quad \varphi \to \varphi_{\infty}, \quad \text{as} \quad \overline{y} \to \infty$$
(7)

The energy eq. (3) for thermal conductivity is modified:

$$k(T) = k_{\infty} \left(1 + \varepsilon \frac{T - T_{\infty}}{\Delta T} \right)$$
(8)

where k_{∞} is termed as ambient liquid conductivity, while ε is the thermal dependence conductivity parameter. We interpolate the flowing suitable variables to acquire the dimensionless variables:

$$u = b\overline{x}f_{y}(y,\tau), \quad v = -\sqrt{vb}f(y,\tau), \quad y = \sqrt{\frac{b}{v}\overline{y}}, \quad \tau = t\omega$$
(9)

$$\theta(y,\tau) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad s(y,\tau) = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \quad \phi(y,\tau) = \frac{\varphi - \varphi_{\infty}}{\varphi_w - \varphi_{\infty}}$$
(10)

Inserting these variables in eqs. (2)-(4) leads to following dimensionless PDE:

$$f_{yyy} + \beta (Sf_{yyy\tau} + f_{yy}^2 - ff_{yyyy}) - (1 + \lambda)(Sf_{y\tau} + f_y^2 - ff_{yy}) - M(1 + \lambda)f_y = 0$$
(11)

$$(1 + \varepsilon \theta)\theta_{yy} + \varepsilon(\theta_y)^2 + \Pr[-S\phi_\tau + f\phi_y + Nb\theta_y\phi_y + Nt(\theta_y)^2 + (Nd)s_{yy}] = 0$$
(12)

$$s_{yy} + \operatorname{Le}(f\phi_y - S\phi_\tau) + \operatorname{Ld}\theta_{yy} = 0$$
(13)

$$\phi_{yy} + \operatorname{Ln}(f\phi_y - S\phi_\tau) + \frac{Nt}{Nb}\theta_{yy} = 0$$
(14)

In previous dimensionless equations β is the Deborah number, M – the combined Hartmann number and porosity parameter (combined parameter), S – the oscillation frequency to stretching rate ratio, Pr – the Prandtl number, Nt – the thermophoresis parameter, Nb – the

Brownian motion parameter, Nd – the modified Dufour number, Le – the regular Lewis number, Ld – the Dufour Lewis number, and Ln – the nano Lewis number which are defined in the following forms:

$$\beta = \lambda_{1}b, \quad M = \frac{\sigma B_{0}^{2}}{\rho b} + \frac{v\varphi^{*}}{kb}, \quad S = \frac{\omega}{b}, \quad \text{Le} = \frac{v}{D_{s}}, \quad Nt = (\rho c)_{p} \frac{D_{T}(T_{f} - T_{\infty})}{(\rho c)_{f}T_{\infty}v}$$
$$Nb = \frac{(\rho c)_{p} D_{B}(C_{f} - C_{\infty})}{(\rho c)_{f}v}, \quad \text{Ln} = \frac{v}{D_{B}}, \quad \text{Ld} = \frac{D_{CT}(T_{w} - T_{\infty})}{\alpha_{m}(C_{w} - C_{\infty})}, \quad Nd = \frac{D_{TC}(C_{w} - C_{\infty})}{\alpha_{m}(T_{w} - T_{\infty})}$$

The boundary conditions are:

$$f_{y}(0,\tau) = \sin \tau, \quad f(0,\tau) = 0, \quad \theta(0,\tau) = 1, \quad s(0,\tau) = 1, \quad \phi(0,\tau) = 1$$
 (15)

$$f_y(\infty,\tau) \to 0, \quad f_{yy}(\infty,\tau) \to 0, \quad \theta(\infty,\tau) \to 0, \quad s(\infty,\tau) \to 0, \quad \phi(\infty,\tau) \to 0$$
(16)

We signify the following quantities associated with local Nusselt number, local Sherwood number, and local nano Sherwood number:

$$\operatorname{Nu}_{x} = \frac{\overline{x}q_{\mathrm{s}}}{k(T_{w} - T_{\infty})}, \quad \operatorname{Sh}_{x} = \frac{\overline{x}j_{\mathrm{s}}}{D_{\mathrm{B}}(C_{w} - C_{\infty})}, \quad \operatorname{Sh}_{xn} = \frac{\overline{x}q_{mn}}{D_{\mathrm{s}}(\varphi_{w} - \varphi_{\infty})}$$
(17)

where q_s is the heat flux at the plate, j_s – the surface mass flux, while q_{mn} correlates the nanoparticle mass flux. Interleave eqs. (9) and (10) in previous expressions get following forms:

$$\operatorname{Nu}_{x}\operatorname{Re}_{x}^{-1/2} = -\theta_{y}(0,\tau), \quad \operatorname{Sh}_{x}\operatorname{Re}_{x}^{-1/2} = -s_{y}(0,\tau), \quad \operatorname{Sh}_{nx}\operatorname{Re}_{x}^{-1/2} = -\phi_{y}(0,\tau)$$
(18)

where $\operatorname{Re}_{x} = u_{w}\overline{x}/v$ is the local Reynolds number.

Analysis of results

 $\tau = 9.5\pi$

The field equations are numerically solved analytically by using homotopy analysis method. This method is originally advised by Liao [29] and later on many investigators computed their results by using this method [30-34]. In fact, homotopy analysis method is a series solution method for which the convergence of obtained solution is strictly depend on appropriate selection of auxiliary constants namely, h_f , h_θ , h_s , and h_{ϕ} . In order to verify our results, the computed solution is compared with already reported continuation presented by Zheng *et al.* [35] and Abbas *et al.* [36], Table 1 is prepared. Here, an excellent accuracy of results has been reported.

 τ [35][36]Present results $\tau = 1.5\pi$ 11.67865611.67865611.678656 $\tau = 5.5\pi$ 11.67870611.67870711.678706

Table 1. Comparison of $f''(0, \tau)$ with [34, 35], when S = 1, M = 12, and $\lambda = 0$

11.678656

Figure 2 displays the *h* curves for velocity, temperature, nanoparticles solutal concentration and concentration profiles for some precise values of engineering parameters like $\lambda = 0.2$, S = 0.4, $\beta = 0.5$, M = 0.4, Pr = 0.5, $\varepsilon = 0.3$, Ld = 0.3, Sc = 0.3, Le = 0.3, Ln = 03, and

11.678656

11.678656

 $\tau = \pi/2$. It is observed that suitable values for preeminent solution can be picked from $-1.5 \le h_{\rm f} \le 0, -1.2 \le h_{\theta} \le 0.0, -1.2 \le h_{\rm s} \le 0, -1.1 \le h_{\phi} \le 0$. In order to vary the associated parameter, all other parameters accomplished some constant values like $\lambda = 0.2$, S = 0.4, $\beta = 0.5$, M = 0.4, $\Pr = 0.5$, $\varepsilon = 0.3$, Ld = 0.3, Le = 0.3, Ln = 0.3, and $\tau = \pi/2$. The profiles temperature, solutal concentration and nanoparticles concentration at different time instants $\tau = \pi/6, \pi, 5\pi/6, \text{ and } \pi/3$ are reflected in fig. 3. The associated profiles are massively depressed as time varies. The temperature, solutal concentration and nanoparticles concentration accentuated with a diminishing behavior by varying time $\pi/6$ to $\pi/3$. However, the concentration profile altered quite marginally as compared to temperature and solute concentration distribution.





Figure 2. The *h*-curves for velocity, temperature, solutal concentration, and nanoparticles concentration profiles

Figure 3. Variation of time with velocity, solutal concentration, and concentration profiles

The behavior of temperature profile, θ , for variation of Deborah number, β , thermophoresis parameter, Nt, variable thermal conductivity parameter, ε , combined parameter, M, and modified Dufour number, Nd, is utilizes in figs. 4(a)-4(e). Figure 4(a) reports the significance of β for temperature profiles while all other parameters keep constants. It is important to mention that minute values of Deborah number ($\beta \le 1$) portrayed the liquid behavior. The higher values exempted the solid like behavior. It is observed that the temperature of nanoparticles for solid case is lesser than viscous case. However, the change in temperature distribution is quite marginal. Moreover, a decreasing trend in associated layer is scrutinized for leading values of β The increasing values of β accompanying the depressed thermal boundary-layer. In order to evaluate the behavior of Nt on θ , fig. 4(b) is presented. A comparative increasing trend is noted in temperature profile as Nt varies from 0 to 1.5. The thermophoresis is interesting phenomenon in which tiny fluid particles migrate from the region of high temperature to low temperature. Due to this transportation, a collection of fluid particles is dragged away from heated surface due to which the temperature of liquid as well as thermal boundary-layer increases. The implication of ε on θ is investigated in fig. 4(c). The variable thermal conductivity results maximum temperature of nanoparticles in the whole region. Further, a thicker thermal boundary-layer has been noted for the leading values of ε . Therefore, presence of variable thermal conductivity may more useful to enhance the heat transportation. Figure 4(d) summarized the effects of M on θ . The combined parameters bring to light joint effects of both Hartmann number and porosity parameter. Higher values of Hartmann number bring Lorentz force in system which enhanced the temperature of fluid. Secondly, the occurrence of porosity medium plays similar role due to permeability medium. Figure 4(e) reports the up shoot of Nd on θ . It is manifested that θ appears to be increase with variation of Nd.



Figures 5(a)-5(d) is sketched to probe the impact of Deborah number, β , ratio of relaxation to retardation time, λ , regular Lewis number, Le, Dufour Lewis number, Ld, and combined parameter, M, on solutal concentration, s. The observation noted from the fig. 5(a) divulge that s escalates by varying β . However, the decreasing trend is relatively slower. The inspirations of ratio of relaxation to retardation time λ on s are specified in fig. 5(b). This figure demonstrates that solutal concentration increases by increasing λ . Figure 5(c) examines the trend of regular Lewis number on solutal concentration s. The progressive values of Lewis number result a decrement in s quite efficiently. Moreover, a depressed solutal concentration boundarylayer has been notified by varying Lewis number. The physical aspect behind this trend is that Lewis number is associated with thermal diffusion to species diffusion. Hence by varying Lewis number, the rate of species diffusion becomes relatively slower and as a result solutal concentration reduced. Figure 5(d) portrays the effect of Dufour Lewis number, Ld, on solutal concentration, s. A substantial growth in s is noted for larger values of Ld.

The collective significance of various parameters like Deborah number, β , ratio of relaxation to retardation time, λ , nano Lewis number, Ln, and Brownian motion parameter, *Nb*, on nanoparticles concentration, ϕ , is attended in figs. 6(a)-6(d). Figure 6(a) accomplished the







consequence of β on ϕ . It is noted that the concentration of nanoparticles demoralized by increasing β . It is also remarked that change in nanoparticles concentration for β is relatively

slower as compared to solutal concentration, *s*. By definition, Deborah number is associated with retardation time, large retardation times materials are generally less viscous in nature which results in increases in velocity and consequently reduces the concentration distribution of nanoparticles. The pivotal behavior of λ on ϕ is illustrated in fig. 6(b). An advancement of concentration profile is observed by varying λ . Figure 6(c) quantify the observations of Ln on ϕ . A decline behavior is examined by increasing Ln. The structure of associated boundary-layer seems to be thicker as Ln varies. The variation of *Nb* on ϕ is visualized in fig. 6(d). It is detected that enlarging values of *Nb* magnifies the concentration distribution which turns down the concentration of nanoparticles. Moreover, the appearance of *Nb* is because of interaction of nanoparticles which are associated with Brownian motion. It is observed that rate of change in concentration distribution.

Table 2 summarized the numerical computations for local Nusselt number, $Nu_x Re_x^{-1/2}$, nanofluid Sherwood number, $Sh_{nx} Re_x^{-1/2}$, and local Sherwood number, $Sh_x Re_x^{-1/2}$ for various parameters. We observed that $Nu_x Re_x^{-1/2}$ risen with larger Deborah number and Prandtl number. However, a lower trend is noted for thermophoresis parameter, Brownian motion parameter and relaxation to retardation time constant. The observations for rate of mass transfer $Sh_x Re_x^{-1/2}$ becomes for greater variation of Brownian motion parameter. An increasing trend of nanofluid Sherwood number $Sh_{nx} Re_x^{-1/2}$ is accomplished by varying Deborah number.

β	Pr	λ	Nt	Nb	Nd	$- heta_y(0, au)$	$-s_y(0, \tau)$	$-\phi_y(0, \tau)$
0.0 0.5 1.5	0.7	0.2	0.2	0.2	0.2	0.661082 0.681833 0.690130	0.51936 0.520409 0.541459	0.779282 0.799285 0.809292
0.2	0.1 0.7 1.0					0.705145 0.797551 0.861138	0.516187 0.728766 0.731677	0.800311 0.913611 0.954499
	0.7	0.0 0.5 1.5				0.822811 0.802723 0.792412	0.758896 0.739601 0.730313	0.936444 0.924341 0.911122
		0.2	0.0 0.8 1.2			0.731395 0.66338 0.616906	0.754331 0.509362 0.33585	0.880266 0.838720 0.817932
			0.2	0.5 0.6 1.0		0.631852 0.612766 0.564873	0.550416 0.647405 0.687644	0. 78231 0.73264 0.66951
				0.2	0.0 0.5 1.0	0.702159 0.681844 0.661326	0.570572 0.560216 0.559771	1.08437 1.08422 1.08404
					0.2	0.712333 0.700844 0.680326	0.565636 0.551216 0.546671	0.96545 0.96001 0.95987

Table 2. Numerical values of local Nusselt number, local Sherwood number and local nano Sherwood number

Conclusion

We proposed a theoretical analysis to simulate the thermophysical aspects of nanoparticles in presence of non-Newtonian fluid. With help of appropriate quantities, the thermophysical parameters from governing equations are diverted in dimensionless forms. Analytical solution has been reported by using homotopy analysis method. Due to variation of time, the temperature, solutal concentration, and nanoparticles concentrations linearly decrease. Convective heat transfer of nanoparticles effectively improved with variable thermal conductivity, modified Dufour number and combined parameter. For the larger Dufour Lewis number, the solute concentration arises. The solutal concentration becomes slower by varying Lewis number and Deborah number. Further, an enhancement in concentration of nanoparticles is observed for relaxation to retardation time parameter.

Acknowledgment

We are grateful to the reviewers for their useful suggestions and comments to enhance the novelty of manuscript. Second and third authors thanks the financial support of Higher Education Commission (HEC) of Pakistan under National Research Programme for Universities (NRPU) project no 5281.

References

- Ahmed, J., et al., A Note on Convective Heat Transfer of an MHD Jeffrey Fluid over a Stretching Sheet, AIP Advances, 5, (2015), ID 117117
- [2] Hayat, T., et al., Unsteady Flow and Heat Transfer of Jeffrey Fluid over a Stretching Sheet, Thermal Science, 18 (2014), 4, pp. 1069-1078
- [3] Das, K., et al., Radiative Flow of MHD Jeffrey Fluid past a Stretching Sheet with Surface Slip and Melting Heat Transfer, Alexandria Engineering Journal, 54 (2015), 4, pp. 815-821
- [4] Ahmad, K., Ishak, A., Magnetohydrodynamic (MHD) Jeffrey Fluid over a Stretching Vertical Surface in a Porous Medium, *Propulsion and Power Research*, 6 (2017), 4 pp. 269-276
- [5] Nam, Z., et al., Influence of Thermal Radiation on Unsteady MHD Free Convection Flow of Jeffrey Fluid over a Vertical Plate with Ramped Wall Temperature, *Mathematical Problems in Engineering*, 2016 (2016), ID 6257071
- [6] Ramesh, G. K, Numerical Study of the Influence of Heat Source on Stagnation Point Flow towards a Stretching Surface of a Jeffrey Nanoliquid, *Journal of Engineering*, 2015 (2015), ID 382061
- [7] Saleem, S., et al., Convective Heat and Mass Transfer in Magneto Jeffrey Fluid Flow on a Rotating Cone with Heat Source and Chemical Reaction, *Communications in Theoretical Physics*, 70 (2018), 5, pp. 534
- [8] Narayana, P. V. S., Babu, D. H., Numerical Study of MHD Heat and Mass Transfer of a Jeffrey Fluid over a Stretching Sheet with Chemical Reaction and Thermal Radiation, *Journal of the Taiwan Institute of Chemical Engineers*, 59, (2016), Feb., pp. 18-25
- [9] Hussain, Q., et al., Heat Transfer Analysis in Peristaltic Flow of MHD Jeffrey Fluid with Variable Thermal Conductivity, Applied Mathematics and Computations (English Edition), 36 (2015), Feb., pp. 499-516
- [10] Babu, D. H., Narayana, P. V. S., Melting Heat Transfer and Radiation Effects on Jeffrey Fluid Flow over a Continuously Moving Surface with Parallel Free Stream, *Journal of Applied and Computational Mechanics*, 5 (2019), 2, pp. 468-476
- [11] Narayana, P. V. S., et al., Numerical Study of a Jeffrey Fluid over a Porous Stretching Sheet with Heat Source/Sink, International Journal of Fluid Mechanics Research, 46 (2019), 2, pp. 187-197
- [12] Choi, S. U. S., Estman, J, Enhancing Thermal Conductivity of Fluids with Nanoparticles. ASME Publications-Fed, 231 (1995), Oct., pp. 99-106
- [13] Buongiorno, J. Convective Transport in Nanofluids, Journal of Heat Transfer, 128 (2006), 3, pp. 240-250
- [14] Liu, T., et al., Unsteady Flow and Heat Transfer of Maxwell Nanofluid in a Finite Thin Film with Internal Heat Generation and Thermophoresis, *Thermal Science*, 22 (2018), 6B, pp. 2803-2813

- [15] Lin, Y., et al., MHD Pseudo-Plastic Nanofluid Unsteady Flow and Heat Transfer in a Finite Thin Film over Stretching Surface with Internal Heat Generation, International Journal of Heat and Mass Transfer, 84, (2015), May, pp. 903-911
- [16] Sheikholeslami, M., et al., Nanofluid Flow and Heat Transfer in a Rotating System in the Presence of a Magnetic Field, Journal of Molecular Liquids, 190 (2014), Feb., pp. 112-120
- [17] Sheikholeslami, M., Numerical Investigation for CuO-H₂O Nanofluid Flow in a Porous Channel with Magnetic Field Using Mesoscopic Method, *Journal of Molecular Liquids*, 249 (2018), Jan., pp. 739-746
- [18] Malik, M. Y., *et al.*, Mixed Convection Flow of MHD Eyring-Powell Nanofluid over a Stretching Sheet: A Numerical, *AIP Advances 5*, (2015), ID 117118
- [19] Hayat, T., et al., Magnetohydrodynamic Flow of Nanofluid over Permeable Stretching Sheet with Convective Boundary Conditions, *Thermal science*, 20 (2016), 6, pp. 1835-1845
- [20] Sheikholeslami, M., Bhatti, M. M., Forced Convection of Nanofluid in Presence of Constant Magnetic Field Considering Shape Effects of Nanoparticles, *International Journal of Heat and Mass Transfer 111*, (2017), Aug., pp. 1039-1049
- [21] Abbasi, F. M., et al., Analysis of Entropy Generation in Peristaltic Nanofluid Flow with Ohmic Heating and Hall Current, Physica. Scripta, 94 (2019), 2, ID 025001
- [22] Reddy, K. V., et al., Thermophoresis and Brownian Motion Effects on Magnetohydrodynamics Electro-Osmotic Jeffrey Nanofluid Peristaltic Flow in Asymmetric Rotating Microchannel, Journal of Nanofluids, 8 (2019), 2, pp. 349-358
- [23] Mahantesh, M., et al., Heat and Mass Transfer Analysis of Carreau Nanofluid Over an Exponentially Stretching Sheet in a Saturated Porous Medium, *Nanofluids*, 8 (2019), 5, pp. 990-997
- [24] Gireesha, B. J., *et al.*, Non-Linear Radiative Heat Transfer and Boundary Layer Flow of Maxwell Nanofluid Past Stretching Sheet, *Journal of Nanofluids*, 8 (2019), 5, pp. 1093-1102
- [25] Babu, D. H., et al., Thermal Radiation and Heat Source effects in MHD Non-Newtonian Nanofluid Flow over a Stretching Sheet, Journal of Nanofluids, 8 (2019), 5, pp. 1085-1092
- [26] Latif, N. A. A., et al., Unsteady MHD Bio-Nanoconvective Anistropic Slip Flow past a Vertical Rotating Cone, *Thermal Science*, 23 (2019), 2A, pp. 427-441
- [27] Abro, K. A., et al., Effects of Carbon Nanotubes on Magnetohydrodynamic Flow of Methanol Based Nanofluids via Atangana-Baleanu and Caputo-Fabrizio Fractional Derivatives, *Thermal Science*, 23 (2019), 2B, pp. 883-898
- [28] Goyal, M., Bhargava, R., Numerical Study of Thermodiffusion Effects on Boundary Layer Flow of Nanofluids over a Power Law Stretching Sheet, *Microfluid Nanofluid*, 17 (2014), 3, pp. 591-604
- [29] Liao, S. J., Advance in the Homotopy Analysis Method, 5 Toh Tuck Link, World Scientific Publishing, Singapore, 2014
- [30] Turkyilmazoglu, M., Analytic Approximate Solutions of Rotating Disk Boundary Layer Flow Subject to a Uniform Suction or Injection, *International Journal of Mechanical Sciences*, 52 (2010), 12, pp. 1735-1744
- [31] Turkyilmazoglu, M., Determination of the Correct Range of Physical Parameters in the Approximate Analytical Solutions of Non-Linear Equations Using the Adomian Decomposition Method, *Mediterr Journal* of Mathematics, 13 (2016), 6, pp. 4019-4037
- [32] Turkyilmazoglu, M., Some Issues on HPM and HAM Methods: A Convergence Scheme, Mathematical and Computer Modelling 53, (2011), 9-10, pp. 1929-1936
- [33] Dinarvand, S., *et al.*, Homotopy Analysis Method for Mixed Convective Boundary Layer Flow of a Nanofluid over a Vertical Circular Cylinder, *Thermal Science*, *19* (2015), 2, pp. 549-561
- [34] Turkyilmazoglu, M., The Analytical Solution of Mixed Convection Heat Transfer and Fluid Flow of a MHD Viscoelastic Fluid over a Permeable Stretching Surface, *International Journal Mechanical Sciences*, 77 (2013), Dec., pp. 263-268
- [35] Zheng, L. C., et al., Unsteady Heat and Mass Transfer in MHD Flow over an Oscillatory Stretching Surface with Soret and Dufour Effects, Acta Mechica Sinica, 29 (2013), 5, pp. 667-675
- [36] Abbas, Z., et al., Hydromagnetic Flow in a Viscoelastic Fluid due to the Oscillatory Stretching Surface, International Journal of Non-Linear Mechanics, 43 (2008), 8, pp. 783-797

Paper submitted: April 28, 2019 Paper revised: July 2, 2019 Paper accepted: July 11, 2019 © 2021 Society of Thermal Engineers of Serbia. Published by the Vinča Institute of Nuclear Sciences, Belgrade, Serbia. This is an open access article distributed under the CC BY-NC-ND 4.0 terms and conditions.