Unsteady mixed nano-bioconvection flow in a horizontal channel with its upper plate expanding or contracting: A revised model

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Abstract:

This paper presents a revised model for the problem international journal of heat and mass transfer 86 (2015) 174–182 titled unsteady mixed nano-bioconvection flow in a horizontal channel with its upper plate expanding or contracting. In this paper, the authors used a two phase model for the nanofluid presented in international journal of thermal sciences 77 (2014) 126-129 in which the nanofluid particle volume at the wall is passively rather than actively controlled. Here, we observed that the boundary conditions are not satisfied in all curves of the nanoparticle volume fraction ϕ i.e. the curves do not match the boundary conditions presented in equations (19). The second note is that all values of ϕ are positive which contradicts with the results presented in international journal of thermal sciences 77 (2014) 126-129. Since the equations are coupled then it can be said that not only the behaviors of ϕ are wrong but also all the results presented in that paper is not correct. The main reason for conducting this investigation is performing a revision for the mentioned paper and presenting a set of curves to show the corrected behaviors for dependent variables.

Keywords: Bioconvection, channel, nanofluid, revised model, unsteady flow

1. **Introduction**:

Nano-bioconvection is the process of combined nanofluids with bioconvection. These systems have been an important to get both enhanced thermal performance and green, sustainable characteristics by making them ideal for the next generation of biofuels. This type of convection is induced due to an unstable density stratification caused by up swimming microorganisms. Specifically, the unstable density stratification produces when the microorganisms which are heavier than water accumulate in the upper portions of the fluid which yields a particular type of hydrodynamic instability produced bioconvection plumes.

Microorganism particles are significantly useful in the production of several commercial and industrial products e.g. bio fuel made from waste, fertilizers, ethanol, etc. The motile micro-organisms are self-urged which enlarges the denseness of ordinary liquids by swimming toward a particular direction within the material in attraction to motivate like gravity, oxygen, daylight whereas nanoparticles cannot swim.

The literature contains the studies highlighting the significant developments in the enhanced modeling of nanofluid bioconvection boundary layer flows, which have been obtained with various complex body forces and other surfaces (wall) effects such as slip, non-isothermal and non-isosolutal conditions, thermal radiation, complex geometries (wavy surfaces). Bioconvection is utilized in modeling oil and gas-bearing sedimentary basins and microbial enhanced oil recovery (EOR) since they are devoted to the exploration of the techniques of different bioconvection problems in suspensions of solid particles. The microbial (EOR) is a new mechanism running in oil and gas industries to improve oil recovery. This process comprises the injection of selected microorganisms into the reservoir and by their, in situ multiplication, they diminish the residual oil left in the reservoir after secondary recovery is exhausted. Hence exploring convection is a rich area of research for scientists [1]. So that many aspects of bioconvection problems in suspensions that contain solid particles were investigated by several researchers. For example, Kuznetsov and Avramenko [2] initiated the exploration of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite depth. Kuznetsov [3, 4] made a series of analysis on the bioconvection of a nanofluid in a suspension containing both nanoparticles and microorganisms. He noticed that the addition of gyrotactic microorganisms into nanofluids is likely to increase its stability as a suspension. He further found that suspensions of gyrotactic microorganisms could exhibit bioconvection, which is a macroscopic motion in the fluid induced by up swimming or the motion of motile microorganisms. This is due to that the motile microorganisms are usually heavier than water so that they can swim in the upward direction in response to stimuli such as gravity, light and chemical attractions. Tham et al. [5] used the Keller-box finite difference method to analyze steady mixed convection boundary layer flow from a curved isothermal surface in a porous medium saturated by a nanofluid containing both nanoparticles and gyrotactic microorganisms in a stream moving vertically upwards. Shaw et al. [6] investigated bioconvection of gyrotactic microorganism closed to the boundary layer region of an inclined semi-infinite porous wall embedded in a porous medium containing water as a base fluid consisting of motile microorganisms. Xu and pop [7] investigated the fully developed mixed convection flow between two paralleled horizontal flat plates filled by a nanofluid containing both nanoparticles and gyrotactic microorganisms.

The problem of the magnetohydrodynamic bioconvection of an incompressible electrically conducting nanofluid near a vertical wavy surface saturated porous medium containing both nanoparticle and gyrotactic microorganisms was investigated by Ahmed and Mahdy [8]. They found that the local Nusselt number, the local Sherwood number, and the local density number of the motile microorganisms decrease by increasing either the Grashof number or the magnetic field parameter. Hady et al. [9] used an explicit finite difference scheme to solve the governing equations for the problem of unsteady thermo bioconvection boundary layer flow of a nanofluid containing gyrotactic microorganisms along a stretching sheet under the influence of magnetic field and viscous dissipation. Their results shown that both magnetic parameter and bioconvection Rayleigh number have positive effect on the dimensionless Nusselt number and density number of the motile microorganisms while the opposite behavior became clear in the case of Grashof number and Eckert number. Uddin et al. [10] explored the computational investigation of Stefan blowing and multiple-slip effects on buoyancy-driven bioconvection nanofluid flow containing both nanoparticles and microorganisms by introducing the Lie group transformation to seek the similarity solutions of such nanobioconvection flow.

Raizah [11] discussed the effects of viscous dissipation and Joule heating on the mixed convective flow of a nanofluid over a stretching surface in the presence of both nanoparticles and gyrotactic microorganisms. The nanofluid is represented by the model that includes both the effects of Brownian motion and thermophoresis. Mosayebidorcheh et al. [12] analyzed the bioconvection nanofluid flow which contains both nanoparticles and gyrotactic microorganisms. They investigated a flow in a horizontal channel considering the model proposed by Kuznestov and Nield (2013). Ahmed et al. [13] described the steady magnetohydrodynamic mixed thermo-bioconvection in a square enclosure filled with a homogeneous and isotropic porous medium in the presences of oxytocic microorganisms.

Kuznetsov and Nield [14, 15] revised their models [16, 17] by considering the nanoparticle volume fraction at the wall is passively controlled. They found that using this approach leads to that all values of nanoparticle volume fraction are negative and they explained this behavior based on the fact that the effect of thermophoresis is such that an elevation which leads to reduce the value of nanoparticle volume fraction at the wall. Raees et al. [18] used the revised models [14, 15] to study the unsteady bioconvection flow inside a horizontal channel using the nanofluids. But their results are questionable because all the curves of nanoparticle volume fraction are not match boundary conditions presented in equations (19) in [18]. Also, all values of the nanoparticle volume fraction are positive which is contrary with the facts presented in [14, 15]. So, the main objective of this paper is to revise this problem by presenting the corrected behaviors of this problem with neutral discussion to present the corrected physical explanations for this important study.

2. Mathematical analysis

Consider the unsteady laminar flow of an incompressible and viscose nanofluid containing gyrotactic microorganisms inside a horizontal channel. To save the reader time, the physical model and coordinates system are found in Raees et al. [18] and not presented here again. The following bullet points, a limitation for the important assumptions in this problem:

- Two phase model is used to simulate the nanofluid.
- Passively controlled boundary conditions for the nanoparticle volume fraction are considered.
- Water is assumed as a based nanofluid.
- Concentration of the nanoparticles is dilute and they not conglomerate.

Taking into account these assumptions, the governing equations in vorticity-stream function formula are expressed as:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\Omega \tag{1}$$

$$\frac{\partial\Omega}{\partial t}+u\frac{\partial\Omega}{\partial x}+\mathrm{v}\frac{\partial\Omega}{\partial \mathrm{y}}=\upsilon_f\left[\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial y^2}\right] \tag{2}$$

$$\frac{\partial \mathrm{T}}{\partial t}+u\frac{\partial T}{\partial x}+\mathrm{v}\frac{\partial T}{\partial \mathrm{y}}=\alpha\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\tau\left(D_B\left[\frac{\partial\emptyset}{\partial \mathrm{x}}\frac{\partial T}{\partial \mathrm{x}}+\frac{\partial\emptyset}{\partial \mathrm{y}}\frac{\partial T}{\partial \mathrm{y}}\right]+\frac{D_T}{T_0}\left[\left(\frac{\partial T}{\partial \mathrm{x}}\right)^2+\left(\frac{\partial T}{\partial \mathrm{y}}\right)^2\right]\right) \tag{3}$$

$$\frac{\partial\emptyset}{\partial t}+u\frac{\partial\emptyset}{\partial x}+\mathrm{v}\frac{\partial\emptyset}{\partial y}=D_B\left[\frac{\partial^2\emptyset}{\partial x^2}+\frac{\partial^2\emptyset}{\partial y^2}\right]+\frac{\mathrm{D_T}}{\mathrm{T_0}}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{4}$$

$$\frac{\partial \mathrm{n}}{\partial t}+u\frac{\partial n}{\partial x}+\mathrm{v}\frac{\partial n}{\partial \mathrm{y}}+\frac{\partial}{\partial \mathrm{y}}(n\hat{v})=D_{\mathrm{n}}\frac{\partial^2 n}{\partial \mathrm{y}^2} \tag{5}$$

Subjected to the following boundary conditions:

$$u=0, v=\frac{dh}{dt}, T=T_2, D_B\frac{\partial\emptyset}{\partial \mathrm{y}}+\frac{D_T}{T_0}\frac{\partial T}{\partial \mathrm{y}}=0, n=n_2\ at\ y=h(t) \tag{6a}$$

$$u=0, v=0, T=T_1,\ \emptyset=\emptyset_0, n=n_1\ at\ y=0 \tag{6b}$$

Where, u, v are the horizontal and vertical velocity component, respectively, x,y are the Cartizian coordinates, ψ is the stream function, Ω is the vorticity.

Introducing the following similarity transformation:

$$\psi(x,y)=\sqrt{\frac{b\upsilon_f}{1-\alpha t}}xf(\eta), u=\frac{bx}{\upsilon_f(1-\alpha t)}f'(\eta), v=-\sqrt{\frac{b\upsilon_f}{1-\alpha t}}f(\eta), \eta=\sqrt{\frac{b}{\upsilon_f(1-\alpha t)}}y, \theta(\eta)=\frac{T-T_0}{T_2-T_0}, \phi(\eta)=\frac{\emptyset-\emptyset_0}{\emptyset_0}, S(\eta)=\frac{n-n_0}{n_2-n_0} \tag{7}$$

Substituting Eq. (7) into Eqs. (2)-(5), the following system of ordinary differential equations is obtained:

$$f''''+ff'''-f'f''-\beta\eta f'''-3\beta f''=0 \tag{8}$$

$$\theta''+\Pr(f\theta'-\beta\eta\theta')+Nb\theta'\phi'+Nt\theta'^2=0 \tag{9}$$

$$\phi'' + \mathrm{Le}(f\phi' - \beta\eta\phi') + \frac{Nt}{Nb}\theta'' = 0 \tag{10}$$

$$S'' + \mathrm{Sc}(fS' - \beta\eta S') - Pe(\,\phi''S + \phi'S') = 0 \tag{11}$$

Also, the converted boundary conditions are given by:

$$f' = 0.f = 0, \theta = 1, Nb\phi' + Nt\theta' = 0, S = 1 \; at \; \eta = 0 \tag{12a}$$

$$f' = 0.f = w, \theta = \delta_\theta, \phi = \delta_\phi, S = \delta_S \; at \; \eta = 1 \tag{12b}$$

In equations (8)-(12), $\beta = \frac{\alpha}{2b}$ is the unsteadiness squeeze parameter, $Pr = \frac{\upsilon_f}{\alpha}$ is the Prandtl number, $Nb = \frac{\tau D_B \emptyset_0}{\alpha}$ is the Brownian motion parameter, $Nt = \frac{\tau \mathrm{D_T}(T_2 - T_0)}{T_0 \alpha}$ is the thermophoresis parameter, $Le = \frac{\upsilon_f}{D_B}$ is the Lewis number, $Pe = \frac{b_c W_c}{D_\mathrm{n}}$ is the Peclet number, $Sc = \frac{\upsilon_f}{D_\mathrm{n}}$ is the Schmidt number and $\delta_\theta = \frac{T_1 - T_0}{T_2 - T_0}$, $\delta_\phi = \frac{\emptyset_1 - \emptyset_0}{\emptyset_0}$, $\delta_S = \frac{n_1 - n_0}{n_2 - n_0}$, $w = \frac{H\alpha}{2\sqrt{b\upsilon_f}}$ are dimensionless constants. In this study the values of $w, \delta_\theta, \delta_\phi$ and δ_S are fixed at 1, 1/2, 0 and 1, respectively.

3. Results and discussion

Equations (8)-(11) together with the boundary conditions (12) are solved numerically using a fourth order Runge-Kutta method with shooting technique. The details of this method and the validation test are found in [11, 19-20] and did not repeated again.

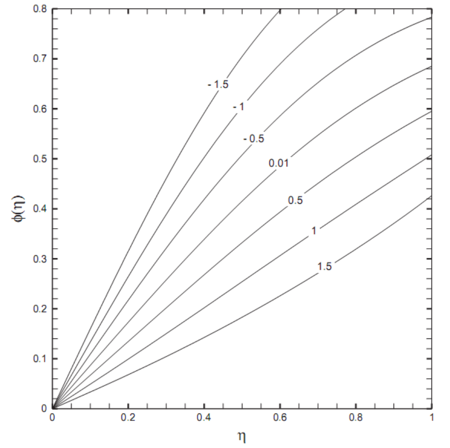
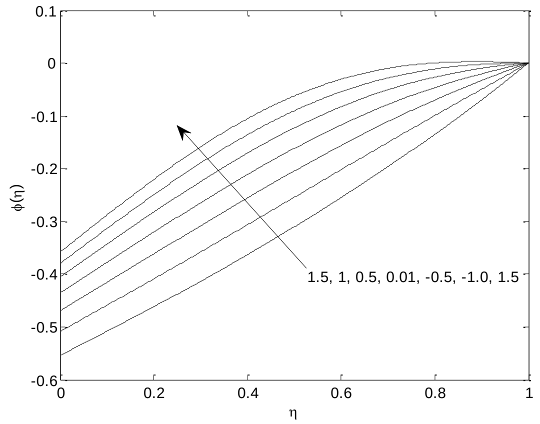
Wrong behaviour Raees et al. [18]	Present (correct behaviour)
	

Fig. 1. Nanoparticle volume fraction profile $\phi(\eta)$ for different values of squeeze parameter β in the case of $w = 1, Nt = Nb = 0.1, Le = Pe = Sc = Pr = 1.$

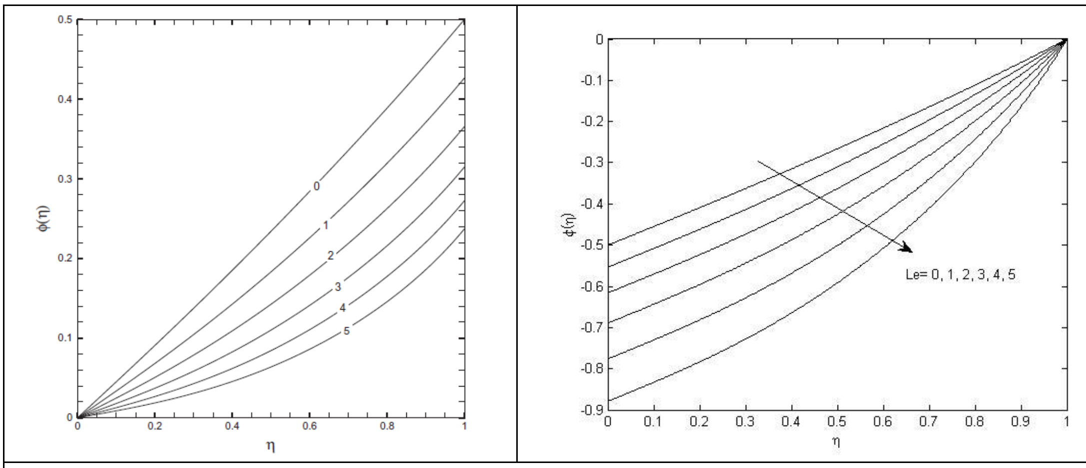


Fig. 2. Nanoparticle volume fraction profile $\phi(\eta)$ for different values of Le in the case of $w = 1, Nt = Nb = 0.1, \beta = 1.5, Pe = Sc = Pr = 1$.

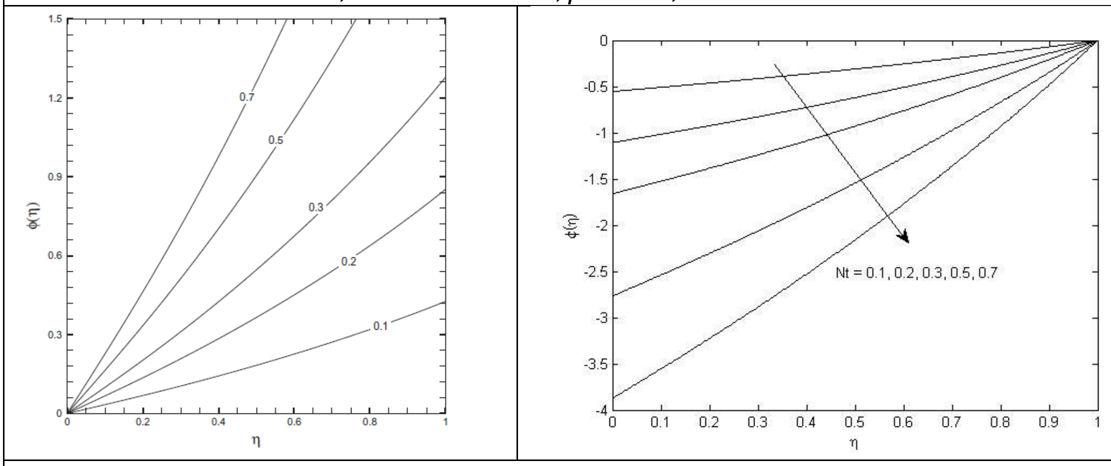


Fig. 3. Nanoparticle volume fraction profile $\phi(\eta)$ for different values of Nt in the case of $w = 1, Nb = 0.1, \beta = 1.5, Le = Pe = Sc = Pr = 1$.

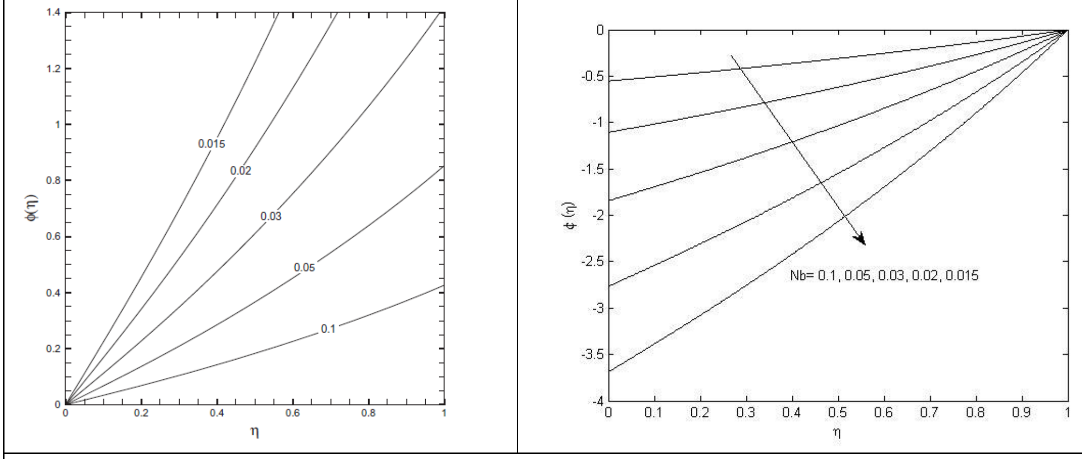


Fig. 4. Nanoparticle volume fraction profile $\phi(\eta)$ for different values of Nb in the case of $w = 1, Nt = 0.1, \beta = 1.5, Le = Pe = Sc = Pr = 1$.

A set of behaviors is presented in figures 1-10 represented comparisons between the nanoparticle volume fraction profiles $\phi(\eta)$ and the density of motile microorganisms profile $S(\eta)$ obtained from this treatments and those obtained by Raees et al. [18]. In figs. 1-4, effects of the squeeze parameter β, Lewis number Le, the thermophoresis

Nt and the Brownian motion parameter Nb on profiles of the nanoparticle volume fraction are illustrated. It is found that the present results are matched the boundary conditions presented in Eq.(12). In fact, Eq.(12) shows that the nanoparticle volume fraction at $\eta = 0$ is unknown and it is calculated from the ordinary differential equation (ODE) presented in Eq. (12a) while it is equal $\delta_\phi = 0$ at $\eta = 1$ and these conditions are not satisfied in the figures presented in Raees et al. [18]. On the other hand, all the values of nanoparticle volume fraction presented in our revised model are negative and this due to that effects of the thermophoresis parameter Nt is such that an elevation which results in a reduction in values of the nanoparticle volume fraction at the wall. This behavior agrees with the results of Kuznetsov and Nield [14, 15] and agrees with a very large number of published papers such that [7, 19-21]. Unfortunately, this observation is, also, not satisfied in Raees et al. [18] as is seen in Figs. 1-4. Moreover, Figs. 1-4 revealed, also, that the increase in Lewis number Le, thermophoresis parameter Nt and Brownian motion parameter Nb causes a reduction in profiles of the nanoparticle volume fraction while it is enhanced as the squeeze parameter β increases.

<table>
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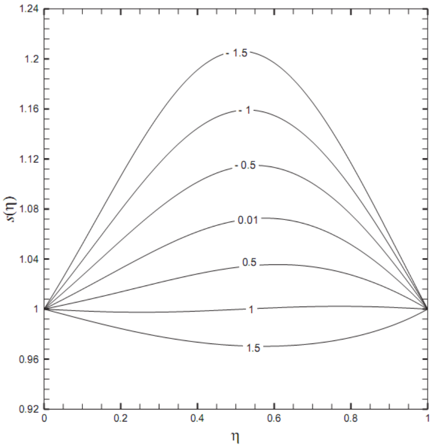
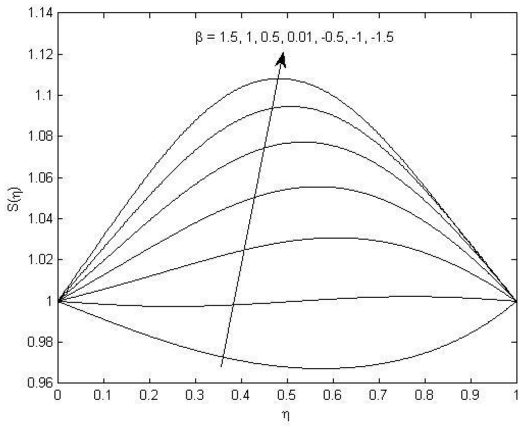
<tr><td>
</td><td>
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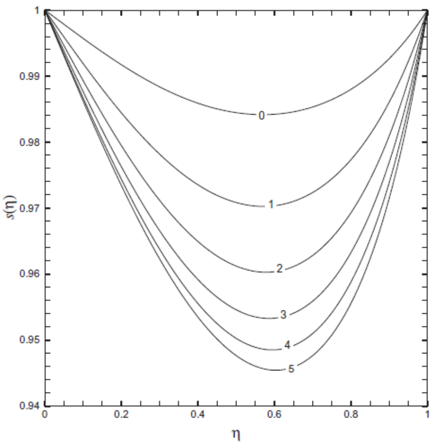
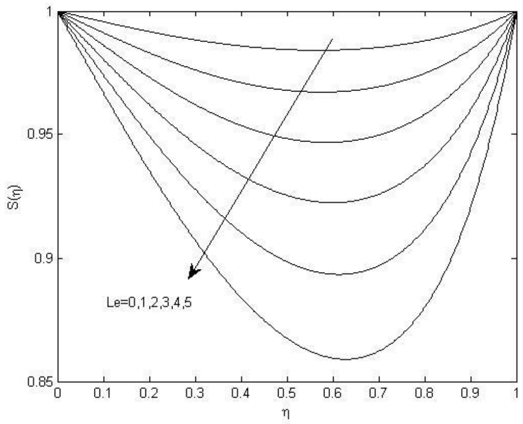
<tr><td colspan="2">Fig. 5. Density of motile microorganisms profile $S(\eta)$ for different values of squeeze parameter β in the case of $w = 1, Nt = Nb = 0.1, Le = Pe = Sc = Pr = 1$.</td></tr>
<tr><td>
</td><td>
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<tr><td colspan="2">Fig. 6. Density of motile microorganisms profile $S(\eta)$ for different values of Le in the case of $w = 1, Nt = Nb = 0.1,\ \beta = 1.5, Pe = Sc = Pr = 1$.</td></tr>
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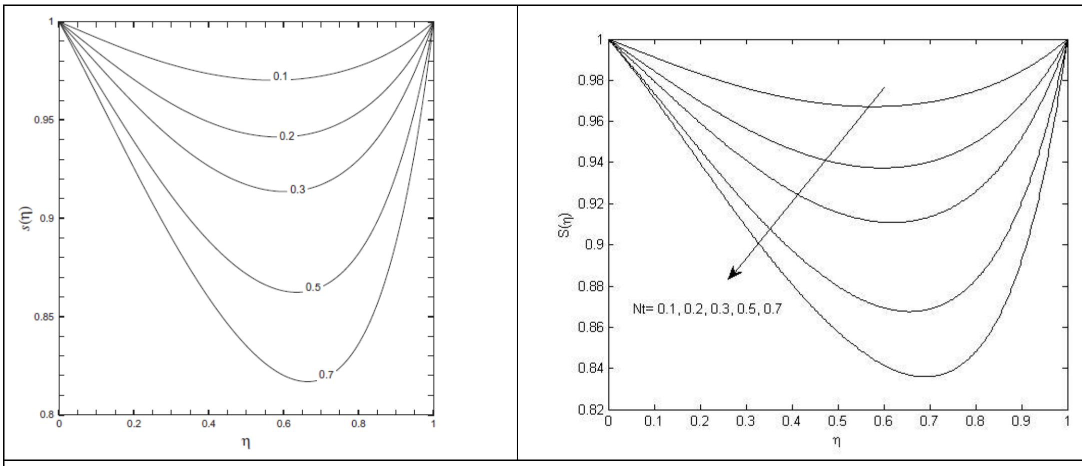

Fig. 7. Density of motile microorganisms profile $S(\eta)$ for different values of Nt in the case of $w = 1, Nb = 0.1,\ \beta = 1.5, Le = Pe = Sc = Pr = 1$.

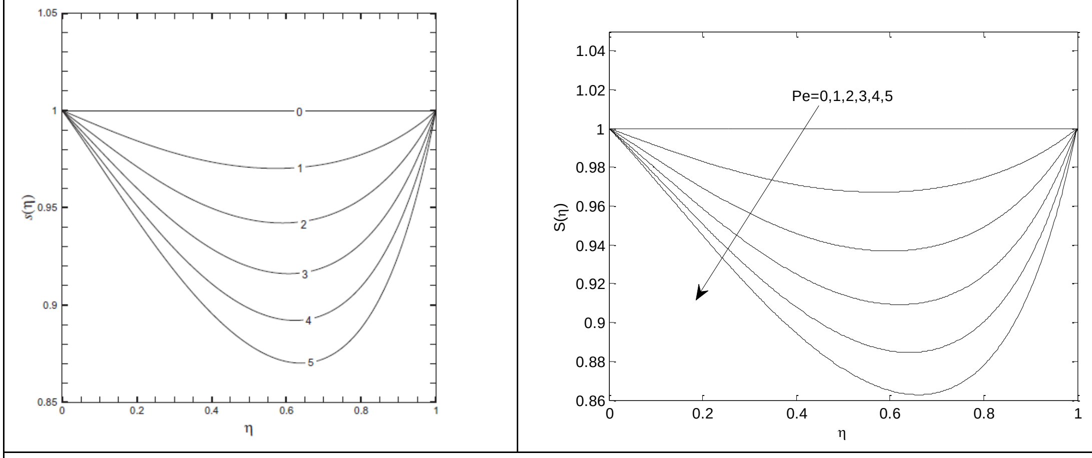

Fig. 8. Density of motile microorganisms profile $S(\eta)$ for different values of Pe in the case of $w = 1, Nt = Nb = 0.1,\ \beta = 1.5, Le = Pe = Sc = Pr = 1$.

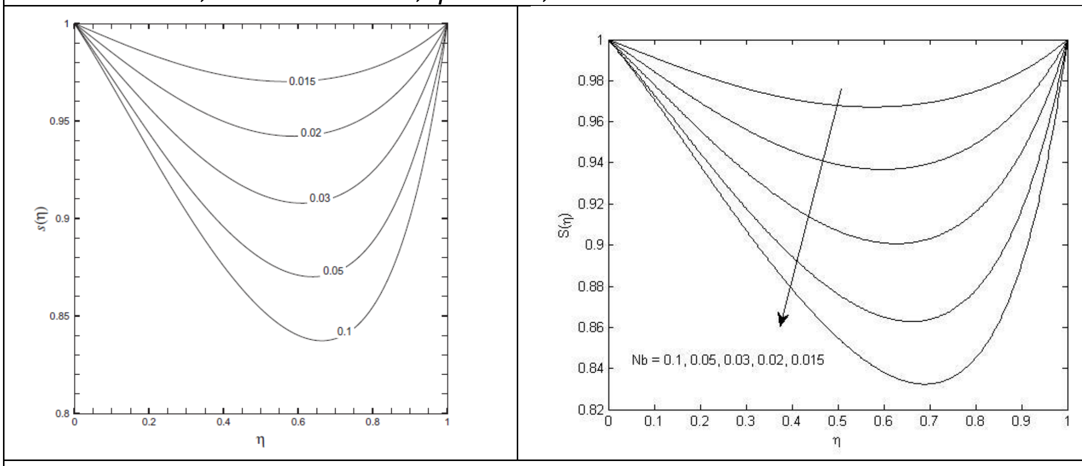

Fig. 9. Density of motile microorganisms profile $S(\eta)$ for different values of Nb in the case of $w = 1, Nt = 0.1,\ \beta = 1.5, Le = Pe = Sc = Pr = 1$.

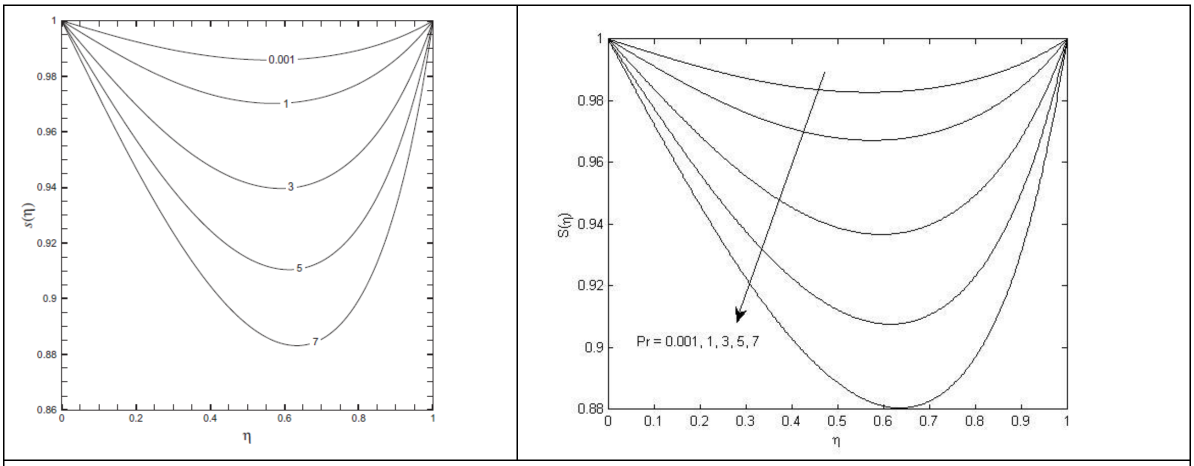

Fig. 10. Density of motile microorganisms profile $S(\eta)$ for different values of Pr in the case of $w = 1, Nb = Nt = 0.1,\ \beta = 1.5, Le = Pe = Sc = Pr = 1$.

The wrong behaviors of nanoparticle volume fraction of Raees et al. [18] affected on behaviors of the density of motile microorganisms profile $S(\eta)$ since the solved equations are coupled. Therefore, the corrected behavior of density of motile microorganisms are presented in Figs. 5-10 for different values of the squeeze parameter β, Lewis number Le, the thermophoresis Nt, Peclet number Pe, the Brownian motion parameter Nb and Prandtl number Pr. It is observed that the behaviors of the density of motile microorganisms $S(\eta)$ for the low values of β, Le, Nt, Pe, Nb and Pr are approach to Raees et al. [18] while the big values of governing parameter gives a big differences between the present behaviors and those obtained by Raees et al. [18]. Like profiles of the nanoparticle volume fraction, the density of motile microorganisms profiles $S(\eta)$ are decreasing functions of Le, Nt, Pe, Nb and Pr, while the increase in the squeeze parameter β enhances the profiles of $S(\eta)$.

4. Conclusions

The problem of unsteady mixed nano-bioconvection flow in a horizontal channel with its upper plate expanding or contracting was re-examined. The behaviors of dependent variables were corrected to satisfy the boundary conditions of the problem. From this investigation, the following conclusions are addressed.

1. Using the two phase model in which the nanoparticle volume fraction on the boundary is passively controlled gives negative values of the nanoparticle volume fraction which has not been achieved in the Ref. [18].

2. For the results to be correct, the behaviors must be met to the boundary conditions of the problem and that did not satisfy in the Ref.[18] and corrected in the present study.

3. The nanoparticle volume fraction and the density of motile microorganisms are decreasing functions of Lewis number, the thermophoresis parameter and the Brownian motion parameter, while they increase as the squeeze parameter increases.

References:

[1] N.S. Khan, T. Gul, M.A. Khan, E. Bonyah, S. Islam, Mixed convection in gravity-driven thin film non-Newtonian nanofluids flow with gyrotactic microorganisms, Results in Physics, 7(Supplement C) (2017) 4033-4049.
[2] A.V. Kuznetsov, A.A. Avramenko, Effect of small particles on this stability of bioconvection in a suspension of gyrotactic microorganisms in a layer of finite depth, International Communications in Heat and Mass Transfer, 31(1) (2004) 1-10.
[3] A. Kuznetsov, The onset of nanofluid bioconvection in a suspension containing both nanoparticles and gyrotactic microorganisms, International Communications in Heat and Mass Transfer, 37(10) (2010) 1421-1425.
[4] A.V. Kuznetsov, Nanofluid bioconvection in water-based suspensions containing nanoparticles and oxytactic microorganisms: oscillatory instability, Nanoscale research letters, 6(1) (2011) 100.
[5] L. Tham, R. Nazar, I. Pop, Steady mixed convection flow on a horizontal circular cylinder embedded in a porous medium filled by a nanofluid containing gyrotactic micro-organisms, Journal of Heat Transfer, 135(10) (2013) 102601.
[6] S. Shaw, P. Sibanda, A. Sutradhar, P. Murthy, Magnetohydrodynamics and soret effects on bioconvection in a porous medium saturated with a nanofluid containing gyrotactic microorganisms, Journal of Heat Transfer, 136(5) (2014) 052601.
[7] H. Xu, I. Pop, Fully developed mixed convection flow in a horizontal channel filled by a nanofluid containing both nanoparticles and gyrotactic microorganisms, European Journal of Mechanics - B/Fluids, 46 (2014) 37-45.
[8] S.E. Ahmed, A. Mahdy, Laminar MHD natural convection of nanofluid containing gyrotactic microorganisms over vertical wavy surface saturated non-Darcian porous media, Applied Mathematics and Mechanics, 37(4) (2016) 471-484.
[9] F.M. Hady, A. Mahdy, R.A. Mohamed, O.A.A. Zaid, Effects of Viscous Dissipation on Unsteady MHD Thermo Bioconvection Boundary Layer Flow of a Nanofluid Containing Gyrotactic Microorganisms along a Stretching Sheet, World Journal of Mechanics, 2016, 6, 505-526.
[10] M.J. Uddin, M.N. Kabir, O.A. Bég, Computational investigation of Stefan blowing and multiple-slip effects on buoyancy-driven bioconvection nanofluid flow with microorganisms, International Journal of Heat and Mass Transfer, 95 (2016) 116-130.
[11] Z. Raizah, MHD mixed bioconvection stagnation-point flow of a nanofluid toward stretching surfaces with viscous dissipation and joule heating effects, Computational Thermal Sciences, 9(5) (2017) 1-15.
[12] S. Mosayebidorcheh, M.A. Tahavori, T. Mosayebidorcheh, D.D. Ganji, Analysis of nano-bioconvection flow containing both nanoparticles and gyrotactic microorganisms in a horizontal channel using modified least square method (MLSM), Journal of Molecular Liquids, 227(Supplement C) (2017) 356-365.
[13] S.E. Ahmed, H.F. Oztop, M. Mansour, N. Abu-Hamdeh, MHD Mixed Thermo-Bioconvection in Porous Cavity Filled by Oxytactic Microorganisms, Thermal Science, V22, (2018) 6, pp. 2711 - 2721.
[14] A.V. Kuznetsov, D.A. Nield, The Cheng–Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid: A revised model, International Journal of Heat and Mass Transfer, 65 (2013) 682-685.
[15] A.V. Kuznetsov, D.A. Nield, Natural convective boundary-layer flow of a nanofluid past a vertical plate: A revised model, International Journal of Thermal Sciences, 77 (2014) 126-129.

[16] A.V. Kuznetsov, D.A. Nield, Natural convective boundary-layer flow of a nanofluid past a vertical plate, International Journal of Thermal Sciences, 49(2) (2010) 243-247.
[17] D.A. Nield, A.V. Kuznetsov, The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid, International Journal of Heat and Mass Transfer, 52(25) (2009) 5792-5795.
[18] A. Raees, H. Xu, S.-J. Liao, Unsteady mixed nano-bioconvection flow in a horizontal channel with its upper plate expanding or contracting, International Journal of Heat and Mass Transfer, 86 (2015) 174-182.
[19] S.E. Ahmed, S.S. Mohamed, M.A. Mansour, A. Mahdy, Heat transfer and entropy generation due to a nanofluid over stretching cylinder: effects of thermal stratification, Computational Thermal Sciences: An International Journal, 9(1) (2017) 29-47.

[20] S.E. Ahmed, Z.A.S. Raizah, A.M. Aly, Entropy generation due to mixed convection over vertical permeable cylinders using nanofluids, Journal of King Saud University - Science, (2017).
[21] H. Xu, I. Pop, Mixed convection flow of a nanofluid over a stretching surface with uniform free stream in the presence of both nanoparticles and gyrotactic microorganisms, International Journal of Heat and Mass Transfer, 75 (2014) 610-623.