ON EXACT AND APPROXIMATE SOLUTIONS OF (2+1)-DIMENSIONAL KONOPELCHENKO-DUBROVSKY EQUATION VIA MODIFIED SIMPLEST EQUATION AND CUBIC B-SPLINE SCHEMES

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This paper studies the analytical and numerical solutions for (2+1)-dimensional Konopelchenko–Dubrovsky equation. It also examines the performance of the modified simplest equation method and the cubic B-spline scheme on this model. Many explicit wave solutions are found by using the analytical technique. These solutions allow studying the physical properties of this model. The comparison between the analytical and numerical solutions are discussed to show which one of cubic B-spline scheme families is more accurate in finding the numerical solutions of this model.

Keywords: Modified simplest equation method; B-spline schemes; Solitary wave solution; Analytical and numerical wave solution.

1 Introduction

Nonlinear partial differential equations are the most suitable technique to express many significant phenomena. It is also able to study the mechanism and physical properties of these phenomena. To this end, exact and numerical solutions are particularly significant. Indeed, a lot of physicists and mathematicians have been investigating the exact and numerical solutions. Up to now, many numerical and solitary schemes have been formulating, such as generalized extended tanh-function method, Khater method, modified auxiliary equation method (modified Khater method), Wronskian technique, linear superposition principle, and Hirota direct method [1-15].

Recently, for studying the exact traveling wave solutions, the modified simplest equation method has been proposed to investigate many kinds of solutions such as rational, exponential, hyperbolic, trigonometric, kink, rogue, lump, bilinear, and solitary [16-20]. Moreover, the complexion solutions had been investigated such that it defines as an interaction of exponential and trigonometric waves [21-25] while the B-spline schemes have been being used to study the numerical solutions of many various forms of nonlinear partial differential equations [26-30].

In this paper, we study (2+1)-dimensional Konopelchenko–Dubrovsky equation derived by Konopelchenko BG and Dubrovsky VG [31]. The classical form of his equation is given by

\[
\phi_t - \phi_{xxx} - 3 \frac{d}{d y} \left( \int \phi_y \, dx \right) + \frac{3}{2} a^2 \phi^2 \phi_x + 3 a \phi_x \int \phi_y \, dx - 6 b \phi \phi_x = 0,
\]

where \( a, b \) are arbitrary constants and \( \phi = \phi(x, y, t) \) is an analytical function in \( x, y, t \). Eq. (1.1) when, \( (a = 0) \) becomes the Kadomtsev-Petviashvili (KP) equation while, when \( (b = 0) \) becomes the modified Kadomtsev-Petviashvili (mKP) equation. With the following dependent variable transformation \[ \alpha = 0, \phi = \frac{2}{b} \ln(\varphi)_{xx}, \] Eq. (1.1) transforms to

\[
(D_x D_t - D_x^2 - 3 D_y^2) \varphi \cdot \varphi = 0.
\]

While, the (2+1)-dimensional Konopelchenko–Dubrovsky system takes the following form [32]-[35].
\[
\begin{aligned}
S_t - S_{xxx} - 6 b S S_x + \frac{3}{2} a^2 S^2 S_x^2 - 3 R_y + 3 a S_x R_y = 0, \\
S_y = R_x,
\end{aligned}
\]
(1.3)
where \( a \) and \( b \) are arbitrary constants and \( S(x, y, t), R(x, y) \) represent a wave function. Using the traveling wave transformation \( S(x, y, t) = S(\xi), R(x, y, t) = R(\xi) \), where \( (\xi = x + y + c t) \) on the system (1.3), obtains
\[
\begin{aligned}
c S' - S'' - 6 b S S' + \frac{3}{2} a^2 S^2 S' - 3 R' + 3 a S' R = 0, \\
S' = R'.
\end{aligned}
\]
(1.3a)
Integration of the second equation in the previous system gives
\[
S = R.
\]
(1.3b)
Substituting (1.3b) into the first equation in the system (1.3a) and then integrate the obtained equation with zero constant of integration, we get
\[
(c - 3) S - \left(3 b + \frac{3}{2} a \right) S^2 + \frac{a^2}{2} S^3 = 0.
\]
(1.4)

The strategy of this paper is organized as follows: Section 2 applies the modified simplest equation method and the B-spline method to (2+1)-dimensional Konopelchenko-Dubrovsky equation. Section 3 represents some of the exact and approximate solutions in three and two-dimensional plot to illustrate more of the physical properties of this model. Section 4 gives a conclusion of our paper.

2 Application
In this part, we apply the modified simplest equation and the B-spline methods [36]-[40] to (2+1)-dimensional Konopelchenko–Dubrovsky equation.

2.1 Modified simplest equation method
According to the general solutions that suggested by the method and balance rule between \( S'' \) and \( S^3 \), we get \( N = 1 \) and the general solution of Eq. (1.4) in the following form:
\[
S(\xi) = \sum_{i=-N}^{N} a_i f(\xi)^i = \frac{a_{i=1}}{f(\xi)} + a_0 + a_1 f(\xi),
\]
(2.1)
where \( a \) is arbitrary constant and \( f(\xi) \) satisfies the following auxiliary equation \[ f''(\xi) = \alpha + \lambda f(\xi) + \mu f(\xi)^2 \], where \( \alpha, \lambda, \mu \) are arbitrary constants. Substituting Eq. (2.1) and its derivatives into Eq. (1.4), and collecting all terms of the same power of \( f(\xi)^i \) where \( i = -3, -2, -1, 0, 1, 2, 3 \). Solving the obtained algebraic equations by Maple or Mathematica softwares, we get:

Family 1
\[
a_0 = \frac{2\lambda}{a}, a_1 = \frac{2\mu}{a}, a_{-1} = \frac{2\alpha}{a}, c = 3 + \lambda^2 - 4\alpha \mu, b = \frac{1}{2} a (-1 + \lambda) \) where \( \mu \) or \( \alpha \neq 0, a \neq 0, \lambda \in \mathbb{R} - \{1\}, \) and \( 3 + \lambda^2 \neq 4\alpha \mu .
\]

Family 2
\[
a_0 = -\frac{\lambda}{a}, a_1 = 0, a_{-1} = -\frac{2\alpha}{a}, c = \frac{1}{2} (6 - \lambda^2 + 4\alpha \mu), b = -\frac{a}{2} \) where \( \alpha \neq 0, a \neq 0, \) and \( 6 - \lambda^2 \neq 4\alpha \mu .
\]

Family 3
\[
a_0 = \frac{\lambda}{a}, a_1 = \frac{2\mu}{a}, a_{-1} = 0, c = \frac{1}{2} (6 - \lambda^2 + 4\alpha \mu), b = -\frac{a}{2} \) where \( \mu \neq 0, a \neq 0, \) and \( 6 - \lambda^2 \neq 4\alpha \mu .
\]
According to the value of parameters in family 1, we get the solitary wave solutions of Eq.(1.1) in the following formulas:

Case 1
When, \( \lambda = 0 \), we get:

When \( \alpha \mu > 0 \)

\[
S_1 = \frac{4 \sqrt{\alpha \mu}}{a} \csc \left[ \sqrt{\alpha \mu} (x + y + \theta + t(3 - 4 \alpha \mu)) \right].
\]

(2.2)

When \( \alpha \mu < 0 \)

\[
S_2 = -\frac{4 \sqrt{-\alpha \mu}}{a} \csc \left[ \sqrt{-\alpha \mu} (x + y + t(3 - 4 \alpha \mu)) + \frac{\log[\theta]}{2} \right].
\]

(2.3)

Case 2
When, \( \alpha = 0 \), we get

When \( \lambda > 0 \)

\[
S_3 = -\frac{2 \lambda}{a(-1 + e^{\lambda (x + y + \theta + t(3 + \lambda^2))})}. \]

(2.4)

When \( \lambda < 0 \)

\[
S_4 = \frac{2}{a} \left( \lambda + \frac{1}{e^{\lambda (x + y + \theta + t(3 + \lambda^2))} + \frac{1}{\mu}} - \mu \right).
\]

(2.5)

Case 3
When \( \lambda \neq 0, \alpha \neq 0, \mu \neq 0 \)

When \( 4 \alpha \mu > \lambda^2 \) and \( \mu > 0 \)

\[
S_5 = \frac{(\lambda^2 - 4 \alpha \mu) \sec \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right]^2}{a(\lambda - \sqrt{-\lambda^2} + 4 \alpha \mu \tan \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right])},
\]

(2.6)

\[
S_6 = \frac{(\lambda^2 - 4 \alpha \mu) \csc \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right]^2}{a(\lambda - \sqrt{-\lambda^2} + 4 \alpha \mu \cot \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right])}.
\]

(2.7)

When \( 4 \alpha \mu > \lambda^2 \) and \( \mu < 0 \)

\[
S_7 = \frac{1}{a} \left[ 3 \lambda + \sqrt{-\lambda^2} + 4 \alpha \mu \tan \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right] \right]
\]

\[
+ \frac{4 \alpha \mu}{\lambda + \sqrt{-\lambda^2} + 4 \alpha \mu \tan \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right]},
\]

(2.8)

\[
S_8 = \frac{1}{a} \left[ 3 \lambda + \sqrt{-\lambda^2} + 4 \alpha \mu \cot \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right] \right]
\]

\[
+ \frac{4 \alpha \mu}{\lambda + \sqrt{-\lambda^2} + 4 \alpha \mu \cot \left[ \frac{1}{2} \sqrt{-\lambda^2} + 4 \alpha \mu (x + y + \theta + t(3 + \lambda^2 - 4 \alpha \mu)) \right]},
\]

(2.9)
According to the value of parameters in family 2, we get the solitary wave solutions of Eq.(1.1) in the following formulas:

Case 1
When, \(\lambda = 0\), we get:

When \(\alpha \mu > 0\)

\[
S_9 = -\frac{2\sqrt{\alpha\mu}}{a}\cot\left[\sqrt{\alpha\mu}(x + y + \theta + t(3 + 2\alpha\mu))\right],
\]
(2.10)

\[
S_{10} = -\frac{2\sqrt{\alpha\mu}}{a}\tan\left[\sqrt{\alpha\mu}(x + y + t(3 + 2\alpha\mu))\right],
\]
(2.11)

When \(\alpha \mu < 0\)

\[
S_{11} = \frac{2\sqrt{-\alpha\mu}}{a}\coth\left[\sqrt{-\alpha\mu}(x + y + t(3 + 2\alpha\mu)) + \frac{\log[\theta]}{2}\right],
\]
(2.12)

\[
S_{12} = \frac{2\sqrt{-\alpha\mu}}{a}\tanh\left[\sqrt{-\alpha\mu}(x + y + t(3 + 2\alpha\mu)) + \frac{\log[\theta]}{2}\right].
\]
(2.13)

Case 2
When \(\lambda \neq 0, \alpha \neq 0, \mu 
eq 0\)

When \(4\alpha \mu > \lambda^2\) and \(\mu > 0\)

\[
S_{13} = \frac{1}{a}\left[\frac{-\lambda + \frac{4\alpha\mu}{\lambda - \sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}}{\sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}\right],
\]
(2.14)

\[
S_{14} = \frac{1}{a}\left[\frac{-\lambda + \frac{4\alpha\mu}{\lambda - \sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}}{\sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}\right].
\]
(2.15)

When \(4\alpha \mu > \lambda^2\) and \(\mu < 0\)

\[
S_{15} = \frac{-1}{a}\left[\frac{\lambda + \frac{4\alpha\mu}{\lambda + \sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}}{\sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}\right],
\]
(2.16)

\[
S_{16} = \frac{-1}{a}\left[\frac{\lambda + \frac{4\alpha\mu}{\lambda + \sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}}{\sqrt{-\lambda^2 + 4\alpha\mu}(x + y + \theta + t(3 + \frac{\lambda^2}{2} + 2\alpha\mu))}\right].
\]
(2.17)

According to the value of parameters in family 3, we get the solitary wave solutions of Eq.(1.1) in the following formulas:

Case 1
When, \(\lambda = 0\), we get:

When \(\alpha \mu > 0\)

\[
S_{17} = \frac{2\sqrt{\alpha\mu}}{a}\tan\left[\sqrt{\alpha\mu}(x + y + \theta + t(3 + 2\alpha\mu))\right],
\]
(2.18)

\[
S_{18} = \frac{2\sqrt{\alpha\mu}}{a}\cot\left[\sqrt{\alpha\mu}(x + y + \theta + t(3 + 2\alpha\mu))\right].
\]
(2.19)

When \(\alpha \mu < 0\)
\[ S_{19} = \frac{2\sqrt{-\alpha \mu}}{a} \Tanh \left( \frac{\sqrt{-\alpha \mu(x + y + t(3 + 2\alpha \mu))} + \Log[\theta]}{2} \right), \quad (2.20) \]
\[ S_{20} = \frac{2\sqrt{-\alpha \mu}}{a} \Coth \left( \frac{\sqrt{-\alpha \mu(x + y + t(3 + 2\alpha \mu))} + \Log[\theta]}{2} \right). \quad (2.21) \]

Case 2

When, \( \alpha = 0 \), we get

When \( \lambda > 0 \)

\[ S_{21} = \frac{\lambda}{a} \left( -1 - \frac{2}{-1 + e^{\lambda(x+y+\theta-\frac{1}{2}(6+\lambda^2))\mu} - 1 + e^{\lambda(x+y+\theta-\frac{1}{2}(6+\lambda^2))\mu}} \right). \quad (2.22) \]

When \( \lambda < 0 \)

\[ S_{22} = \frac{1}{a} \left[ \lambda + 2\mu(-1 + \frac{1}{1 + e^{\lambda(x+y+\theta-\frac{1}{2}(6+\lambda^2))\mu}}) \right]. \quad (2.23) \]

Case 3

When \( \lambda \neq 0, \alpha \neq 0, \mu \neq 0 \)

When \( 4\alpha \mu > \lambda^2 \) and \( \mu > 0 \)

\[ S_{23} = \frac{\sqrt{-\lambda^2 + 4\alpha \mu}}{a} \Tan \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha \mu(x + y + \theta + t(3 - \frac{\lambda^2}{2} + 2\alpha \mu))} \right]. \quad (2.24) \]
\[ S_{24} = \frac{\sqrt{-\lambda^2 + 4\alpha \mu}}{a} \Coth \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha \mu(x + y + \theta + t(3 - \frac{\lambda^2}{2} + 2\alpha \mu))} \right]. \quad (2.25) \]

When \( 4\alpha \mu > \lambda^2 \) and \( \mu < 0 \)

\[ S_{25} = \frac{1}{a} \left[ 2\lambda + \sqrt{-\lambda^2 + 4\alpha \mu} \Tan \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha \mu(x + y + \theta + t(3 - \frac{\lambda^2}{2} + 2\alpha \mu))} \right] \right]. \quad (2.26) \]
\[ S_{26} = \frac{1}{a} \left[ 2\lambda + \sqrt{-\lambda^2 + 4\alpha \mu} \Coth \left[ \frac{1}{2} \sqrt{-\lambda^2 + 4\alpha \mu(x + y + \theta + t(3 - \frac{\lambda^2}{2} + 2\alpha \mu))} \right] \right]. \quad (2.27) \]

2.2 Cubic B-spline scheme

In this section, we apply the cubic B-spline numerical scheme to (2+1)-dimensional Konopelechenko-Dubrovsky equation to study the numerical solution of this model. The numerical solution that obtained by this scheme can be written as a linear combination of cubic B-splines basis functions. Consider the following grid \( a = \xi_0 < \xi_1 < \cdots < \xi_{n-1} < \xi_n = b \) as the uniform partition of solution where \( \xi_{i+1} - \xi_i = \frac{b-a}{n} = h \), where \( i = 0, \ldots, n - 1 \). The numerical solution of Eq. (1.4) has the following general form

\[ S(\xi) = \sum_{i=-1}^{n+1} c_i B_i(\xi), \quad (2.28) \]

where \( c_i, (i = -1, \ldots, n + 1) \) are arbitrary constants and \( B_i(\xi), (i = -1, \ldots, n + 1) \) are cubic B-spline function which satisfy the following condition

\[ B_i(\xi) = \frac{1}{h^3} \begin{cases} 
(\xi - \xi_{i-2})^3, & \xi \in [\xi_{i-2}, \xi_{i-1}], \\
(\xi - \xi_{i-2})^3 - 4(\xi - \xi_{i-1})^3, & \xi \in [\xi_{i-1}, \xi_i], \\
(\xi_{i+2} - \xi)^3 - 4(\xi_{i+1} - \xi)^3, & \xi \in [\xi_i, \xi_{i+1}], \\
(\xi_{i+2} - \xi)^3, & \xi \in [\xi_{i+1}, \xi_{i+2}], \\
otherwise & \text{otherwise}.
\end{cases} \quad (2.29) \]
where, the coefficient of of $B_i(\xi), B'_i(\xi), B''_i(\xi)$ has the following shown value in the table 1

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\xi_{i-2}$</th>
<th>$\xi_{i-1}$</th>
<th>$\xi_i$</th>
<th>$\xi_{i+1}$</th>
<th>$\xi_{i+2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i(\xi)$</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$B'_i(\xi)$</td>
<td>0</td>
<td>$\frac{3}{h}$</td>
<td>0</td>
<td>$-\frac{3}{h}$</td>
<td>0</td>
</tr>
<tr>
<td>$B''_i(\xi)$</td>
<td>0</td>
<td>$\frac{6}{h^2}$</td>
<td>$-12$</td>
<td>$\frac{6}{h^2}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Values of $B_i(\xi), B'_i(\xi), B''_i(\xi)$

According to these values of $B_i(\xi), B'_i(\xi), B''_i(\xi)$, we get

$$S(\xi) = c_{i-1} + 4c_i + c_{i+1},$$

(2.30)

$$S'(\xi) = \frac{3}{h}c_{i-1} - \frac{3}{h}c_{i+1},$$

(2.31)

$$S''(\xi) = \frac{6}{h^2}c_{i-1} - \frac{12}{h^2}c_i + \frac{6}{h^2}c_{i+1}.$$ 

(2.32)

Substituting Eqs. (2.30)-(2.32) into Eq. (1.4) with the following initial condition that obtained from Eq. (2.13)

$$S = -\frac{2}{3}\tanh(2\xi),$$

(2.13)

where,

$$a = -4; \mu = 1; \alpha = -6; \theta = 1; \lambda = 0; c = -5; b = 3,$$

$S(0) = 0,$

$$S(1) = -\frac{2\tanh[2]}{3},$$

$$S'(0) = -\frac{4}{3},$$

$$S'(1) = -\frac{4}{3}\text{Sech}[2]^2.$$ 

We get a system of equations. Solving this system, we get

<table>
<thead>
<tr>
<th>Value of $\xi$</th>
<th>Approximate</th>
<th>Exact</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0$</td>
<td>$-1.734723475 \times 10^{-18}$</td>
<td>0</td>
<td>$1.734723475976 \times 10^{-18}$</td>
</tr>
<tr>
<td>$\xi = 0.1$</td>
<td>$-0.0721748973954985$</td>
<td>$-0.13158354681660267$</td>
<td>$0.05940864942110417$</td>
</tr>
<tr>
<td>$\xi = 0.2$</td>
<td>$-0.13997442420104717$</td>
<td>$-0.2532993081701499$</td>
<td>$0.1133248396910273$</td>
</tr>
<tr>
<td>$\xi = 0.3$</td>
<td>$-0.200969688883744$</td>
<td>$-0.35803304466535685$</td>
<td>$0.15706335577698244$</td>
</tr>
<tr>
<td>$\xi = 0.4$</td>
<td>$-0.2549185485599881$</td>
<td>$-0.4426911801785659$</td>
<td>$0.1877726316185782$</td>
</tr>
<tr>
<td>$\xi = 0.5$</td>
<td>$-0.3033871007264205$</td>
<td>$-0.5077294373038432$</td>
<td>$0.20434233657742273$</td>
</tr>
<tr>
<td>$\xi = 0.6$</td>
<td>$-0.349402279038176$</td>
<td>$-0.5557697380081035$</td>
<td>$0.2063675101042859$</td>
</tr>
<tr>
<td>$\xi = 0.7$</td>
<td>$-0.453963142954458$</td>
<td>$-0.5902344321348416$</td>
<td>$0.19280683075323546$</td>
</tr>
<tr>
<td>$\xi = 0.8$</td>
<td>$-0.543963142954458$</td>
<td>$-0.6144457029376476$</td>
<td>$0.1604825598319855$</td>
</tr>
<tr>
<td>$\xi = 0.9$</td>
<td>$-0.529365211731331$</td>
<td>$-0.6312040085641788$</td>
<td>$0.101837968328478$</td>
</tr>
<tr>
<td>$\xi = 1.0$</td>
<td>$-0.6426850533838779$</td>
<td>$-0.6426850533838779$</td>
<td>$0.0$</td>
</tr>
</tbody>
</table>

Table 2. Values of exact and approximate solutions
2.3 Figure

Fig. 1 Three dimensional and contour plots of Eq. (2.2), when $[\alpha = 4; \mu = 2; \theta = 1; a = -6; y = 3]$. 

Fig. 2 Three and two-dimensional plots of Eq. (2.3), when $[\alpha = -4; \mu = 2; \theta = 1; a = -6; y = 3]$. 

Fig. 3 Three-dimensional and contour plots of Eq. (2.4), when $[\alpha = 0; \mu = 2; \theta = 1; \lambda = 3; a = -6; y = 3]$. 
3 Conclusion

In this paper, we used the modified simplest equation method and the cubic B-spline scheme to (2+1)-dimensional Konopelchenko-Dubrovsky equation. We succeed in obtaining analytical and numerical solutions of the model. We obtained different forms of solutions such as shock waves, singular, solitary waves, periodic singular waves, plane waves, and others. We obtained novel and distinct, solitary wave solutions of this model. Some of our obtained solutions can be reduced to the known solutions in some instances. We also obtained the approximate solutions and discuss both solutions to show the absolute value of the error table (2). The results show the effectiveness of the Adomian decomposition method for interval near zero. Some solitary and approximate solutions are sketched to investigate more of the physical properties of this model Figs. (1)- (5). The performance of both methods shows useful and powerful in studying many of nonlinear partial differential equations.

Acknowledgment:

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Abha, KSA for funding this work through Research Group under grant number (R.G.P-1/151/40).


Paper submitted: January 31, 2019

Paper revised: June 20, 2019

Paper accepted: July 5, 2019