COMBINED TECHNICAL SYSTEM

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Technical systems are important systems frequently used by applied sciences. Proper operation of technical systems is very important. Therefore, the statistically calculated reliability of a technical system is an important indicator for the system. Technical systems occur in different structures depending on the connection types of the components that constitute the system. The connection diagrams of components can be encountered in a highly complex situation. In such cases, the reliability of the system is difficult to calculate. There is no single method in the literature to calculate the reliability of a technical system. The methods in the literature differ according to the connection types of the systems. In this study, a method and a MATLAB program have been proposed for calculating the reliability of k-out-of-n F systems and consecutive k-out-of-n F systems. The proposed method can also be used for different connections.

Key words: *technical systems, reliability of system, mean time to failure*

Introduction

The reliability of the *k-out-of-n F* systems was successfully calculated in the early periods of the literature on the analysis of technical systems. When the distributions of the components of the system are given in the calculation, the distribution of the t^{th} order statistic is sufficient to calculate the reliability of the system. The reliability of consecutive *k-out-of-n F* systems can be calculated using the system signature. A consecutive *k-out-of-n F* system consists of an ordered sequence of n components such that the system fails if and only if at least k consecutive components fail. The first report for this system was presented by [1]. For further references [2-9]. A closed recurring water supply system with n water pumps in a thermo-electric plant and vacuum system in an electronic accelerator are good examples for *consecutive-k-out-of-n F* system. The following illustrative example is presented [10, 11]. In the related literature, system signature and order statistics are used together in the calculation of the reliability of technical systems. There are many studies in the literature on the system signature. In these studies, different methods of calculating the system signature can be seen. The system signature of a technical system consisting of independent n components is a probability vector with n components. The k^{th} component of the vector shows the possibility that the k^{th} failed component will fail the system. Let $S_i = (s_1, s_2, \ldots s_n)$ be the system signature of system consisting of independent n components. The reliability of the system is defined by the probability of $R = P\{U > t\}$at time $t > 0$. This probability can be calculated [12]:

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$$P(U > t) = \sum_{i=1}^{n} s_i P(U_{in} > t) = \sum_{i=1}^{n} s_i \sum_{j=0}^{i-1} \binom{n}{j} \left[F(t)\right]^j \left[1 - F(t)\right]^{(n-j)} \tag{1}$$

where U_{in} is the order statistics of lifetimes of the components, U is the lifetimes of the system, and $F(t)$ is the common distribution of the lifetimes of the components. Let X be working time of the component. The $q = F(t) = Pr\{X < t\}$ shows the probability of the components' failure at time $t > 0$ while $p = 1 - F(t) = Pr\{X > t\}$ indicates the probability of the components' operation. For example, the system signature of a technical system consisting of parallel connected n components is $S = (0,...0, 1)$ and its reliability $R = 1 - q^n$ is from the previous equation. The system signature of a technical system consisting of serial connected n components is $S = (1, 0,...0)$ and its reliability is calculated as $R = p^n$. In this example, the reliability of the components is expressed as probability independently at time.

Technical systems under stress

Continuous operating times of technical systems in real applications are not only associated with the operating times of the components that make up the system. A system that will start to work for the first time has a operating life calculated with the help of various tests during the production phase. However, this period may be less than expected as a result of external pressures from the start of the operation of the system. Therefore, the average operating time of the system should be calculated taking into account the stress to which it is exposed. These types of technical systems are called stress-strength models. Stress-strength models have an important position in reliability analysis. In this model, the reliability of the system is represented by the variable, Y, which shows the durability of the system, and the variable, X, which indicates the stress applied to the system, with the probability of $Pr\{X < Y\}$. There are many studies on stress-durability models in the literature. Some results on reliability estimate for the case where the multiple number of variable operating pressure by given [13]. Studies for stress-durability models on samples with multivariate exponential Weinman distribution conducted [14]. More information about the developments in this area provided [15]. In addition, the model is discussed for systems consisting of several components. The operating systems in a system consisting of n components where the minimum component k exceeds a common X stress studied [16]. Taking into consideration the risk of operating under stress of the components, the operation probability of the system is calculated [17].

System reliability studies show that both the components and the system-related stress and endurance random variables change over time, which brings a more realistic approach to system reliability. For example, the average life span when used in groundwater extraction of a water pump and the average life expectancy when used in salt water is different. The effect that causes this difference is the pressure applied of salt water to the operating system.

Hazard function has an important role in the systems under stress. The Hazard function refers to the risk of purification of a running system. For more information about the Hazard function [18].

Hazard ratio can be obtained with the help of the distribution function that represents the operating time of a technical system. If the hazard rate is known, the distribution function:

$$F(t) = 1 - \exp\left\{ -\int_0^t \lambda(u)\,\mathrm{d}u \right\} \tag{2}$$

Let us show the performance of the technical system with $Z(t)$. The following $p(t)$ probability will indicate the possibility of the system running forwards:

$$p(t) = Pr\{Z(t) > t + h | Z(t) > t\} = \frac{1 - H(t+h)}{1 - H(t)} \tag{3}$$

Here, $H(t) = Pr\{Z(t) < t\}$ is the probability. As a result, the probability of $p(t)$:

$$p(t) = \exp\left\{-\int_t^{t+h} \lambda(u)\mathrm{d}u\right\} \tag{4}$$

The importance of this review in systems operating under stress is that the $\lambda(t)$ hazard ratio under stress can be chosen more easily depending on the system. For example $\lambda(t) = 1/(EY - EX)$, can be selected under the assumption that the mean of stress and durability has changed over time. This case will directly reflect the average operating time of the technical system.

Different parameters are not widely used in the literature to examine different technical systems. Hazard ratio can be used in different ways to calculate important indicators of technical systems. The odds ratio with similar thinking can also be used to calculate the reliability of the technical system. Here, if we design the odds ratio as the ratio of the probability that the system will fail to operate, we create an important parameter for the system.

Survival signature

The system signat ure is an important parameter for the reliability of technical systems in the literature. However, the components of the technical system must have the same distribution in order to identify the system signature. Otherwise the system signature becomes meaningless. When the recent studies in the literature are followed, it has been shown that a parameter similar to the system signature, survival signature, can be defined for technical systems consisting of components with different distribution. The use of survival signature is not only as practical as a system signature, but it is an important material for technical systems with different distribution components. For the creation of survival signature, the possible components are formed by grouping the components with different distributions. Suppose there is K different distribution in a technical system. Let us show the number of components with the same distribution as m_k, $k = 1,...,K$. Let us show the number of machines that work with l_k, $k = 1,...,K$. When the system is in the $L = (l_1,...,l_K)$ state, let us show the probability of the system's operation by $\Phi(l_1,...,l_k)$. Let us show the set of state vectors of k type components with $S_{l_k}^k$. Let us show all the state vectors of the components that make up the technical systems with $S_{l_1,...l_K}$. The technical system's reliability, including the structure function ℓ of the technical system:

$$R = Pr\{U > t\} = \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \Phi(L) Pr\left\{\bigcap_{k=1}^{K} \{C_t^k = l_k\}\right\} \tag{5}$$

Here, C_t^k and $\Phi(L)$ show the number of components t running from the k type components and the survival signature, respectively:

$$\Phi(L) = \left\{\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1}\right\} \sum_{X \in S_{l_1,\cdots,l_K}} \ell(X) \tag{6}$$

It seems that, the fact that the components that make up the system in a technical system have different distributions make the calculation of the system reliability quite difficult. In the next section, we give a different technique to be used in technical systems consisting of components with different distributions.

Combining process

Let q be the probability of failure of the systems. In order to combine two technical systems connected in parallel or in serial, $u = q/p$ ratios of the systems have been used. In case of parallel and serial connection, the combined process can be made according to the following equations:

$$f_p(u,v) = \left\{\left(\frac{1}{u}\right)\left(\frac{1}{v}\right) + \frac{1}{u} + \frac{1}{v}\right\}^{-1} \tag{7}$$

$$f_s(u,v) = uv + u + v \tag{8}$$

where u and v are the q/p ratio of the first and second systems, respectively. After all the combined processes, $1 - F = p = 1/(1 + u)$ is the reliability of the system obtained from the q/p ratio. Mean time to failure:

$$MTTF = \int_0^\infty p\mathrm{d}t \tag{9}$$

Illustrative example

Let $\ell = \min\{\max\{A_1, A_2\}A_3\}$ be the connection of three different systems. In this example, let A_1, A_2, A_3 be *3-out-of-5 F*, *consecutive-3-out-of-5 F*, and *4-out-of-6 F* systems, respectively. Let distribution of each components of the systems be as Exp(0.1). In this case, the mean time to failure of the system can be calculated as $MTTF = 6.9486$. The graph of the reliability of the system according to time can be obtained fig. 1.

The program used in the calculations is given in the *Appendix*.

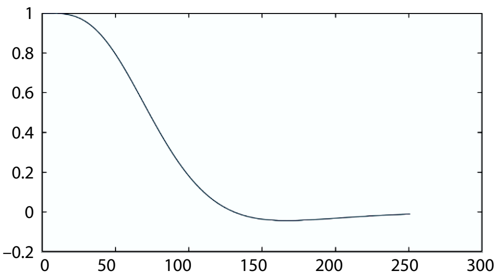

Figure 1. The reliability of the system according to time

Conclusion

Calculation of the mean time to failure of a technical system whose connections are highly complex is quite important for the reliability of the technical system. The results of the study and the program used are very important in terms of examining complex technical systems. In the example, three different technical systems have been combined according to the ℓ rule and the reliability of the system has been calculated according to time. In addition, the mean time to failure of the ℓ system has been obtained with the help of numerical integral calculus.

Appendix

```
%l-out-of-n F system
n=input('n=?')
l=input('l=?')
%consecutive k-out-of-m F system
m=input('m=?')
k=input('k=?')
%ll-out-of-nn F system
nn=input('nn=?')
ll=input('ll=?')
mttf=0;pson=0;t=(0:0.1:25);
A1p=zeros(1,length(t));u1=zeros(1,length(t));
A2q=zeros(1,length(t));u2=zeros(1,length(t));
A3p=zeros(1,length(t));u3=zeros(1,length(t));
u12=zeros(1,length(t));u=zeros(1,length(t));
p=zeros(1,length(t));q=zeros(1,length(t));
Ap=zeros(1,l);App=zeros(1,ll);
for i=1:length(t)
 q(i)=1-exp(-0.1*t(i));
 p(i)=1-q(i);
end
for i=1:length(t)
 for j=1:l
 Ap(j)=nchoosek(n,j-1)*q(i)^(j-1)*p(i)^(n-j+1);
 end
 A1p(i)=sum(Ap);
 Ap=zeros(1,l);
 u1(i)=(1-A1p(i))/A1p(i);
end
for i=1:length(t)
 A2q(i)=(m-k+1)*q(i)^k;
 u2(i)=A2q(i)/(1-A2q(i));
end
for i=1:length(t)
 for j=1:ll
 App(j)=nchoosek(nn,j-1)*q(i)^(j-1)*p(i)^(nn-j+1);
 end
 A3p(i)=sum(App);
 App=zeros(1,ll);
 u3(i)=(1-A3p(i))/A3p(i);
end
for i=1:length(t)
 u12(i)=((1/u1(i))*(1/u2(i))+1/u1(i)+1/u2(i))^(-1);
end
for i=1:length(t)
 u(i)=(u12(i))*(u3(i))+u12(i)+u3(i);
end
for i=1:length(t)
pson(i)=1/(1+u(i));
end
mttf=(sum(pson))*0.1
plot(pson)
```

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