THERMO DIFFUSION ASPECTS IN JEFFREY NANOFLUID OVER PERIODICALLY MOVING SURFACE WITH TIME DEPENDENT THERMAL CONDUCTIVITY

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Abstract: *Double diffusion flow of Jeffrey fluid in presence of nanoparticles is studied theoretically under time dependent thermal conductivity. The considered nanoparticles are evaporated over convectively heated surface which moves periodically in its own plane. The appropriate dimensionless variables are employed to obtain the dimensionless forms of governing equations. We computed the analytical solution of nonlinear differential equations by utilizing homotopy analysis method. The present investigation reveals the features of various emerging parameters like Deborah number, combined parameter, oscillation frequency to stretching rate ratio, Prandtl number, Lewis number, thermophoresis parameter, Brownian motion parameter, nano Lewis number, modified Dufour parameter and Dufour solutal Lewis number. A useful enhancement in movement of nanoparticles is observed by utilizing the combined magnetic and porosity effects. Unlike traditional studies, present analysis is confined with the unsteady transportation phenomenon from periodically moving surfaces. Such computation may be attributable in flow results from tensional vibrations due to stretching and elastic surfaces. The simulation presented here can be attractable significance in the bioengineered nanoparticles manufacturing. It is observed that the heat transportation of nanoparticles may efficiently enhance through the utilization of variable thermal conductivity. The solutal concentration decreases with increasing Deborah number and Lewis number. It is further noted that the nano Lewis number causes reduction of nanoparticles concentration.*

KEYWORDS: *Jeffrey nanofluid, double diffusion flow, time dependent thermal conductivity, oscillatory stretching surface.*

1. INTRODUCTION

Substantial explorations pertaining to the flow of non-Newtonian fluids are proposed in recent century because of their convenient applications in many chemical industries. The general applications include polymer, biological solutions, manufacturing crude soft material, greases, glues, chemicals, paints, petroleum and oil reservoir engineering. The exclusive aspect of these fluid models is the existence of nonlinear relationship between shear stress and deformation rate and consequently referred power law model. Unlike viscous fluid models, the rheology of non-Newtonian fluids is not easy to entertain. Therefore, in modern era, various models are enlarged by engineers to thrash out the diverse rheological properties. Jeffrey fluid is probably one of model which attained special attraction by researchers as it profitably forecast the relaxation and retardation time effects. Owing to such interesting rheological behavior, various attempts for flow of Jeffrey fluid have been made using various flow features. Jawad et al. [1] reported the Jeffrey fluid flow over convectively heated surface in presence of applied magnetic field. The analytical computation performed by Hayat et al. [2] for unsteady flow of Jeffrey fluid considered over stretching surface. The slip flow and melting heat feature in radiative flow of Jeffrey fluid has been examined by Das et al. [3]. Another investigation concentrated by Ahmad et al. [4] on stretching flow of Jeffrey fluid in presence of porous medium. Zin et al. [5] simulated numerical computations on free convection flow of Jeffrey fluid in presence of wall ramped temperature over vertical moving surface. In the work of Ramesh and co-workers [6], results were suggested for stagnation point flow of Jeffrey fluid over vertical moving surface. The addition of heat source/sink in chemical reactive flow of Jeffrey fluid specified by vertical cone was analyzed by Saleem et al. [7]. The study of chemically reactive Jeffrey fluid in presence of heat and mass transport phenomenon has been numerically treated by Narayan et al. [8]. Hussain et al. [9] studied peristaltic transport of Jeffrey fluid in presence of variable thermal conductivity. They performed the analytical computations for the dimensionless problem. Babu et al. [10] investigated the melting heat transfer effects in the radiative flow of Jeffrey fluid caused by a parallel moving surface. A numerical based continuation dealing with the flow of Jeffrey fluid over a porous stretched surface was performed by Narayana et al. [11].

Recent advancement in the field of nanotechnology has been impelled in the field of chemical, mechanical, biomedical products, industrial applications and to improve the energy consumptions. The term "nanofluid" is compliance in the era of scientific research recently and attracted the interest of researchers. Nanofluids are frequently used enhance the heat and mass transportation, nuclear systems, cooling of objects, solar generation systems, biomedical applications, storage of energy, cooling of reactors etc. These are mixture of tiny particles immersed in the base fluid which improve the thermal properties due to interaction of nanoparticles. The concept of nanofluid was theoretical formulated in 1995 by Choi [12] in which he experimentally showed that the enhancement in poor thermal conductivity can be efficiently improved by addition of tiny sized particles. The Brownian and thermophoresis aspect in flow of nanofluid has been examined by Buongiorno [13]. Liu et al. [14] pointed out impact of heat absorption in flow of rate type nanofluid over finite thin film. The numerical treatment regarding flow of pseudo-plastic fluid in presence of nanoparticles was reported by Lin and co workers [15]. Sheikholeslami et al. [16] theoretically justified the impact of magnetic field in presence of nanoparticles distribution over rotating system. In more continuation, Sheikholeslami [17] utilized the CuO-H_2O nanoparticles in channel flow by means of using mesoscopic method. The numerical results for flow of Eyring Powell nanofluid has been accomplished by Malik et al. [18]. Hayat et al. [19] examined the stretching flow of nanofluid over convectively heated surface analytically. The enhancement of heat transfer for forced convection flow of nanofluid was originated by Sheikholeslami and Bhatti [20]. The peristaltic transport of nanofluid in presence of Hall current and entropy generation was explored by

Abbasi et al. [21]. The electro-Osmotic flow of Jeffrey nanofluid over rotating micro-channel has been directed by Reddy et al. [22]. Reddy et al. [23] performed numerical computations on the characteristics of Carreau nanofluid over stretching surface in presence of nanoparticles. Gireesha et al. [24] explained the nonlinear thermal radiation flow of Maxwell nanofluid caused by stretching surface. Babu et al. [25] performed the numerical computations based on Runge-Kutta method for Jeffrey nanofluid over a moving stretched sheet. The bioconvection flow of viscous nanofluid with various slip features over a rotating cone has been numerically analyzed by Latif et al. [26]. Abro et al. [27] examined the characteristics of heat transfer in electrically conducting flow of single as well as multiple carbon nanotubes in flow of Casson fluid under the influence of magnetic field. Goyal and Bhargava [28] numerically investigated the thermodiffusion features in the nanofluid flow over a nonlinear stretched surface.

The aim of current investigation is to report the transportation solutal concentration in flow of non-Newtonian nanoparticles over periodically moving convectively heated surface. The combined magnetic and porosity aspects are also implemented. These theoretical computations can be valuable in manufacturing processes, enhancement of energy transport and heat resources. An efficient analytical method, homotopy analysis method, is occupied to determine the solution of dimensionless equations. The characteristics of various parameters are evaluated graphically.

2. FLOW PROBLEM

We assume two-dimensional and unsteady flow of Jeffrey nanofluid over in presence of variable thermal conductivity. The flow is engaged due to the periodically stretching surface which governed the oscillatory velocity i.e. $u = b\overline{x}\sin\omega t.$ The constant b denotes maximum rate of stretched while ω expresses the frequency. Further, the aspect of thermophoresis and Brownian motion are also utilized in aspect of thermo diffusion effects. The effects of magnetic field are carried out in the transverse directions. Let T_w is the temperature of the surface, C_w is the solutal concentration while ϕ_w represents the nanoparticles concentration while T_∞, C_∞ and ϕ_∞ are respective ambient values for related profiles

The dimensional equations concerned the mass, momentum, heat and diffusion of concentrations for nanoparticles are examined as [2, 28]:

$$\frac{\partial u}{\partial \overline{x}} + \frac{\partial v}{\partial \overline{y}} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial \overline{x}} + v\frac{\partial u}{\partial \overline{y}} = \frac{\nu}{1+\lambda}\left[\frac{\partial^2 u}{\partial \overline{y}^2} + \lambda_1\left(\frac{\partial^3 u}{\partial \overline{y}^2 \partial t} + u\frac{\partial^3 u}{\partial \overline{x}\partial \overline{y}^2} - \frac{\partial u}{\partial \overline{x}}\frac{\partial^2 u}{\partial \overline{y}^2} + \frac{\partial u}{\partial \overline{y}}\frac{\partial^2 u}{\partial x \partial \overline{y}} + v\frac{\partial^3 u}{\partial \overline{y}^3}\right)\right] - \frac{\sigma_e B_0^2}{\rho_f}u - \frac{\nu\varphi^*}{k^*}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial \overline{x}} + v\frac{\partial T}{\partial \overline{y}} = \frac{1}{(\rho c)_p}\frac{\partial}{\partial \overline{y}}\left(k(T)\frac{\partial T}{\partial \overline{y}}\right) + \tau_1\left[D_T\frac{\partial \phi}{\partial \overline{y}}\frac{\partial T}{\partial \overline{y}} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial \overline{y}}\right)^2\right] + DK_{TC}\frac{\partial^2 C}{\partial \overline{y}^2}, \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial \overline{x}} + v\frac{\partial C}{\partial \overline{y}} = D_s\frac{\partial^2 C}{\partial \overline{y}^2} + DK_{CT}\frac{\partial^2 T}{\partial \overline{y}^2}, \tag{4}$$

$$\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial \overline{x}} + v\frac{\partial \varphi}{\partial \overline{y}} = D_B\frac{\partial^2 \varphi}{\partial \overline{y}^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial \overline{y}^2}, \tag{5}$$

where u is horizontal velocity component, v is the vertical component, ν represents kinematic viscosity, λ is ratio of relaxation to retardation time, λ_1 retardation constant, σ_e electrical conductivity, ρ_f notify base fluid density, B_0 governed with magnetic field strength, φ^* signify porous medium, k^* imply permeability of porous medium, T entails temperature, $\tau_1 = (\rho c)_p / (\rho c)_f$ denotes ratio between heat capacity of nanoparticles material to heat capacity of fluid, k is the thermal conductivity, DK_{TC} is Dufour diffusivity, DK_{CT} is Soret diffusivity, C is solutal concentration, φ nanoparticles volume fraction, D_s is the solutal diffusivity and D_B denotes the Brownian diffusion coefficient. The initial and boundary conditions concerned to the present flow field are articulated as:

$$u = b\bar{x}\sin\omega t,\;\; v = 0,\;\; T = T_w, C = C_w, \varphi = \varphi_w \;\text{ at }\; \bar{y} = 0,\;\; t > 0, \tag{6}$$

$$u \to 0,\;\; \frac{\partial u}{\partial \bar{y}} \to 0,\;\; T \to T_\infty,\; C \to C_\infty, \varphi \to \varphi_\infty \;\text{ as }\; \bar{y} \to \infty. \tag{7}$$

The energy equation (3) for thermal conductivity is modified as

$$k(T) = k_\infty\left(1 + \varepsilon\frac{T - T_\infty}{\Delta T}\right), \tag{8}$$

where k_∞ in above equation is termed as ambient liquid conductivity while ε is the thermal dependence conductivity parameter. We interpolate the flowing suitable variables to acquire the dimensionless variables

$$u = b\bar{x}f_y(y,\tau),\quad v = -\sqrt{\nu b}\,f(y,\tau),\; y = \sqrt{\frac{b}{\nu}}\bar{y},\; \tau = t\omega, \tag{9}$$

$$\theta(y,\tau) = \frac{T - T_\infty}{T_w - T_\infty}, s(y,\tau) = \frac{C - C_\infty}{C_w - C_\infty}, \phi(y,\tau) = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty}. \tag{10}$$

Inserting of above variables in Eqs. (2)-(4) leads to following dimensionless PDE's

$$f_{yyy} + \beta\left(Sf_{yyy\tau} + f_{yy}^2 - ff_{yyyy}\right) - (1+\lambda)\left(Sf_{y\tau} + f_y^2 - ff_{yy}\right) - M(1+\lambda)f_y = 0, \tag{11}$$

$$(1+\varepsilon\theta)\theta_{yy} + \varepsilon\left(\theta_y\right)^2 + \Pr\left[-S\phi_\tau + f\phi_y + Nb\theta_y\phi_y + Nt\left(\theta_y\right)^2 + (Nd)s_{yy}\right] = 0, \tag{12}$$

$$s_{yy} + Le\left(f\phi_y - S\phi_\tau\right) + Ld\theta_{yy} = 0, \tag{13}$$

$$\phi_{yy} + Ln\left(f\phi_y - S\phi_\tau\right) + \frac{Nt}{Nb}\theta_{yy} = 0. \tag{14}$$

In above dimensionless equations, β the Deborah number, M the combined Hartmann number and porosity parameter (combined parameter), S the oscillation frequency to stretching rate ratio, $\Pr$ stands for Prandtl number, Nt the thermophoresis parameter, Nb the Brownian motion parameter, Nd the modified Dufour number, Le the regular Lewis number, Ld the Dufour Lewis number and Ln is the nano Lewis number which are defined in the following forms

$$\beta = \lambda_1 b,\;\; M = \sigma B_0^2 / \rho b + \nu\varphi^* / kb,\;\; S = \omega / b,\;\; Le = \nu / D_s,\;\; Nt = (\rho c)_p D_T\left(T_f - T_\infty\right) / (\rho c)_f T_\infty \nu,$$

$Nb = (\rho c)_p D_B (C_f - C_\infty)/(\rho c)_f \nu,\; Ln = \nu / D_B,\; Ln = \nu / D_B,\; Ld = D_{CT}(T_w - T_\infty)/\alpha_m (C_w - C_\infty),$
$Nd = D_{TC}(C_w - C_\infty)/\alpha_m (T_w - T_\infty).$

The boundary conditions are

$$f_y(0,\tau) = \sin\tau,\; f(0,\tau) = 0, \theta(0,\tau) = 1, s(0,\tau) = 1, \phi(0,\tau) = 1, \tag{15}$$

$$f_y(\infty,\tau) \to 0,\; f_{yy}(\infty,\tau) \to 0,\; \theta(\infty,\tau) \to 0, s(\infty,\tau) \to 0, \phi(\infty,\tau) \to 0. \tag{16}$$

We signify the following quantities associated with local Nusselt number, local Sherwood number and local nano Sherwood number

$$Nu_x = \frac{\bar{x} q_s}{k(T_w - T_\infty)}, Sh_x = \frac{\bar{x} j_s}{D_B(C_w - C_\infty)}, Sh_{xn} = \frac{\bar{x} q_{mn}}{D_s(\varphi_w - \varphi_\infty)}, \tag{17}$$

where q_s notify heat flux at the plate, j_s is surface mass flux while q_{mn} correlates the nano particle mass flux. Interleave Eqs. (9)-(10) in above expressions get following forms

$$Nu_x \mathrm{Re}_x^{-1/2} = -\theta_y(0,\tau),\; Sh_x \mathrm{Re}_x^{-1/2} = -s_y(0,\tau),\; Sh_{nx} \mathrm{Re}_x^{-1/2} = -\phi_y(0,\tau), \tag{18}$$

where $\mathrm{Re}_x = u_w \bar{x} / \nu$ is the local Reynold number.

5. ANALYSIS OF RESULTS

The field equations are numerically solved analytically by using homotopy analysis method. This method is originally advised by Liao [29] and later on many investigators computed their results by using this method [30-34]. In fact, homotopy analysis method is a series solution method for which the convergence of obtained solution is strictly depend on appropriate selection of auxiliary constants namely, h_f, h_θ, h_s and h_ϕ. In order to verify our results, the computed solution is compared with already reported continuation presented by Zheng et al. [35] and Abbas et al. [36], Table 1 is prepared. Here, an excellent accuracy of results has been reported.

Table 1: Comparison of $f''(0,\tau)$ with [34, 35] when $S = 1$ $M = 12$ and $\lambda = 0$.

τ	Zheng et al. [35]	Abbas et al. [36]	Present results
$\tau = 1.5\pi$	11.678656	11.678656	11.678656
$\tau = 5.5\pi$	11.678706	11.678707	11.678706
$\tau = 9.5\pi$	11.678656	11.678656	11.678656

Fig. 2 displays the $h-$curves for velocity, temperature, nanoparticles solutal concentration and concentration profiles for some precise values of engineering parameters like $\lambda = 0.2,$, $S = 0.4,$ $\beta = 0.5,$ $M = 0.4,$ $\mathrm{Pr} = 0.5,$ $\varepsilon = 0.3,$ $Ld = 0.3,$ $Sc = 0.3,$ $Le = 0.3,$ $Ln = 0.3$ and $\tau = \pi/2$. It is observed that suitable values for preeminent solution can be picked fro $-1.5 \le h_f \le 0,$ $-1.2 \le h_\theta \le 0.0,$ $-1.2 \le h_s \le 0,$ $-1.1 \le h_\phi \le 0.$

In order to vary the associated parameter, all other parameters accomplished some constant values like $\lambda = 0.2,$ $S = 0.4,$ $\beta = 0.5,$ $M = 0.4,$ $\mathrm{Pr} = 0.5,$ $\varepsilon = 0.3,$ $Ld = 0.3,$ $Le = 0.3,$ $Ln = 0.3$ and $\tau = \pi/2$. The profiles temperature, solutal concentration and nanoparticles concentration at different time instants $\tau = \pi/6, \pi, 5\pi/6$ and $\pi/3$ are reflected in Fig. 3. The associated profiles are massively depressed as time varies. The temperature, solutal concentration and nanoparticles concentration accentuated with a diminishing behavior by varying time $\pi/6$ to $\pi/3$. However, the concentration profile altered quite marginally as compared to temperature and solute concentration distribution.

The behavior of temperature profile θ for variation of Deborah number β, thermophoresis parameter Nt, variable thermal conductivity parameter ε, combined parameter M and modified Dufour number Nd is utilizes in Fig. 4(a-e). Fig. 4(a) reports the significance of Deborah number β for temperature profiles while all other parameters keep constants. It is important to mention that minute values of Deborah number $(\beta \leq 1)$ portrayed the liquid behavior. The higher values exempted the solid like behavior. It is observed that the temperature of nanoparticles for solid case is lesser than viscous case. However, the change in temperature distribution is quite marginal. Moreover, a decreasing trend in associated layer is scrutinized for leading values of β. The increasing values of β accompanying the depressed thermal boundary layer. In order to evaluate the behavior of thermophoresis parameter Nt on θ, Fig. 4(b) is presented. A comparative increasing trend is noted in temperature profile as Nt varies from 0 to 1.5. The thermophoresis is interesting phenomenon in which tiny fluid particles migrate from the region of high temperature to low temperature. Due to this transportation, a collection of fluid particles is dragged away from heated surface due to which the temperature of liquid as well as thermal boundary layer increases. The implication of variable thermal conductivity parameter ε on θ is investigated in Fig. 4(c). The variable thermal conductivity results maximum temperature of nanoparticles in the whole region. Further, a thicker thermal boundary layer has been noted for the leading values of ε. Therefore, presence of variable thermal conductivity may more useful to enhance the heat transportation. Fig. 4(d) summarized the effects of combined parameter M on θ. The combined parameters bring to light joint effects of both Hartmann number and porosity parameter. Higher values of Hartmann number bring Lorentz force in system which enhanced the temperature of fluid. Secondly, the occurrence of porosity medium plays similar role due to permeability medium. Fig. 4(e) reports the up shoot of modified Dufour number Nd on θ. It is manifested that θ appear to be increase with variation of Nd.

Fig. 5(a-d) is sketched to probe the impact of Deborah number β, ratio of relaxation to retardation time λ, regular Lewis number Le, Dufour Lewis number Ld and combined parameter M on solutal concentration s. The observation noted from the Fig. 5(a) divulge that s escalates by varying β. However, the decreasing trend is relatively slower. The inspirations of ratio of relaxation to retardation time λ on s are specified in Fig. 5(b). This figure demonstrates that solutal concentration increases by increasing λ. Fig. 5(c) examine the trend of regular Lewis number Le on solutal concentration s. The progressive values of Le result a decrement in s quite efficiently. Moreover, a depressed solutal concentration boundary layer has been notified by varying Le. The physical aspect behind this trend is that Lewis number is associated with thermal diffusion to species diffusion. Hence by varying Le, the rate of species diffusion becomes relatively slower and as a result solutal concentration reduced. Fig. 5(d) portrays the effect of Dufour Lewis number Ld on solutal concentration s. A substantial growth in s is noted for larger values of Ld.

The collective significance of various parameters like Deborah number $\beta,$ ratio of relaxation to retardation time λ nano Lewis number Ln and Brownian motion parameter Nb on nanoparticles concentration ϕ is attended in Fig. 6(a-d). Fig. 6(a) accomplished the consequence of β on $\phi.$ It is noted that the concentration of nanoparticles demoralized by increasing β. It is also remarked that change in nanoparticles concentration for β is relatively slower as compared to solutal concentration $s.$ By definition, Deborah number is associated with retardation time, large retardation times materials are generally less viscous in nature which results in increases in velocity and consequently reduces the concentration distribution of nanoparticles. The pivotal behavior of λ on ϕ is illustrated in Fig. 6(b). An advancement of concentration profile is observed by varying $\lambda.$ Fig. 6(c) quantify the observations of Ln on ϕ. A decline behavior is examined by increasing Ln. The structure of associated boundary layer seems to be thicker as Ln varies. The variation of Nb on ϕ is visualized in Fig. 6(d). It is detected that enlarging values of Nb magnifies the concentration of nanoparticles. Mathematically, Nb is materialized in $1/Nb$ form in the concentration distribution which turns down the concentration of nanoparticles. Moreover, the appearance of Nb is because of interaction of nanoparticles which are associated with Brownian motion. It is observed that rate of change in concentration distribution is restively weaker as compared to the solutal concentration distribution.

Table 2 is summarized the numerical computations for local Nusselt number $Nu_x \mathrm{Re}_x^{-1/2},$ nanofluid Sherwood number $Sh_{nx} \mathrm{Re}_x^{-1/2}$ and local Sherwood number $Sh_x \mathrm{Re}_x^{-1/2}$ for various parameters. We observed that $Nu_x \mathrm{Re}_x^{-1/2}$ risen with larger Deborah number and Prandtl number. However, a lower trend is noted for thermophoresis parameter, Brownian motion parameter and relaxation to retardation time constant. The observations for rate of mass transfer $Sh_x \mathrm{Re}_x^{-1/2}$ becomes for greater variation of Brownian motion parameter. An increasing trend of nanofluid Sherwood number $Sh_{nx} \mathrm{Re}_x^{-1/2}$ is accomplished by varying Deborah number.

6. FINAL REMARKS

We proposed a theoretical analysis to simulate the thermo-physical aspects of nanoparticles in presence of non-Newtonian fluid. With help of appropriate quantities, the thermo-physical parameters from governing equations are diverted in dimensionless forms. Analytical solution has been reported by using homotopy analysis method. Due to variation of time, the temperature, solutal concentration and nanoparticles concentrations linearly decrease. Convective heat transfer of nanoparticles effectively improved with variable thermal conductivity, modified Dufour number and combined parameter. For the larger Dufour Lewis number, the solute concentration arises. The solutal concentration becomes slower by varying Lewis number and Deborah number. Further, an enhancement in concentration of nanoparticles is observed for relaxation to retardation time parameter.

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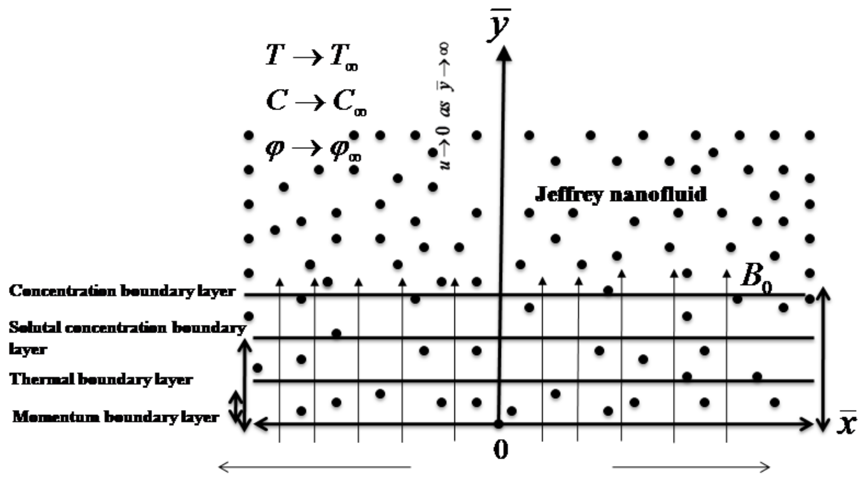


Fig. 1: Geometry of problem.

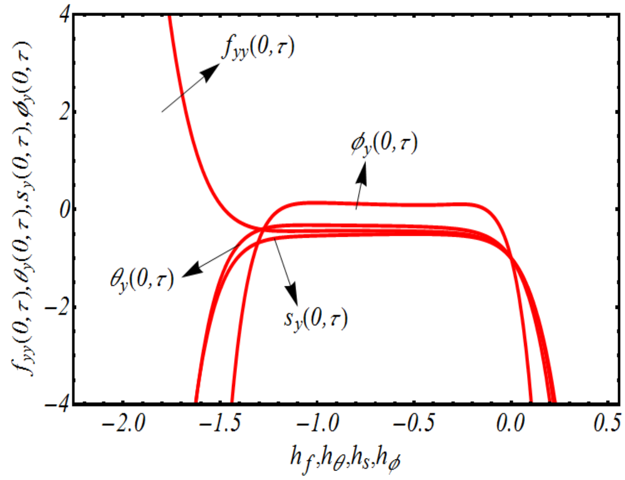


Fig. 2: *h*-curves for velocity, temperature, solutal concentration and nanoparticles concentration profiles.

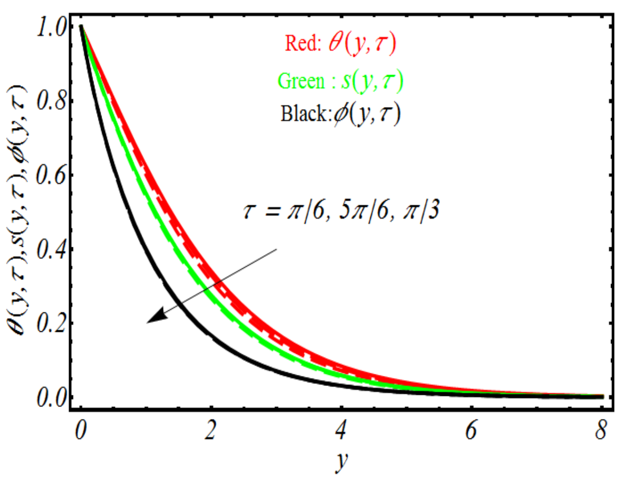


Fig. 3: Variation of time with velocity, solutal concentration and concentration profiles.

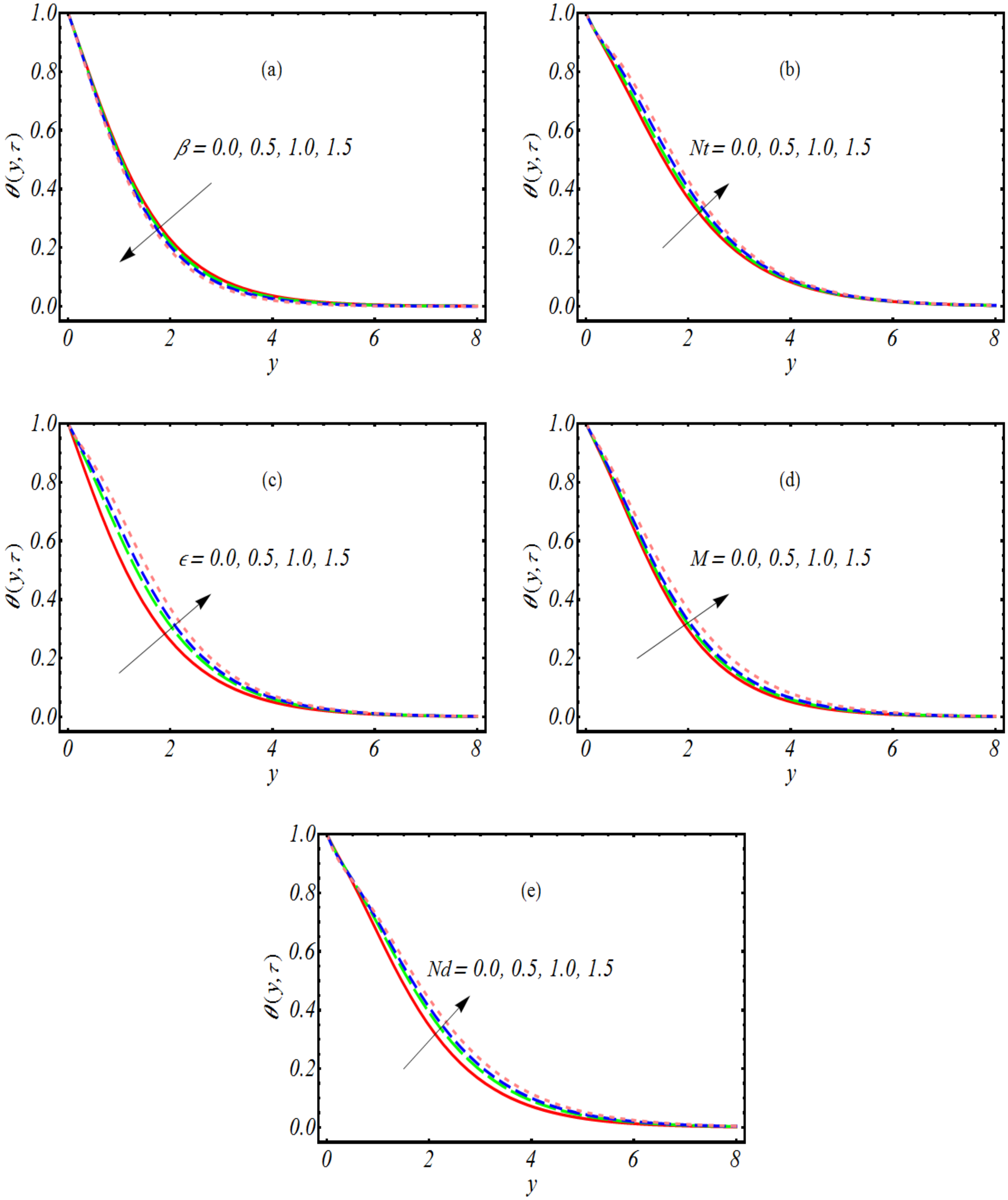


Fig. 4: Temperature distribution for (a) β, (b) Nt, (c) ε, (d) M and (e) Nd.

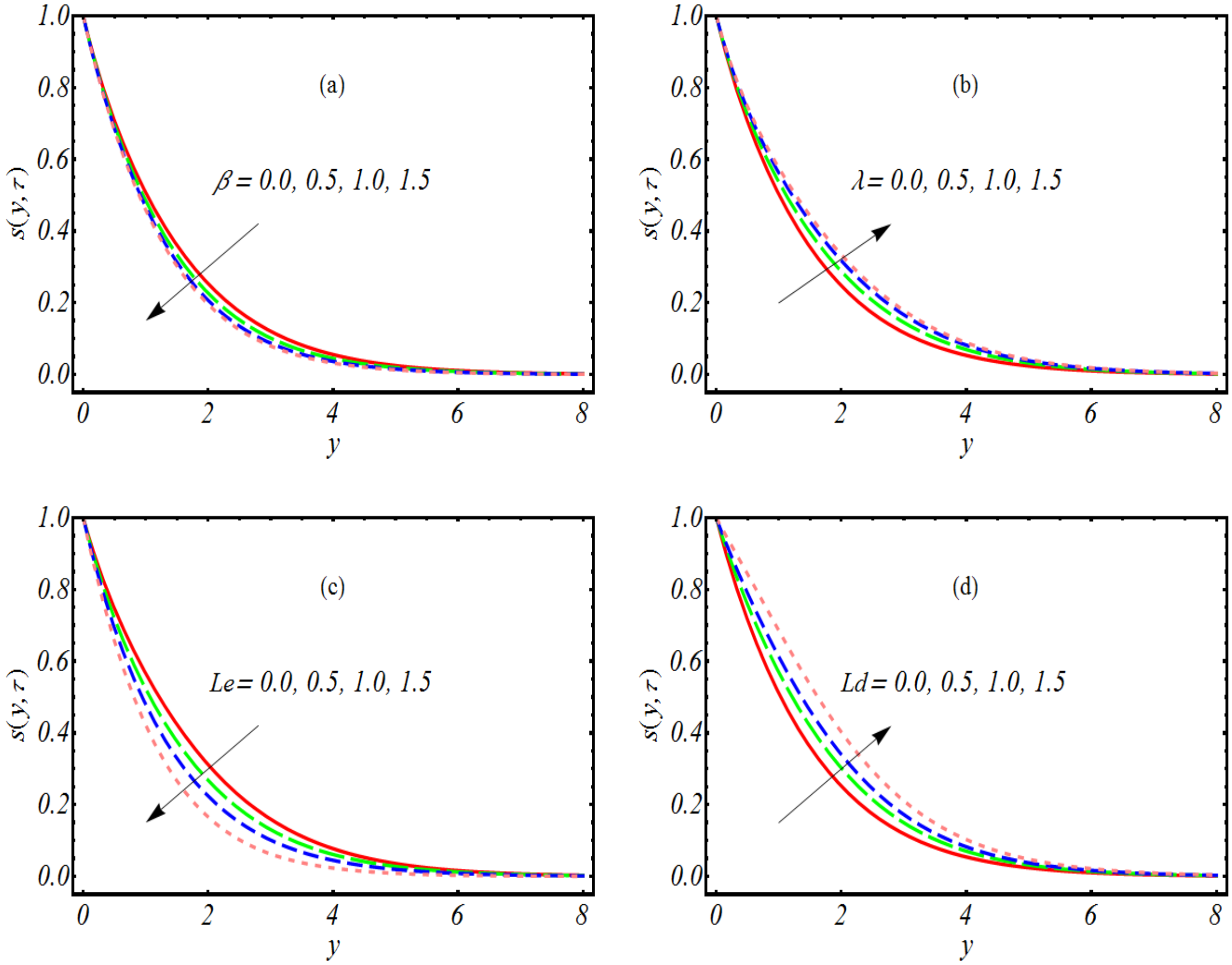
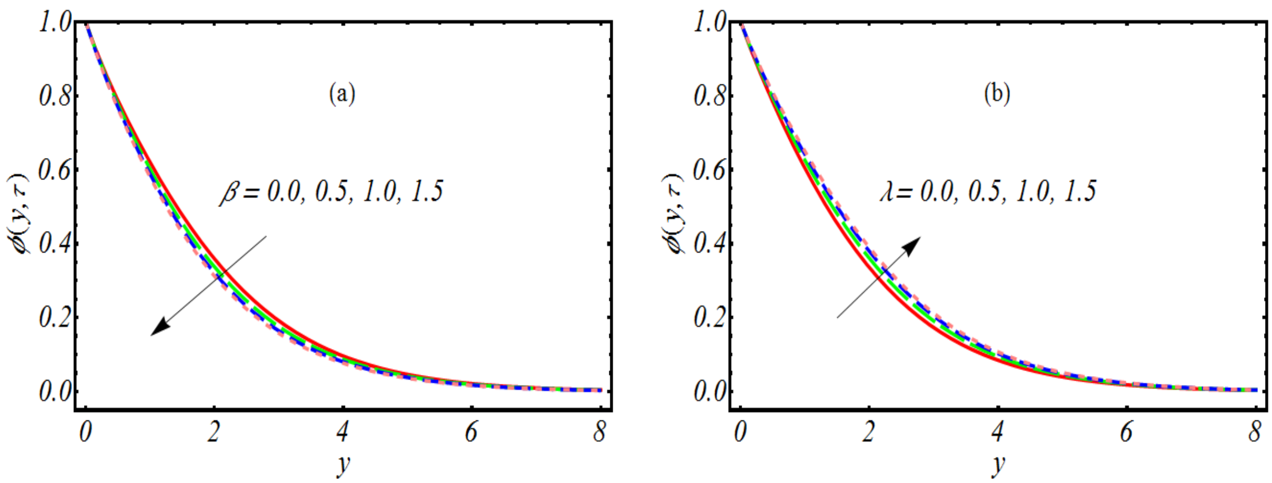


Fig. 5: Solutal concentration for (a) β, (b) λ, (c) Le and (d) Ld.



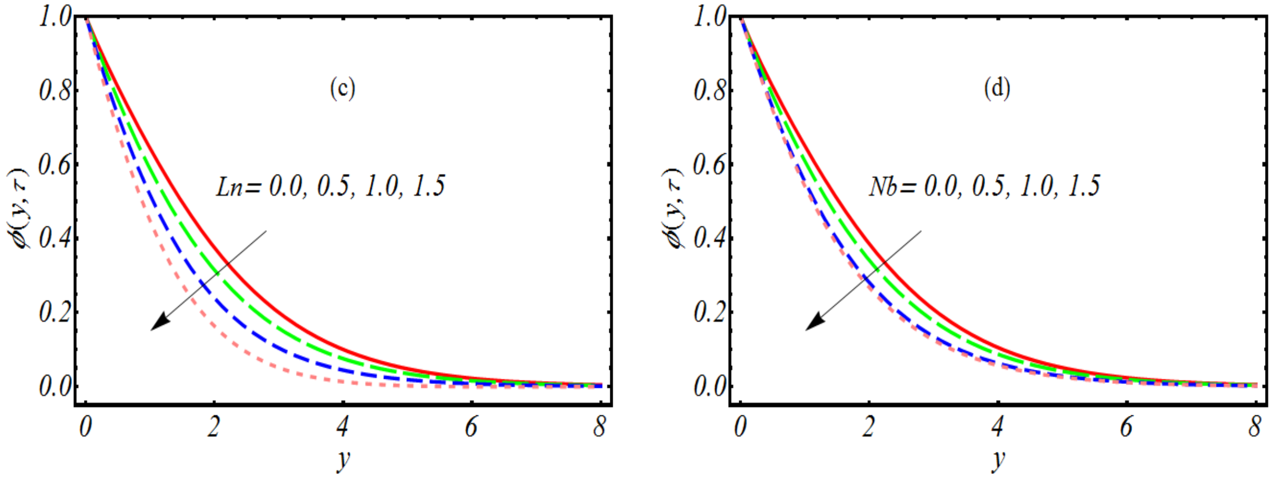


Fig. 6: Concentration distribution for (a) β, (b) λ, (c) Ln and (d) Nb.

Table 2: Numerical values of local Nusselt number, local Sherwood number and local nano Sherwood number.

β	Pr	λ	Nt	Nb	Nd	$-\theta_y(0,\tau)$	$-s_y(0,\tau)$	$-\phi_y(0,\tau)$
0.0	0.7	0.2	0.2	0.2	0.2	0.661082	0.51936	0.779282
0.5						0.681833	0.520409	0.799285
1.5						0.690130	0.541459	0.809292
0.2	0.1					0.705145	0.516187	0.800311
	0.7					0.797551	0.728766	0.913611
	1.0					0.861138	0.731677	0.954499
	0.7	0.0				0.822811	0.758896	0.936444
		0.5				0.802723	0.739601	0.924341
		1.5				0.792412	0.730313	0.911122
		0.2	0.0			0.731395	0.754331	0.880266
			0.8			0.66338	0.509362	0.838720
			1.2			0.616906	0.33585	0.817932
			0.2	0.5		0.631852	0.550416	0. 78231
				0.6		0.612766	0.647405	0.73264
				1.0		0.564873	0.687644	0.66951
				0.2	0.0	0.702159	0.570572	1.08437
					0.5	0.681844	0.560216	1.08422
					1.0	0.661326	0.559771	1.08404

					0.2	0.712333 0.700844 0.680326	0.565636 0.551216 0.546671	0.96545 0.96001 0.95987

Nomenclature

b maximum rate of stretched	ω frequency	T_w surface temperature
C_w is the solutal concentration	ϕ_w nanoparticles concentration	ν represents kinematic viscosity,
λ ratio of relaxation to retardation time	λ_1 retardation constant,	σ_e electrical conductivity,
ρ_f base fluid density	B_0 magnetic field strength	φ^* porous medium
k^* permeability of porous medium	T temperature	α nanofluid thermal diffusivity
DK_{CT} Soret diffusivity	DK_{TC} Dufour diffusivity	C solutal concentration
φ nanoparticles volume fraction	Ln nano Lewis number D_s solutal diffusivity	D_B Brownian diffusion coefficient
β Deborah number,	M combined parameter	$\Pr$ Prandtl number,
S oscillation frequency to stretching rate ratio	Nt thermophoresis parameter	Nd modified Dufour number
Nb Brownian motion parameter	Le regular Lewis number	Ld Dufour Lewis number

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