

## INFLUENCE OF WATER/DIESEL EMULSIFIED FUEL ON DIESEL ENGINE CHARACTERISTICS

by

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*The paper deals with water/diesel emulsion effects on injection and combustion characteristics of a bus Diesel engine. The influences of water content in water/diesel emulsion are investigated by numerical simulation. Higher content of water in water/diesel emulsion increases the injection pressure and decreases the in-cylinder gas pressure, in-cylinder gas temperature, and rate of heat release, this leads to lower engine torque and power. The obtained results indicate a possibility of essential reduction of  $NO_x$  and soot emissions by increasing water content.*

Key words: *Diesel engine, water/diesel emulsion, pump injection timing, heat release rate, harmful emissions*

### Introduction

In recent years many experimental and numerical investigations related to the control of the injection and combustion processes have been done in order to reduce harmful diesel engine emissions. In order to meet ever stronger exhaust emission requirements, many efforts are put into improvement of injection and combustion processes, aftertreatment technology, and usage of alternative fuels. For this reason, many existing Diesel engines have been optimized and modified. In this context and by assuming that the modifications of the existing Diesel engines have to be inexpensive, water addition seems to be a quite promising technique.

In last decades, water addition to mineral diesel before or after injection has been quite intensively investigated [1-4]. Till today, various methods of water addition have been developed, such as:

- water injection into the intake manifold [5],
- water injection directly into the combustion chamber, and
- water/fuel emulsion injection.

In order to inject water into the intake manifold, multi-point and mono-point water injection systems have been developed [3]. In a multi-point water injection system, the amount of water injected into the air stream depends on pressure that is adjusted by the air pressure in the water tank, which is controlled by an electronic regulator. The injection procedure starts immediately after the intake valve is open and finishes just before the intake valve is closed. In a mono-point injection system, water is added to the air stream with one injector, located just before or after the turbocharger [7]. Both systems for water injection into intake manifold can be effectively employed in an existing Diesel engine to reduce  $NO_x$  emission and smoke index simultaneously without loss of power and any negative effect on fuel consumption [6].

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Various strategies have been proposed till now to inject water directly into the combustion chamber, with the aim of reducing  $\text{NO}_x$  emissions and keeping the water quantity small in comparison to water injection into the intake manifold [3, 8]. By this method, water is supplied by using separate injectors, which enables precise control of water and fuel ratio [6]. Another possibility is given by a stratified fuel/water injection system, where water is fed into the injector's dead volume when fuel injection does not take place [3, 9]. When injection begins, fuel and water are injected into the combustion chamber in a stratified condition. Injecting water in this way delays slightly the ignition but this delay can be kept at a low level, even if a large quantity of water is injected, and significant  $\text{NO}_x$  reduction can be achieved. An important advantage of direct water injection is the possibility to change the water to fuel ratio in dependence on engine parameters (speed and load) or during engine warm-up (cold start).

In recent years, water/fuel emulsion has also been investigated with the aim to achieve improvements in fuel economy and reduction of  $\text{NO}_x$  and PM emissions [10-20]. The emulsion fuel is determined by an appropriate water/diesel ratio and a corresponding stabilizing agent. Abu-Zaid [10] concluded that torque, power, specific fuel consumption, and thermal efficiency, by increasing the percentage of water up to 20%, increase. He also concluded that the addition of 2% surfactant to the emulsions seems to have almost negligible effects on the engine performance parameters measured. Attia and Kulchitsky [11] reported that the emulsion structure has a clear effect on the engine performances. Large water droplet size have a great impact on the emissions of  $\text{NO}_x$ , while smaller water droplets influence emissions of unburned hydrocarbon (HC) and smoke levels of exhaust gases. Azimi *et al.* [12] investigated experimentally the effect of water/diesel emulsion on a tractor engine performance and they found out that water addition increases CO and HC emission, meanwhile,  $\text{NO}_x$ , smoke, and noise emissions can be reduced. On the basis of experimentally results, Baskar and Kumar [13] concluded that using water/diesel emulsion and oxygen enriched air can lead to improvements in combustion, performance, and emission characteristics. An exception are NO emissions, which can increase. Chaitanya *et al.* [14] investigated experimentally the influence of water/diesel emulsion, prepared by various surfactant and the obtained results show that  $\text{NO}_x$  emissions can be reduced by up to 38% with Span 20 and Tween 20 as surfactant up to 2% of water content. On the basis of experimentally obtained results, Dubey and Saxena [15] concluded that the benefits of adding water to diesel fuel results in substantial reduction of  $\text{NO}_x$  and particulate matter (PM) emissions. The water/diesel emulsion, however, results in less effective power and engine efficiency [16]. Gheroghe *et al.* [17] concluded, that the main reason to use water/diesel emulsion is the lower in-cylinder temperature due to the micro-explosions, and consequently lower level of  $\text{NO}_x$  and  $\text{SO}_2$ . Ithnin *et al.* [18] showed with experimental investigation of water/diesel emulsion that besides of reduction of  $\text{NO}_x$  and PM emissions, at higher engine loads the CO emission is comparable with diesel fuel, meanwhile,  $\text{CO}_2$  emission is slightly higher. Samec *et al.* [2] performed numerical simulation of water/n-heptane emulsified fuel combustion process in a four cylinder air cooled truck Diesel engine.

In this work a numerical simulation of the combustion process in a six cylinder water cooled bus Diesel engine by using water/diesel emulsion is presented. The used mathematical programs, the in-house developed mathematical model BKIN for simulation of injection process and AVL BOOST for simulation of combustion process, were verified with experiments in our previous research [21-23]. The effects of various content of water in water/diesel emulsion are investigated. The main attention is focused on the influence of water content on the injection and combustion characteristics.

### Numerical simulation model

The whole injection process in the mathematical model BKIN is described by fifteen quantities: the fuelling, the pressures in the barrel chamber, in the delivery valve chamber, in the snubber valve chamber and in the injector chamber as well as the lifts and velocities of the delivery valve, snubber valve and of the needle, the vapor volume in the delivery valve chamber, snubber valve chamber, HP tube and injector chamber [22]. These quantities are related to each other and to time by fifteen ODE containing an unknown parameter - the residual pressure. Since this quantity is not known in advance, iterations are needed to solve this system of differential equations. In the first iteration, it is proposed to start with an estimated residual pressure somewhat higher than the evaporation pressure. After the first integration is completed, the new residual pressure is calculated. This value is then used as an estimate of the residual pressure for the next iteration and so on. This procedure is terminated when the difference of the calculated residual pressure in two successive iterations is smaller than some prescribed tolerance. It is assumed that the total initial volume of vapor in the HP system is distributed proportionally to the system dead volumes.

The employed mathematical model of used AVL BOOST software is based on the first law of thermodynamics, which can be for a Diesel engine written:

$$\frac{d(m \cdot u)}{d\alpha} = -p_c \frac{dV}{d\alpha} + \frac{dQ_f}{d\alpha} - \sum \frac{dQ_w}{d\alpha} - h_{bb} \frac{dm_{bb}}{d\alpha} \quad (1)$$

where the symbol  $\alpha$  denotes the angle of crankshaft rotation,  $m_c$  – the mass of the mixture in the cylinder,  $u$  – the specific internal energy,  $p_c$  – the in-cylinder pressure,  $V$  – the cylinder volume,  $Q_f$  – the fuel energy,  $Q_w$  – the heat transfer through the liner,  $h_{bb}$  and  $m_{bb}$ , are the enthalpy and mass of the mixture that escapes through the gap between the piston and the liner, respectively.

The simulation software, used in this work is based on the following assumptions. For internal mixture preparation it is assumed that: the fuel added to the cylinder charge is immediately combusted, the combustion products mix instantaneously with the rest of the cylinder charge and form a uniform mixture, and as a consequence, the air/fuel ratio of the charge diminishes continuously from a high value at the start of combustion to the final value at the end of combustion. Furthermore, the flow in the tube is treated as a 1-D flow. This means that the pressures, temperatures, and flow velocities, obtained from the solution of the gas dynamic equations, represent mean values over the cross-section of the tubes. The flow losses at particular locations are considered by taking into account the discharge coefficients. The employed software also offers several submodels to account for various possibilities of flow modeling. In this work, the options described in the following have been selected by keeping in mind the objective of a good compromise between the speed of computation and accuracy with respect to the experimentally obtained results.

To determine heat release,  $Q_c$ , the Chmela and Orthaber mixture controlled combustion model was used [21]. The MCC combustion model divides combustion into two stages. The first stage is premixed combustion  $Q_{PMC}$  that occurs after the ignition delay interval ends. The second part of the combustion is the mixing controlled combustion  $Q_{MCC}$  that follows the premixed part of the combustion and begins when all the fuel/air mixture from the first part of the combustion has burned:

$$\frac{dQ_c}{d\alpha} = \frac{dQ_{MCC}}{d\alpha} + \frac{dQ_{PMC}}{d\alpha} \quad (2)$$

In the premixed part of combustion the fuel that was vaporized and mixed with fresh air during the ignition delay interval is burned. Because of the large amount of excess air at this stage,

the fuel/air mixture burns rapidly. To calculate the heat released during the premixed combustion, the Vibe function was used; meanwhile, the heat release during the mixing controlled combustion is determined with respect to injected amount of fuel and local density of turbulent kinetic energy in the cylinder [21].

### Experimental equipment and test procedure

The experiments were performed separately on the injection system of the engine, placed on its own test bed, and on the whole engine, mounted onto another test bed. The fuel injection M system was mounted on Friedman-Maier type 12H100\_h test bed for an in-line high pressure pump, fig. 1.

The schematic diagram of the engine test bed and the combustion chamber of the tested engine are presented in fig. 2. The test bed consisted of an engine and electro-dynamometer Zöllner A-350AC, air-flow rate meter RMG, fuel consumption dynamic measuring system AVL, and emissions analyzers. Furthermore, using a data acquisition system the instantaneous injection pressure and needle lift, the instantaneous in-cylinder gas pressure and in-cylinder temperature, and other important parameters were measured, fig. 2.

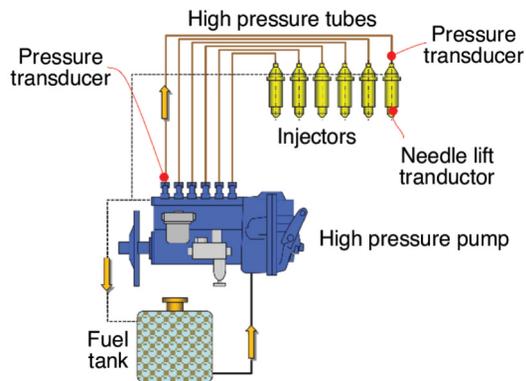


Figure 1. The fuel injection system test bed

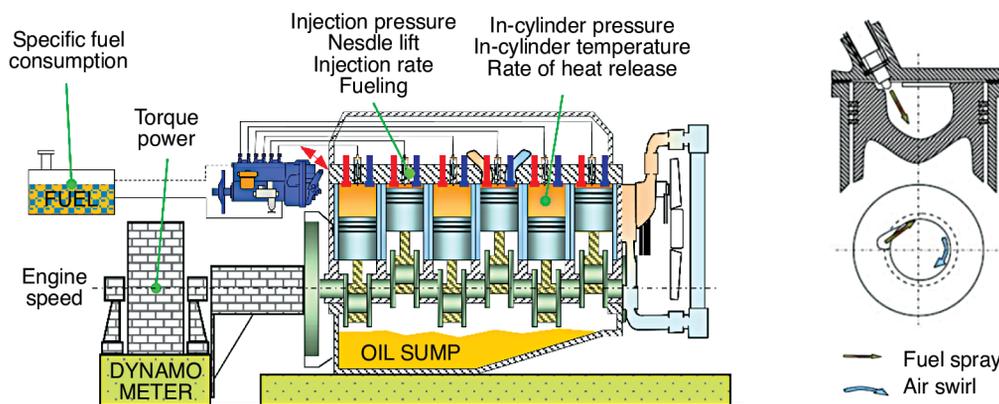


Figure 2. The bus Diesel engine test bed

The tested engine is a bus Diesel engine MAN D 2566 MUM with a mechanically controlled fuel injection system. The engine has completed 500000 km and has undergone general renovation. The main specifications of the tested engine and fuel injection system are given in tab. 1, while the properties of the tested fuels, diesel, emulsion with 20% of water (Dw20), and emulsion with 40% of water (Dw40), are given in tab. 2.

To verify the numerical simulation used for our engine simulation model, the injection and combustion characteristics were measured at various engine operating regimes. For example, fig. 3 shows the comparison of the injection pressure (at the end of high pressure tube), needle lift, and in-cylinder gas pressures only at rated condition by usage of mineral diesel fuel.

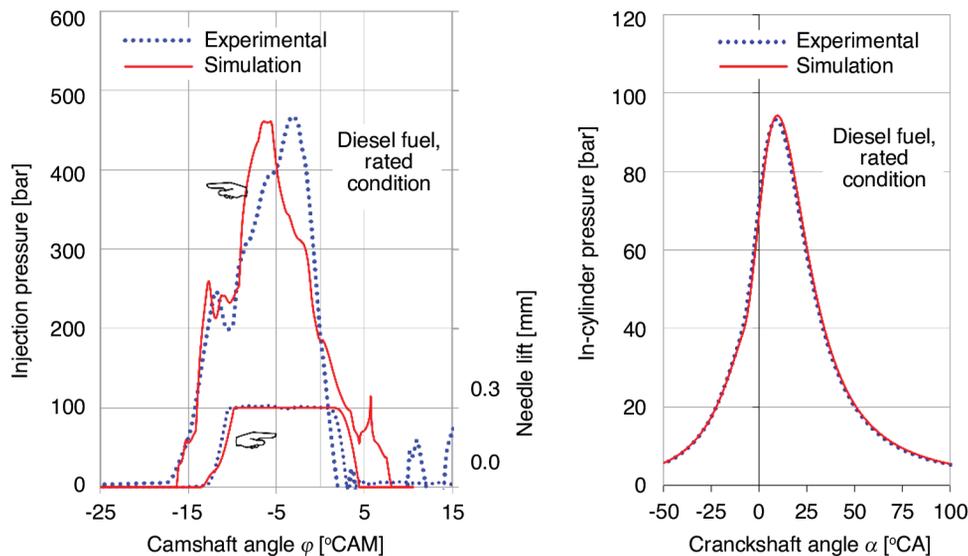
**Table 1. Test engine main specifications**

Engine model	MAN D 2566 MUM
Engine type	4-stroke, six cylinder in line, water cooled
Displacement	11413 cm <sup>3</sup>
Compression rate	17.5
Bore and stroke	125 mm × 155 mm
Max power	162 kW at 2200 rpm
Fuel injection system	Direct injection M system
Pump plunger (diameter × lift)	9.5 mm × 8 mm
Fuel tube length (length × diameter)	1024 mm × 1.8 mm
Injection nozzle (number × hole diameter)	1 × 0.68 mm
Needle lift (max)	0.3 mm
Needle opening pressure	175 bar
Start of delivery (pump injection timing)	23 °CA bTDC

A good agreement in the injection pressure, needle lift, and in-cylinder gas pressures shows that the numerical simulation models are suitable to analyze the influence of fuel on injection and combustion parameters. A similar agreement between experiment and numerical simulation was also obtained for engine power and effective specific fuel consumption and other engine characteristics.

**Table 2. Fuel properties**

Fuel	Diesel	DW20	DW40
Density at 20 °C [kgm <sup>-3</sup> ]	828	862	895
Kinematic viscosity at 20 °C [mm <sup>2</sup> s <sup>-1</sup> ]	3.35	2.88	2.41
Bulk modulus of elasticity at 40 °C, 300 bar [MPa]	1600	1660	1750
Calorific value [MJkg <sup>-1</sup> ]	42.9	32.4	30.5



**Figure 3. Injection pressure, needle lift, and in-cylinder pressure at full load**

## Results and discussion

To analyze the influence of fuel properties on bus Diesel engine characteristics, the most important injection and combustion characteristics, obtained with mineral diesel and water/diesel emulsions, are compared and discussed in this section.

### *Influence of amount of water addition to mineral diesel on injection characteristics*

The diagrams in figs. 4 and 5 show that the injection pressure, calculated in the injector chamber, increases with increasing water content in diesel/water emulsion. The difference between the injection pressures of different fuels arises due to different fuel density, viscosity, bulk modulus and sound velocity [3]. Furthermore, these diagrams show that the injection delay (time between start of delivery and start of injection) decreases with higher content of water. This means that injection timing is advanced by introducing water. For the difference of injection timing, the differences in viscosity and in bulk modulus, which affect the speed of sound,

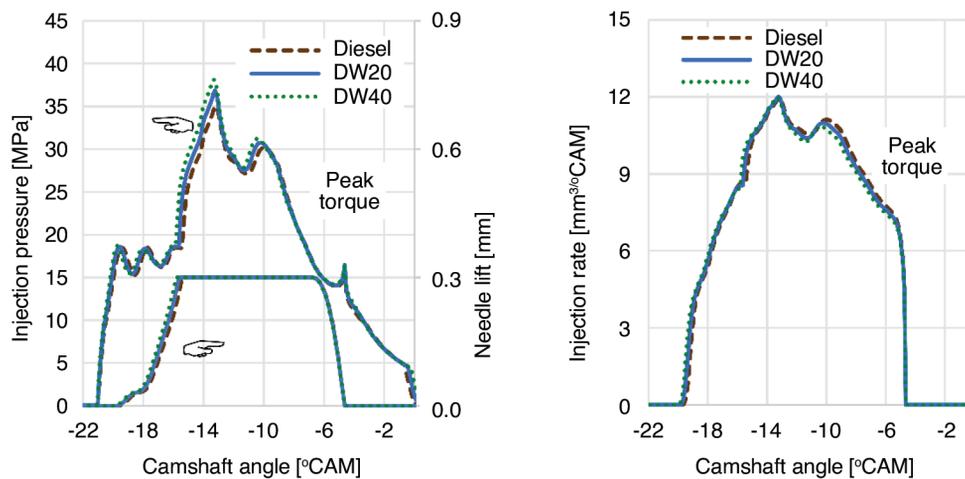


Figure 4. Injection characteristics at peak torque

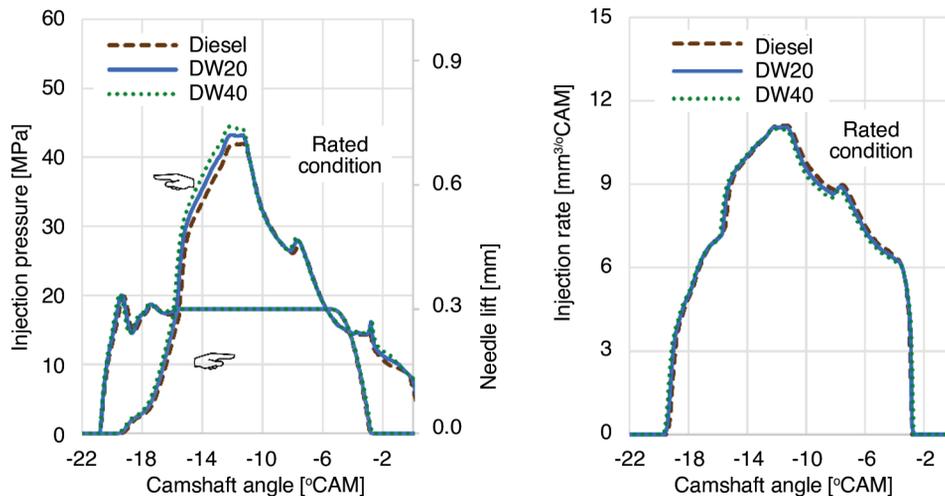


Figure 5. Injection characteristics at rated conditions

are responsible. A higher bulk modulus, caused by increasing the content of water, leads to a more rapid pressure wave propagation from the pump to the needle nozzle and an earlier needle lift. This effect is slightly reduced by lower viscosity of water/diesel emulsions, which in general leads to increased fuel losses during injection process, to slower evolution of pressure and thus to retarded injection timing [4]. However, the parameters that affect advanced injection prevail practically at all full load operating regimes.

For all operating regimes at full loads, figs. 4 and 5, it holds, that with increasing content of water in water/diesel emulsion fuel the injection delay, the mean injection rate, and the pressure squareness decrease. Meanwhile the injection timing and the mean as well as maximal injection pressure increase. In spite of higher mean injection pressure, the pressure squareness decreases when using fuels with higher part of water because of higher peak injection pressure.

The parts of fuelling of different injection phases with respect to fuelling per stroke, are presented in fig. 6 as:

- during first 10% of injection duration (A1)
- from end of A1 till the end of the opening phase (A2)
- open needle phase (B)
- from start of closing phase till the begin of C1 (C2)
- during the last 10% of injection duration (C1).

The presented results show, that the fuelling during needle opening phase (A1 + A2) as well as the fuelling during needle closing phase (C2 + C1) are the smallest at peak torque condition with the higher content of water in mineral diesel, fig. 6.

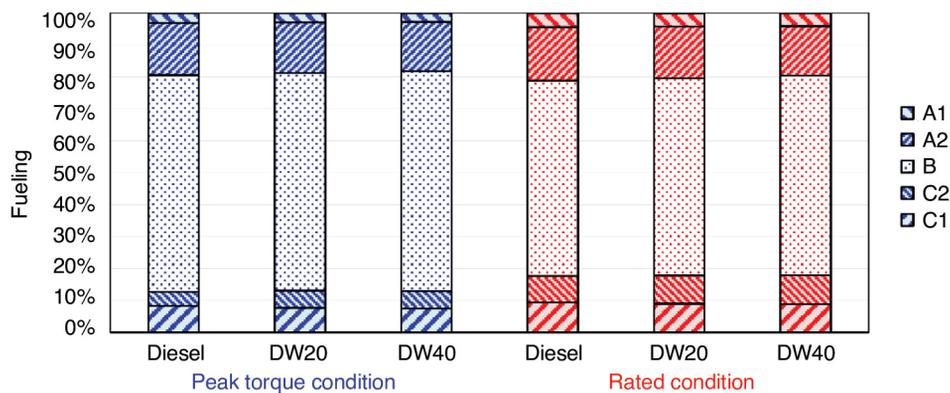


Figure 6. Fuelling in different phases of injection

In order to be able to compare the behavior of different fuels, the relative fuellings in several phases of injection at peak torque and rated condition, were calculated. The most interesting phases (A1 and C1) are compared in fig. 7. One can see that by increasing water content, all of the compared quantities decrease. This offers a good opportunity to reduce  $\text{NO}_x$  as well as smoke and PM emissions.

From fig. 7 it seems that a suitable fuel regarding the harmful  $\text{NO}_x$  and smoke emissions is the employed emulsion with 40% of water, because of the smallest fuel quantity at the start as well as at the end of injection and because of higher mean injection pressure as well as higher mean injection rate. A similar result obtained with water/diesel emulsion has also been confirmed experimentally [2, 3].

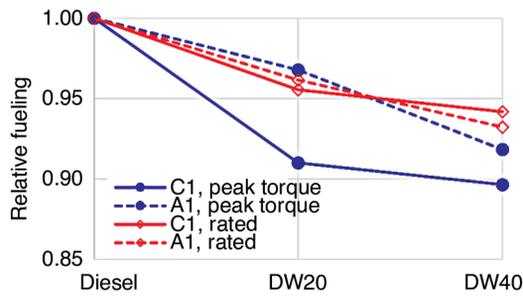


Figure 7. Relative fuelling of injection

#### *Influence of amount of water addition to mineral diesel on combustion characteristics*

Influence of water addition on the in-cylinder gas pressure, rate of heat release, and in-cylinder gas temperature history is presented in figs. 8 and 9. At all tested regimes, the heat release rate peaks are lower for higher content of water. At peak torque condition, the heat release rate peak decreased by about 7% and 28% when using Dw20 and DW40,

respectively. At rated condition, the heat release rate peak decreased by about 11% and 38% for DW20 and DW40, respectively. Furthermore, it can be seen that with higher content of water and at higher engine speeds the peak of heat release is reached earlier. The reduction of the in-cylinder temperature at peak torque condition was about 5% and 18% for DW20 and DW40, respectively. At rated condition the in-cylinder temperature decreased by about 5% and 20% for DW20 and DW40, respectively.

The in-cylinder gas pressure decreases by higher content of water due to lower calorific value of water/diesel emulsion. The reduction of the in-cylinder pressure at peak torque condition was about 1% and 11% for DW20 and DW40, respectively. At rated condition the in-cylinder pressure decreased by about 3% and 16% for DW20 and DW40, respectively. This leads to lower engine power and engine torque, as well as to higher specific fuel consumption, fig. 10. At peak torque condition, the engine torque and power decreased by about 14% and 41%, while specific fuel consumption increased by about 32% and 99% for DW20 and DW40, respectively. At rated condition, the engine torque and power decreased by about 9% and 37%, while specific fuel consumption increased by about 23% and 85% for DW20 and DW40, respectively.

The lower in-cylinder gas temperature of water/diesel emulsion can be attributed to two reasons. Firstly, evaporation of water/diesel emulsion in the intake system lowers the intake

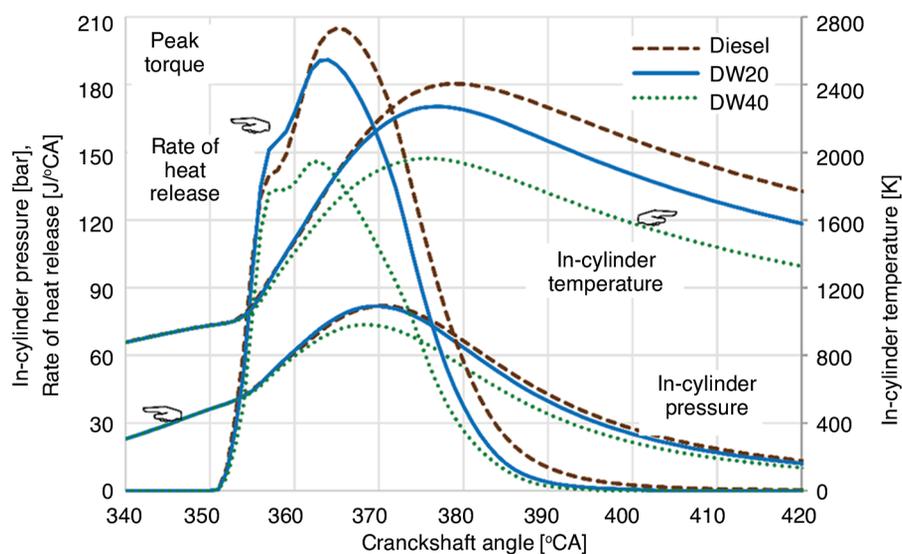


Figure 8. Combustion characteristics at peak torque condition

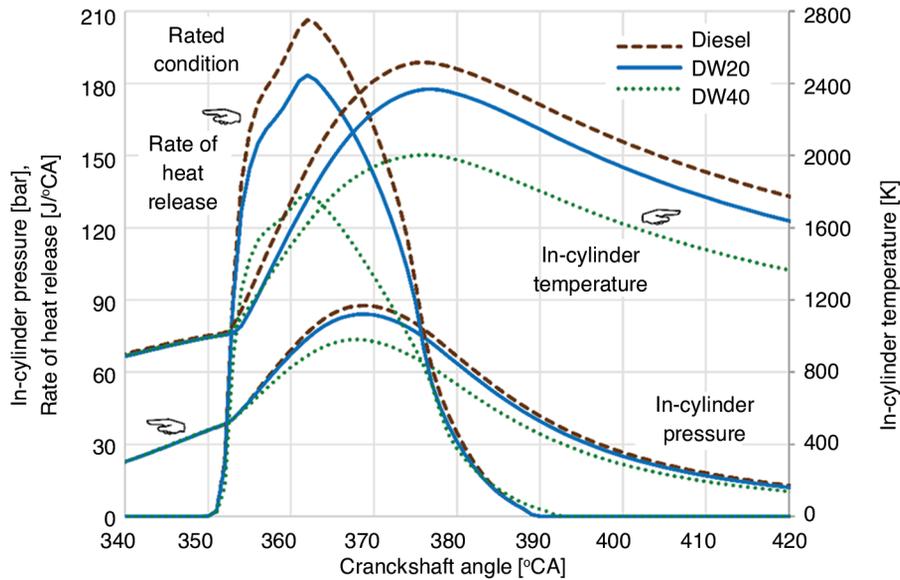


Figure 9. Combustion characteristics at rated condition

mixture temperature. Secondly, the latent heat of vaporization of water/diesel emulsion is higher than of diesel fuel, but the heating value is lower than of diesel fuel. Thus, more heat is needed for water/diesel emulsion than for diesel fuel vaporization, while the energy released by water/diesel is lower than that from the same fuel volume of a diesel fuel. As a result, the in-cylinder gas temperature can be lower for higher content of water in water/diesel emulsion.

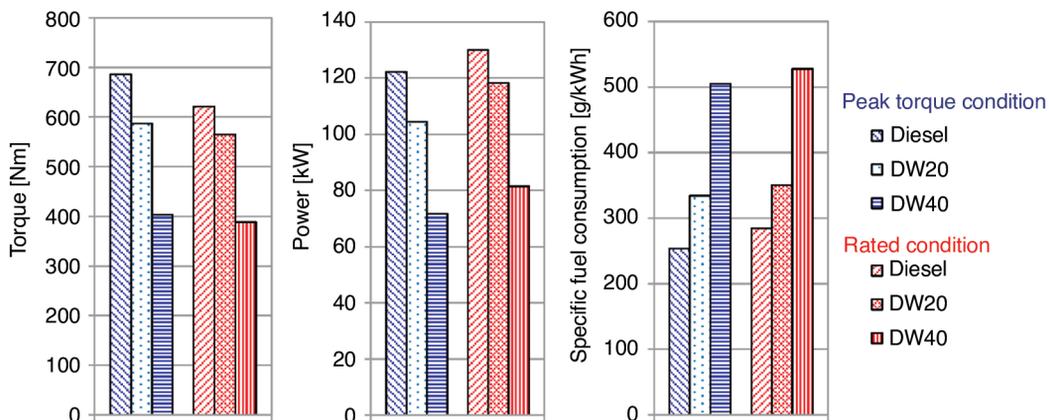


Figure 10. Engine characteristics at peak torque and rated condition

By taking into account the computed maximum heat release rate and maximal in-cylinder gas temperature, the  $\text{NO}_x$  emission should be lower by increasing the water content. It is known, however, that the maximum temperature is not the only important parameter for formation of  $\text{NO}_x$  emissions [23]. The in-cylinder conditions during the first part of injection and combustion process also influence to a great extent the  $\text{NO}_x$  formation. The moments, at which the

maximum in-cylinder gas temperature and heat release rate occur, proved to be more important than the maximum values of the in-cylinder gas temperature and heat release rate. Earlier peaks prolong the period with conditions favourable for NO<sub>x</sub> formation. Therefore, the increase of water content in water/diesel emulsion reduces the NO<sub>x</sub> emission.

### Conclusions

The influence of water/diesel emulsion on the most important injection and combustion characteristics of a bus Diesel engine with M fuel injection system were investigated by numerical simulation. For this reason a mathematical model BKIN for numerical simulation of injection processes and engine model of BOOST AVL software for combustion characteristics were verified by experiments. According to the results, the following conclusions for diesel engines with mechanically controlled injection system can be made:

- With increasing content of water in water/diesel emulsion fuel the injection delay, the mean injection rate and the pressure squareness decrease, meanwhile the injection timing and the mean as well as maximal injection pressure increase.
- Regarding the partial fuellings in different phases of injection process the numerical results show that the fuelling at the beginning of injections (fuelling during needle lifting, fuelling during first 10% of injection period) as well as the fuelling at the end of injections (fuelling during needle closing, fuelling during last 10% of injection period) are the lowest when using DW40, these facts offer a potential to reduce harmful smoke and NO<sub>x</sub> emissions.
- Lower in-cylinder pressure, in-cylinder gas temperature, and rate of heat release, as well as their history through crankshaft angle enable lower NO<sub>x</sub> emission by higher addition of water in water/diesel emulsion.
- The DW20 seems to be a rather acceptable emulsion. The DW40 emulsion usage results in the engine torque and power decreased up to 40% and specific fuel consumption increased up to 100%.

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### References

- [1] Subramanian, K. A., A Comparison of Water-Diesel Emulsion and Timed Injection of Water Into the Intake Manifold of a Diesel Engine for Simultaneous Control of NO and Smoke Emissions, *Energy Conversion and Management*, 52 (2011), 2, pp. 849-857
- [2] Samec, N., et al., Numerical and Experimental Study of Water/Oil Emulsified Fuel Combustion in a Diesel Engine, *Fuel*, 81 (2002), 16, pp. 2035-2044
- [3] Kegl, B., et al., *Green Diesel Engines. Biodiesel Usage in Diesel Engines*, Springer Verlag, London, 2013
- [4] Prasad, S. H., et al., Effect of Introduction of Water Into Combustion Chamber of Diesel Engines - A Review, *Energy and Power*, 5 (2015), 1A, pp. 28-33
- [5] Tesfa, B., et al. Water Injection Effects on the Performance and Emission Characteristics of a CI Engine Operating with Biodiesel, *Renewable Energy*, 37 (2012), 1, pp. 333-344
- [6] Sahin, Z., et al., Experimental Investigation of the Effects of Water Adding to the Intake Air on the Engine Performance and Exhaust Emissions in a DI Automotive Diesel Engine, *Fuel*, 115 (2014), Jan., pp. 884-895
- [7] Tazua, X., et al., Experimental Study of Inlet Manifold Water Injection on Combustion and Emission of an Automotive Direct Injection Diesel Engine, *Energy*, 35 (2010), 9, pp. 3628-3639
- [8] Chybowski, L., et al., An Overview of Systems Supplying Water Into the Combustion Chamber of Diesel Engines to Decrease the Amount of Nitrogen Oxides in Exhaust Gas, *Journal of Marine Science and Technology*, 20 (2015), 3, pp. 393-405
- [9] Sudrajad, A., et al., Experimental Study of Exhaust Emissions of W/O Emulsion Fuel in DI Single Cylinder Diesel Engine, *Modern Applied Science*, 5 (2011), 5, pp. 73-79

- [10] Abu-Zaid, M., Performance of Single Cylinder, Direct Injection Diesel Engine Using Water Fuel Emulsions, *Energy Conversion & Management*, 45 (2004), 5, pp. 697-705
- [11] Attia, A. M. A., Kulchitskiy, A. R., Influence of the Structure of Water-in-Fuel Emulsion on Diesel Engine Performance, *Fuel*, 116 (2014), Jan., pp. 703-708
- [12] Azimi, M., et al., The Effect of Water-Diesel Emulsion Usage on a Tractor Engine Performance and Emission, *Russian Agricultural Sciences*, 42 (2016), 6, pp. 488-492
- [13] Baskar, P., Kumar, A. S., Experimental Investigation on Performance Characteristics of a Diesel Engine Using Diesel-Water Emulsion with Oxygen Enriched Air, *Alexandria Engineering Journal*, 56 (2017), 1, pp. 137-146
- [14] Chaitanya, P. S., et al. Investigation on Effect of Water Emulsified with Diesel by Surfactant Addition on Performance and Emission Characteristics of Diesel Engine, *International Journal of Chemical Sciences*, 14 (2016), 4, pp. 2835-2844
- [15] Dubey, M., Saxena, V., Impact of Emulsified Water/Diesel Mixture on Engine Performance and Environment, *International Journal of Engineering Trends and Technology*, 36 (2016), 9, pp. 461-466
- [16] Fahd, M. E. A., et al., Experimental Investigation of the Performance and Emission Characteristics of Direct Injection Diesel Engine by Water Emulsion Diesel Under Varying Engine Load Condition, *Applied Energy*, 102 (2013), Feb., pp. 1042-1049
- [17] Gheorghe, M., et al. Diesel-Water Emulsion Effects on Diesel Engine Emissions, *Journal of Environmental Protection and Ecology*, 14 (2013), 2, pp. 566-571
- [18] Ithnin, A. M., et al., An Overview of Utilizing Water-in-Diesel Emulsion Fuel in Diesel Engine and its Potential Research Study, *Journal of the Energy Institute*, 87 (2014), 4, pp. 273-288
- [19] Hasannuddin, A. K., et al., Effect of Emulsion Fuel on Engine Emissions - A Review, *Clean Technologies and Environmental Policy*, 18 (2016), 1, pp. 17-32
- [20] Nguyen, K. B., et al., Effect of Double Injection on Combustion, Performance and Emissions of Jatropa Water Emulsion Fueled Direct-Injection Diesel Engine, *Energy*, 80 (2015), Feb., pp. 746-755
- [21] Lešnik, L., et al., The Influence of Biodiesel Fuel on Injection Characteristics, Diesel Engine Performance, and Emission Formation, *Applied Energy*, 111 (2013), Nov., pp. 558-570
- [22] Kegl, B., Numerical Analysis of Injection Characteristics Using Biodiesel Fuel, *Fuel*, 85 (2006), 17-18, pp. 2377-2387
- [23] Kegl, B., Influence of Biodiesel on Engine Combustion and Emission Characteristics, *Applied Energy*, 88 (2011), 5, pp. 1803-1812