STRESS DISTRIBUTION CAUSED BY CO-PHASE LOCALLY SPATIALLY CURVED LAYERS IN AN INFINITE ELASTIC BODY UNDER BI-AXIAL COMPRESSION

by

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In the present paper, the stress distribution is studied in an infinite elastic body, reinforced by an arbitrary number of non-intersecting co-phase locally spatially curved filler layers under bi-axial compression is studied. It is assumed that this system is loaded at infinity with uniformly distributed normal forces with intensity $p_1(p_3)$ acting in the direction which is parallel to the layers' location planes. It is required to determine the self-equilibrated stresses within, caused by the spatially local curving of the layers. The corresponding boundary and contact value problem is formulated within the scope of geometrically non-linear exact 3-D equations of the theory of elasticity by utilizing of the piece-wise homogeneous body model. The solution the formulated problem is represented with the series form of the small parameter which characterizes the degree of the aforementioned local curving. The boundary-value problems for the zeroth and the first approximations of these series are determined with the use of the exponential double Fourier transform. The original of the sought values is determined numerically. Consequently, in the present investigation, the effect of the local curving on the considered interface stress distribution is taken into account within the framework of the geometrical non-linear statement. The numerical results related to the considered interface stress distribution and to the influence of the problem parameters on this distribution are given and discussed.

Key words: *stress distribution, local curving, double Fourier transform, geometrical Non-linearity*

Introduction

According to experimental results given in [1-3], in many cases surface and internal instability (buckling) of fibrous and layered composite materials can be taken as the initial stage of fracture of these materials under compression along reinforcing components. Two subject areas have emerged in the mechanics of fractures of compressed composite materials due to the bulging of reinforcing elements. The first area includes various approximate design models (for example, distribution of the compressive load between the filler and the binder, application of one and 2-D applied theories to stability analysis, *etc*). The studies in [1, 4, 5] are a few of the first ones in this area, with recent investigations carried out in [6-9].

Such cases can occur where the material of the reinforcing layer is nanomaterial, for instance nanocarbon structures or graphene films [10-12].

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The second area uses the 3-D geometrically non-linear exact equations of elasticity (or viscoelasticity) theory to study the fracture mechanisms for composite materials. Consistent consideration of these investigations has been made in [13, 14] and a review of the related works is given in [15]. Note that the corresponding stability loss problems are considered in [16, 17] and these results are also detailed in the monograph [18]. It follows from these references that the investigations regarding the second area have been studied within the scope of the piecewise homogeneous body model as well as within the scope of the continuum approach.

Moreover, the analysis of the foregoing investigations shows that the study of the self-balanced stresses in the composites with the locally curved reinforcing layers has been made mainly for the cases where the mentioned curving is plane one and it is considered the case where the plane-strain state occurs.

Consequently, up to now the influence of the spatiality of the local curving of the reinforcing layers on the distribution of the self-balanced stresses does not studied.

The present paper attempts to fill this gap and studies the self-balanced stresses caused by the spatially local curving of the filler layer.

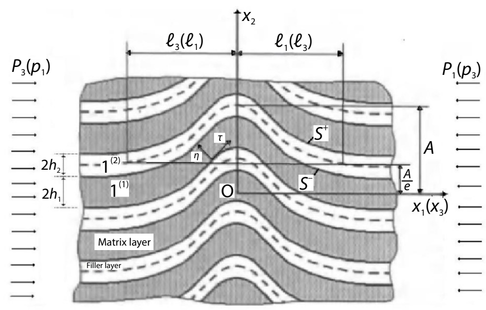

Figure 1. The structure of the composite material with cophasically locally curved layers

Problem formulation and solution method

We consider an infinite elastic body, reinforced by an arbitrary number of non-intersecting cophasically locally curved filler layers as shown in fig. 1, $x_1^{(k)} = \text{constant}$ ($x_3^{(k)} = \text{constant}$). We associate the corresponding Cartesian system $O^{(k)}x_1^{(k)}x_2^{(k)}x_3^{(k)}$ (k = 1, 2) which are obtained from $Ox_1x_2x_3$ see fig. 1, by parallel transfer along the Ox_2 axis, with the middle surface of each layer of the filler and matrix. The positions of the points of the layers we determine through the Lagrange co-ordinates in these systems.

The thickness of each filler layer will be assumed constant. Furthermore, the matrix and filler materials are homogeneous, isotropic and linearly elastic. Now we investigate the stress deformation state in the above body under loading at *infinity* by uniformly distributed normal forces with intensity $p_1(p_3)$ acting in the direction of the $Ox_1(Ox_3)$ axis. The composite structure in the direction of the Ox_2 axis has period $2(h_1 + h_2)$, where $2h_1$ is a thickness of the matrix layer and $2h_2$ is a thickness of the filler layer. Taking this periodicity into consideration, among the layers we single out two of them, *i. e.* $1^{(1)}$, $1^{(2)}$, and the entire solution procedure can be carried out on these two layers.

For each layer we write the geometrical non-linear equilibrium equations, constitutive and geometrical relations:

$$\frac{\partial}{\partial x_j^{(k)}}\left[\sigma_{jn}^{(k)}\left(\delta_i^n+\frac{\partial u_i^{(k)}}{\partial x_n^{(k)}}\right)\right]=0\,,\quad \sigma_{ji}^{(k)}=\lambda^{(k)}\theta^{(k)}\delta_i^j+2\mu^{(k)}\varepsilon_{ji}^{(k)}$$

$$2\varepsilon_{ij}^{(k)}=\frac{\partial u_i^{(k)}}{\partial x_j^{(k)}}+\frac{\partial u_j^{(k)}}{\partial x_i^{(k)}}+\frac{\partial u_n^{(k)}}{\partial x_i^{(k)}}\frac{\partial u_n^{(k)}}{\partial x_j^{(k)}}\,,\ i;j;n=1,2,3\,,\ \ k=1,2 \tag{1}$$

In eq. (1), the conventional notation is used and values related with the filler and matrix layers are denoted by upper indices (2) and (1), respectively.

We denote the upper surface of the $1^{(2)\text{th}}$ layer by S^{+} and the lower by S^{-}, see fig. 1, which will be used below.

Assume that the initial spatial curving of the filler layer is given through the equation of the middle surface, $1^{(2)\text{th}}$ layer, as:

$$x_2^{(2)} = F(x_1^{(2)}, x_3^{(2)}) = \varepsilon f(x_1, x_3) == A\mathrm{e}^{-(\frac{x_1}{\ell_1})^2 - \gamma^2 (\frac{x_3}{\ell_1})^2} = \varepsilon \ell_1 \mathrm{e}^{-(\frac{x_1}{\ell_1})^2 - \gamma^2 (\frac{x_3}{\ell_1})^2} \tag{2}$$

where ε is a dimensionless small parameter ($0 \le \varepsilon \ll 1$), $\varepsilon = A/\ell_1$, and $\gamma = \ell_1/\ell_3$. The geometrical meaning of ℓ_1 and ℓ_3 is illustrated in fig. 1. In eq. (2) the spatiality of the curving is characterized through the parameter γ.

Moreover, we suppose that the functions $F(x_1^{(2)}, x_3^{(2)})$ and their first-order derivatives are continuous and satisfy the following conditions:

$$\left(\frac{\partial F}{\partial x_1^{(2)}}\right)^2 + \left(\frac{\partial F}{\partial x_3^{(2)}}\right)^2 << 1 \tag{3}$$

It is asumed that complete contact conditions on the interfaces between the constituents are satisfied.

The quantities characterizing the stress-strain state of the arbitrary components of the systems considered are presented in the power series form with respect to the parameter ε:

$$\left\{\sigma_{ij}^{(k)}; \varepsilon_{ij}^{(k)}; u_i^{(k)}\right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{\sigma_{ij}^{(k),q}, \varepsilon_{ij}^{(k),q}, u_i^{(k),q}\right\} \tag{4}$$

Using the complete system of equation in eq. (1), we obtain the corresponding boundary-value problem for the determination of the values of each approximation in the eq. (4). Here we consider the determination of the values by using only the zeroth and first approximations. The all numerical results are obtained within these approximations.

According to [14, 16, 17], in the case under consideration the values related to the zeroth approximation are determined:

$$\begin{gathered} \sigma_{33}^{(1),0} \neq 0\,,\ \sigma_{33}^{(2),0} \neq 0\,;\ \sigma_{11}^{(1),0} \neq 0\,,\ \sigma_{11}^{(2),0} \neq 0 \\ \sigma_{12}^{(k),0} = \sigma_{13}^{(k),0} = \sigma_{23}^{(k),0} = \sigma_{22}^{(k),0} = 0 \end{gathered} \tag{5}$$

where it is obtained the following expressions for the stresses $\sigma_{33}^{(1),0}$, $\sigma_{33}^{(2),0}$; $\sigma_{11}^{(1),0}$ and $\sigma_{11}^{(2),0}$:

$$\sigma_{11}^{(k),0} = \frac{E^{(k)}(p_1 + \nu^{(k)} p_3)}{E^{(3)}(1-(\nu^{(k)})^2)} - \frac{E^{(k)}\nu^{(3)}(p_3 + \nu^{(k)} p_1)}{E^{(3)}(1-(\nu^{(k)})^2)} \tag{6}$$

$$\sigma_{33}^{(k),0} = \frac{E^{(k)}(p_3 + \nu^{(\underline{k})} p_1)}{E^{(3)}(1-(\nu^{(k)})^2)} - \frac{E^{(k)}\nu^{(3)}(p_1 + \nu^{(k)} p_3)}{E^{(3)}(1-(\nu^{(k)})^2)},\ \ \sigma_{ij}^{(k),0} = 0 \text{ for } ij \neq 11;33 \tag{7}$$

Note that the values regarding the zeroth approximation are determined by the use of the corresponding linear equations. Moreover, it is assumed that $\partial u_i^{(k),0}/\partial x_j^{(k)} \ll 1$ and they neglected in the equations related with the first approximation.

Consider the determination of the values of the first approximation for which the following equations and relations are obtained from eqs. (1), (4), (6), and (7).

The equilibrium equations:

$$\frac{\partial \sigma_{ji}^{(k),1}}{\partial x_j^{(k)}} + \sigma_{11}^{(k),0} \frac{\partial^2 u_i^{(k),1}}{\partial (x_1^{(k)})^2} + \sigma_{33}^{(k),0} \frac{\partial^2 u_i^{(k),1}}{\partial (x_3^{(k)})^2} = 0 \tag{8}$$

The mechanical and geometrical relations:

$$\sigma_{ji}^{(k),1} = \lambda^{(k)} \theta^{(k),1} \delta_i^j + 2\mu^{(k)} \varepsilon_{ji}^{(k),1}, \ \theta^{(k),1} = \varepsilon_{11}^{(k),1} + \varepsilon_{22}^{(k),1} + \varepsilon_{33}^{(k),1}$$
$$\varepsilon_{ji}^{(k),1} = \frac{1}{2}\left(\frac{\partial u_i^{(k),1}}{\partial x_j^{(k)}} + \frac{\partial u_j^{(k),1}}{\partial x_i^{(k)}} \right) \tag{9}$$

The contact conditions:

$$\begin{gathered}
\sigma_{22}^{(1),1}\Big|_{x_2^{(1)}=\mp h_1} = \sigma_{22}^{(2),1}\Big|_{x_2^{(2)}=\pm h_2} \\
-\frac{\mathrm{d}f}{\mathrm{d}x_1^{(1)}}\sigma_{11}^{(1),0} + \sigma_{12}^{(1),1}\Bigg|_{x_2^{(1)}=\mp h_1} = -\frac{\mathrm{d}f}{\mathrm{d}x_1^{(2)}}\sigma_{11}^{(2),0} + \sigma_{12}^{(2),1}\Bigg|_{x_2^{(2)}=\pm h_2} \\
-\frac{\mathrm{d}f}{\mathrm{d}x_3^{(1)}}\sigma_{33}^{(1),0} + \sigma_{23}^{(1),1}\Bigg|_{x_2^{(1)}=\mp h_1} = -\frac{\mathrm{d}f}{\mathrm{d}x_3^{(2)}}\sigma_{33}^{(2),0} + \sigma_{23}^{(2),1}\Bigg|_{x_2^{(2)}=\pm h_2} \\
u_i^{(1),1}\Big|_{x_2^{(1)}=\mp h_1} = u_i^{(2),1}\Big|_{x_2^{(2)}=\pm h_2}, \quad i = 1,2,3
\end{gathered} \tag{10}$$

For the solution of the problem in eqs. (6)-(10), the double Fourier transformation with respect to the co-ordinates x_1 and x_3 is employed:

$$\varphi_{13F}(s_1, x_2, s_3) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \varphi(x_1, x_2, x_3)\, \mathrm{e}^{-i(s_1x_1 + s_3x_3)} \mathrm{d}x_1 \mathrm{d}x_3 \tag{11}$$

The double Fourier transforms of the sought values are determined by the use of the method developed in the papers [16, 17] after which the original of those can now be represented:

$$\left\{\sigma_{ij}^{(k)}; \varepsilon_{ij}^{(k)}; u_i^{(k)}\right\} = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left\{\sigma_{ij13F}^{(k),q}, \varepsilon_{ij13F}^{(k),q}, u_{i13F}^{(k),q}\right\} \mathrm{e}^{i(s_1x_1 + s_3x_3)} \mathrm{d}s_1 \mathrm{d}s_3 \tag{12}$$

To simplify the matters, we consider the calculation of the integral for $\sigma_{22}^{(k)}$ that is the integral:

$$\sigma_{22}^{(k)} = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \sigma_{2213F}^{(k)}(s_1, x_2, s_3)\, \mathrm{e}^{i(s_1x_1 + s_3x_3)} \mathrm{d}s_1 \mathrm{d}s_3 \tag{13}$$

Introduce the notation:

$$\varphi(x_1, x_2, x_3) = \sigma_{22}^{(k)}(x_1, x_2, x_3), \ \varphi_{13F}(s_1, x_2, s_3) = \sigma_{2213F}^{(k)}(s_1, x_2, s_3)$$
$$\varphi_{3F}(x_1, x_2, s_3) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \varphi_{13F}(s_1, x_2, s_3) \mathrm{e}^{is_1x_1} \mathrm{d}s_1 \tag{14}$$

Using symmetry and eq. (14) the integral eq. (13):

$$\varphi(x_1, x_2, x_3) = \frac{1}{\pi^2}\int_0^{+\infty}\int_0^{+\infty} \varphi_{13F}(s_1, x_2, s_3) \cos(s_1x_1)\cos(s_3x_3) \mathrm{d}s_1 \mathrm{d}s_3 \tag{15}$$

First, the integral eq. (15) is replaced by a corresponding definite integral by using the following approximation:

$$\int_0^{+\infty}\int_0^{+\infty} \varphi_{13F}(s_1,x_2,s_3)\cos(s_1x_1)\cos(s_3x_3)\mathrm{d}s_1\mathrm{d}s_3 \approx \int_0^{s_{3*}}\int_0^{s_{1*}} \varphi_{13F}(s_1,x_2,s_3)\cos(s_1x_1)\cos(s_3x_3)\mathrm{d}s_1\mathrm{d}s_3 \tag{16}$$

The values of s_{1*} and s_{3*} in eq. (16) are defined from the convergence requirement. For the integrals the Gauss integration algorithm is employed. All these procedures are made automatically in personal computer by using the programmes written in MATLAB.

Numerical results and discussion

We consider the distribution of the normal stress σ_{nn}/p_1 acting on the interface surface between the filler and matrix layers. We denote the Young's moduli through $E^{(1)}$, $E^{(2)}$, and the Poisson ratios through $\nu^{(1)}$, $\nu^{(2)}$. We assume that $p_3 = p_1$, $\gamma = 1$, $\nu^{(1)} = \nu^{(2)} = 0.3$, $E^{(1)}/E^{(2)} = 100$, $x_2/h_2 = 1.0$, and $x_3/h_2 = 0$. The influence of the geometrical non-linearity on the mentioned distribution will be characterized through the parameter $\sigma_{11}^{(2),0}/\mu^{(2)}$, $\mu^{(2)} = E^{(2)}/2[1 + \nu^{(2)}]$. Thus, within the scope of the foregoing assumptions we analyze the numerical results and begin this analysis with those regarding the dependence between σ_{nn}/p_1 and x_1/h_2.The graphs of this dependence constructed for various values of $\sigma_{11}^{(2),0}/\mu^{(2)}$ are given in fig. 2.

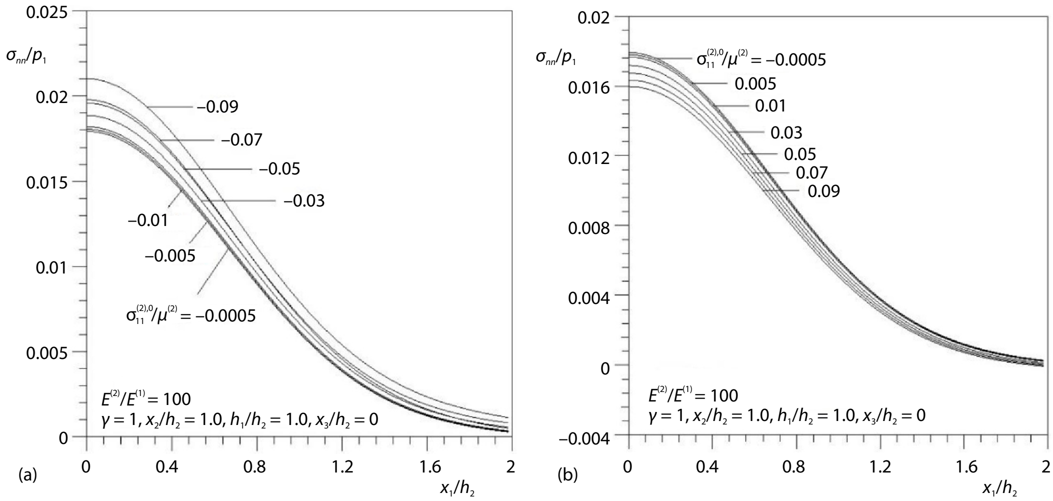

Figure 2. The graphs of the dependencies between σ_{nn}/p_1 and x_1/h_2; (a) compressing, (b) stretching

According to the well-known mechanical consideration, the values of σ_{nn}/p_1 must approach zero with x_1/h_2. This prediction is proved by the graphs given in fig. 2. At the same time, these graphs show that because of the geometrical non-linearity, the absolute values of σ_{nn}/p_1 decrease (increase) with $\sigma_{11}^{(2),0}/\mu^{(2)}$ under tension (compression) of the considered material. In the qualitative sense these results agree with the corresponding ones given in the monograph [14]. Consequently, the results illustrate the reliability and validity of the employed algorithm and personal computer programs.

From fig. 3(a) it can easily be seen that as the values of γ is decrease, $|\sigma_{nn}/p_1|$ approaches a limit in which γ is equal to zero, *i. e.* approaches the case where the curving of the filler layer is a plane one. According to the mechanical consideration, σ_{nn}/p_1 must approach the certain limit value with the ratio h_1/h_2 and this limit value is σ_{nn}/p_1 which is obtained for the case where the filler layer is in an infinite body. This prediction is proven with the graph given in fig. 3(b).

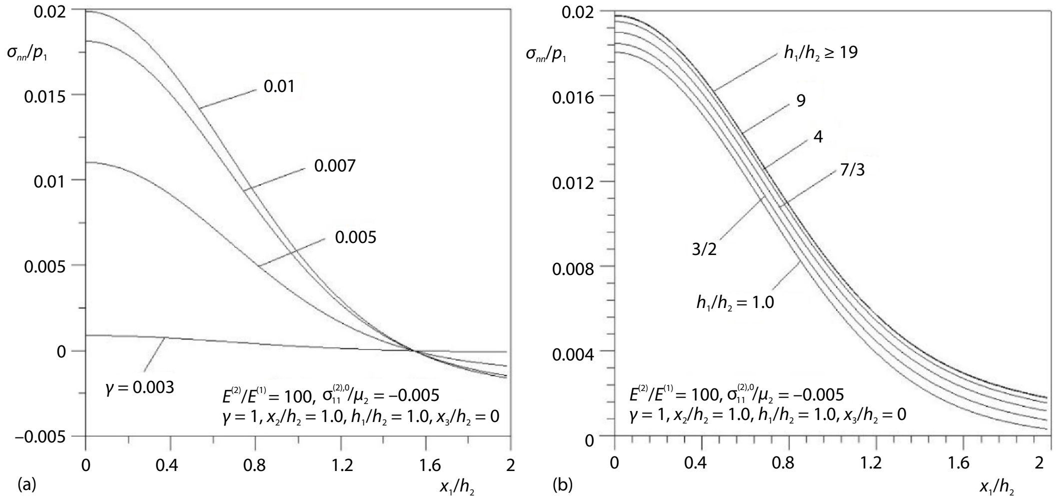

Figure 3. The graphs of the dependencies; (a) between σ_{nn}/p_1 and $\gamma(= \ell_1/\ell_3)$, (b) dependencies between σ_{nn}/p_1 and h_1/h_2

Conclusion

Thus, in the present paper, within the framework of the piecewise homogeneous body model with the use of the 3-D geometrically non-linear exact equations of the theory of elasticity, the stress distribution in an infinite elastic body reinforced by an infinite number alternating two co-phase locally spatially curved filler layers under bi-axial loading of that, has been investigated. A method for solving the problem by employing the double Fourier transformation was employed. Numerical results on the self-balanced normal stress caused by the spatial local curving of the reinforcing layer under stretching, as well as under compressing of the mentioned body have been presented and analyzed.

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