VOLATILITY MEASUREMENT OF THE WORLD INDICES USING DIFFERENT ENTROPY METHODS

by

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In this paper, we show that the application of different entropy methods for world indices. To do this, we use the world indices such as Istanbul Stock Indices (BIST 30), Brazil Index (Bovespa), Germany Index (DAX), Britain Index (FTSE100), South Korea (KOSPİ), Japan Index (Nıkkei 225), United States Index (SP 500), and China Index (SHANGAI) that have been investigated over all of 8 years (2010-2018). We obtain Shannon, Tsallis, Renyi and at last the approximate entropy. Consequently, we provide computational results for these entropies for weekly and monthly data.

Key words: *Shannon entropy, Tsallis entropy, Renyi entropy, approximate entropy*

Introduction

The history of the word entropy can be traced back to 1865 when the German physicist Rudolf Clausius tried to give a new name to irreversible heat loss, what he previously called equivalent-value. The word entropy was chosen because in Greek, entropies mean content transformative or transformation content Laidler [1]. Tsallis [2] suggested an entropy method, which defines the statistical properties of complicated structure. Rao *et al*. [3] determined the cumulative residual entropy, generalized measure of uncertainty which applied in reliability and image alignment and non-additive measures of entropy. Shafe [4] suggested a new way of defining entropy of a system, which gives a general form that is non-extensive like Tsallis entropy, but is linearly dependent on component entropies, like Renyi entropy, which is extensive, checked it numerically with the Tsallis and Shannon entropies and indicated constraints on the energy spectra imposed by the properties of the Lambert function, which are absent in the Shannon form. Pincus [5] indicated the utility of approximate entropy (ApEn), a model-independent measure of sequential irregularity, towards this goal, via several distinct applications, both empirical data and model-based, designed cross-ApEn, a related two-variable measure of asynchrony that provides a more robust and ubiquitous measure of bivariate correspondence than does correlation, and the resultant implications to diversification strategies, and supplied analytic expressions for and statistical properties of ApEn, and compare ApEn to non-linear measures, correlation and spectral analyses, and other entropy measures. Ubriaco [6] indicated that new entropy has the same properties with the Shannon entropy except additive and given that this entropy function satisfies the Lesche and thermodynamic stability criteria. Rompolis [7] proposed a new method of implementing the principle of maximum entropy to retrieve the risk neutral density of future stock, or any other asset, returns from European call and put

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prices. Wang *et al*. [8] defined the market efficiency in foreign exchange (FX) markets by using the multi-scale approximate entropy (MApEn) to assess the randomness in FX market, separated 17 daily FX rates from 1984-2011 into these periods by two global events, Southeast Asia currency crisis and American sub-prime crisis and submitted that the developed FX markets is more efficient than emerging FX markets, and that the financial crisis promotes the market efficiency in FX markets significantly, especially in emerging markets, like China, Hong Kong, Korea and African market. In statistical mechanics the interpretation of entropy is the measure of uncertainty about the system that remains after observing its macroscopic properties (pressure, temperature or volume) [9]. Van Erven *et al*. [10] considered the most important properties of Renyi divergence and Kullback-Leibler divergence, including convexity, continuity, limits of σ-algebras, and the relation of the special order 0 to the Gaussian dichotomy and contiguity and indicated how to generalize the Pythagorean inequality to orders different from 1. Niu and Wang [11] used to study the complexity of financial time series since the financial market is a complex evolved dynamic system and considered multi scale entropy in the complexity of a time series and applied to the financial market. Dadu and Toma [12] obtained some integrated techniques for modelling financial data and solving decision making problems, based on risk theory and information theory, examined several risk measures and entropy measures and compared with respect to their analytical properties and effectiveness in solving real problems. Sati and Gupta [13] described a generalized cumulative residual entropy based on the non-additive Tsallis entropy. Sheraz *et al.* [14] used entropy approach for volatility markets. Stosic *et al.* [15] considered the effects of financial crises on FX markets, where entropy evolution is measured for different exchange rates, using the time-dependent block entropy method and indicated empirical results suggest that financial crises are associated with significant increase of exchange rate entropy, reflecting instability in FX market dynamics. Ponta and Carbone [16] performed such entropy measure on the time series of prices and volatilities of six financial markets on tick-by-tick data sampled every minute for six years of data from 1999-2004 and indicated that the entropy of the volatility series depends on the individual market. Khammar and Jahanshahi [17] provided the weighted form of this measure and call it weighted cumulative residual Tsallis entropy (WCRTE), reproduced ageing classes and shown that it can uniquely determine the survival function and Rayleigh distribution. In this study, the method of based on entropy approach was used for world indices (Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500 and Shangai) for the period 2010-2018.

Material and method

The Shannon entropy

The Shannon entropy of probability measure p on finite set X:

$$S_n(P) = -\sum_{i=1}^{n} p_i \ln p_i \tag{1}$$

where $p_i \geq 0$, $i = 1, 2..., n$, $\sum_{i=1}^{n}, p_i = 1$, and $0\ln 0 = 0$. Given a continuous probability distribution with a density function $f(x)$, we can define The Shannon entropy:

$$H = \int_{-\infty}^{+\infty} f(x) \ln f(x) \mathrm{d}x \tag{2}$$

where $\int_{-\infty}^{+\infty} f(x)\mathrm{d}x$ and $f(x) \geq 0$. The Shannon entropy in formation theory applications, the answer is given by the asymptotic equipartition property. There is $T \subseteq S^n$ with:

$$|T| \leq \mathrm{e}^{n[H(\rho)+\varepsilon]} \tag{3}$$

such that sampling n times from p yields an element of T with probability $\succ 1-\varepsilon$, and $\varepsilon \to 0$ as $n \to \infty$.

The Tsallis entropy

For any positive real number α, the Tsallis Entropy of order α of probability measure p on finite set X is defined as [1-3, 5]:

$$H_\alpha(p)=\begin{cases}\dfrac{1}{\alpha-1}\left(\sum\limits_{i\in X} p_i^\alpha\right), & \text{if } \alpha \neq 1\\ -\sum\limits_{i\in X} p_i \ln p_i, & \text{if } \alpha = 1\end{cases} \tag{4}$$

The characterization of the Tsallis entropy is the same as that of the Shannon entropy except that for the Tsallis entropy, the degree of homogeneity under convex linearity condition is α instead of 1.

Renyi entropy

For $\beta \in [0, \infty]$, the Renyi entropy of order β:

$$H_\beta(\rho)=\frac{1}{1-\beta}\log\left(\sum_{i\in S}\rho_i^\beta\right) \tag{5}$$

The scaling factor is conventional: it makes H_β non-negative for all β, and ensures $H_\beta(u_n) = \log n$, where u_n is the uniform distribution on an n element set.

The main property which the Renyi entropies have in common with Shannon entropy is additivity:

$$H_\beta(\rho\times r)=H_\beta(\rho)+H_\beta(r) \tag{6}$$

Interesting special cases.

For $\beta = 0$, we obtain the max entropy, which is cardinality of the support of ρ:

$$H\ (\rho)=\log|\{i\in S\mid \rho(i)\quad 0\}| \tag{7}$$

For $\beta = 1$, we recover Shannon entropy:

$$H_1(\rho)=\lim_{\beta\to 1}H_\beta(\rho)= \\ =\frac{d}{d\beta}\left\{\frac{1}{1-\beta}\log\left[\sum_i \rho(i)^\beta\right]\right\}_{\beta=1}=-\sum_i \rho(i)\log\rho(i) \tag{8}$$

For $\beta = \infty$, we obtain the min entropy:

$$H_\infty(\rho)=-\log\max_i \rho(i)=\log\min_i\frac{1}{\rho(i)} \tag{9}$$

Results

Data set

We use the weekly and monthly closing prices of Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500, and Shangai which receive from www.bloomberg.com for the period 2010-2018. Tables 1 and 2 summarize statistics of Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500, and Shangai data. Tables 1 and 2 show different mean values for data set, and also the corresponding standard deviations are different. Skewness of weekly data set is positive, that is, this data is skewed right. Skewness of monthly data sets are positive except Dax and Ftse 100 which means that this data is skewed right. Skewness of monthly Dax and Ftse 100 data sets are negative, that is, this data sets are skewed left. The kurtosis of weekly and monthly data sets are lower. The Jarque-Bera (JB) test shows that the normality of each series distribution is strongly rejected at 0.05 level, which means all price index distributions are non-normal. Tables 3 and 4 summarize statistics of Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500, and Shangai return data. Tables 3 and 4 show different mean values for data set, and the corresponding standard deviations are different. Skewness of weekly return data set is negative, indicating that this data is skewed left. Further, skewness of monthly return data sets are negative except Bovespa which indicates that this return data is skewed left. Skewness of monthly Bovespa return data sets are positive which comes to mean that this return data sets are skewed right. The kurtosis of return weekly data sets are high, the kurtosis of return monthly data sets are lower except Dax, Kospi, and Shangai return data set. The JB test shows that the normality of each series distribution is strongly rejected at 0.05 level which denotes that all price index distributions are non-normal. Graphical representations of the data are shown in figs. 1-4.

Table 1. Weekly data summary statistics

	Bist30	Bovespa	Dax	FTSE 100	Kospi	Nıkkei 225	SP 500	Shanghai
Mean	94488.34	60519.83	9214.157	6417.438	2027.053	14975.17	1836.515	2784.008
Median	93034.39	58497.83	9405.300	6483.580	1990.850	15215.71	1880.050	2810.310
Maximum	147880.2	89504.03	13478.86	7778.790	2574.760	24120.04	2929.670	5166.350
Minimum	60285.82	38031.22	5189.930	4838.090	1567.120	8160.010	1022.580	1979.210
Std. Dev	19601.16	10595.74	2339.422	707.6052	201.3713	4696.954	510.3082	550.7637
Skewness	0.618652	0.648448	0.078429	0.013614	0.715300	0.103326	0.246675	0.748537
Kurtosis	2.902065	2.986035	1.728166	2.064463	3.436001	1.689581	2.027903	4.091844
Jarquera Bera	29.33397	32.03066	31.26955	16.67994	42.59075	33.51149	22.62848	65.37675
Probability	0.000000	0.000000	0.000000	0.000239	0.000000	0.000000	0.000012	0.000000

Table 2. Monthly data summary statistics

	Bist30	Bovespa	Dax	FTSE 100	Kospi	Nıkkei 225	SP 500	Shanghai
Mean	95357.98	60837.29	9296.311	6432.202	2036.012	15202.74	1863.567	2770.087
Median	93207.24	58595.23	9554.035	6507.385	1999.050	15598.35	1927.120	2776.385
Maximum	146553.9	89504.00	13229.57	7748.760	2566.460	24120.04	2913.980	4611.740
Minimum	61542.00	40406.00	5502.020	4916.870	1594.580	8434.610	1030.710	1979.210
Std. Dev	19915.53	10959.39	2318.987	702.8878	200.0051	4796.152	525.7761	541.7506
Skewness	0.572688	0.683698	–0.009006	–0.064093	0.666934	0.050553	0.206051	0.672032
Kurtosis	2.829147	3.054830	1.689266	2.075994	3.523419	1.644333	1.957182	3.705514
Jarquera Bera	6.034840	8.427506	7.732572	3.915985	9.239264	8.316248	5.657841	10.36917
Probability	0.048927	0.014791	0.020936	0.141141	0.009856	0.015637	0.059077	0.005602

Table 3. Weekly return data summary statistics

	Bist30	Bovespa	Dax	FTSE 100	Kospi	Nıkkei 225	SP 500	Shanghai
Mean	0.001163	–0.000521	0.001527	0.000613	0.000533	0.001732	0.002027	–0.000448
Median	0.003267	–0.001793	0.004069	0.002781	0.002680	0.003202	0.002861	0.001345
Maximum	0.094031	0.105207	0.101666	0.072373	0.075672	0.088073	0.071284	0.090738
Minimum	–0.144992	–0.165617	–0.137974	–0.102819	–0.093009	–0.117658	–0.074603	–0.142910
Std. Dev	0.033002	0.029721	0.026448	0.020061	0.019960	0.026975	0.019070	0.029509
Skewness	–0.562653	–0.178657	–0.561212	–0.514189	–0.768233	–0.544738	–0.538185	–0.729680
Kurtosis	4.111159	4.937430	5.161167	5.573864	5.516813	4.494312	5.060694	6.130819
Jarquera Bera	47.51876	73.74487	112.6791	145.9644	165.2065	64.97854	102.6956	226.7034
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 4. Monthly return data summary statistics

	Bist30	Bovespa	Dax	FTSE 100	Kospi	Nıkkei 225	SP 500	Shanghai
Mean	0.004950	0.002747	0.006235	0.002489	0.002387	0.006971	0.008617	–0.001285
Median	0.005071	0.001745	0.006901	0.007051	0.006421	0.012544	0.011000	0.000493
Maximum	0.142395	0.156733	0.116139	0.077947	0.082796	0.111541	0.102307	0.187058
Minimum	–0.142070	–0.126206	–0.213096	–0.075437	-0.143579	–0.123916	–0.085532	–0.256813
Std. Dev	0.064923	0.058015	0.047415	0.033581	0.037212	0.050164	0.034230	0.064275
Skewness	–0.009671	0.102453	-0.807550	-0.133047	–0.835759	–0.467918	-0.327667	–0.275306
Kurtosis	2.345779	2.743198	6.208546	2.790950	5.325116	2.981422	3.517714	5.371802
Jarquera Bera	1,909856	0.481204	57.52728	0.510513	36.55895	3.906099	3.109641	26.43176
Probability	0.384840	0.786154	0.000000	0.774718	0.000000	0.141841	0.211227	0.000002

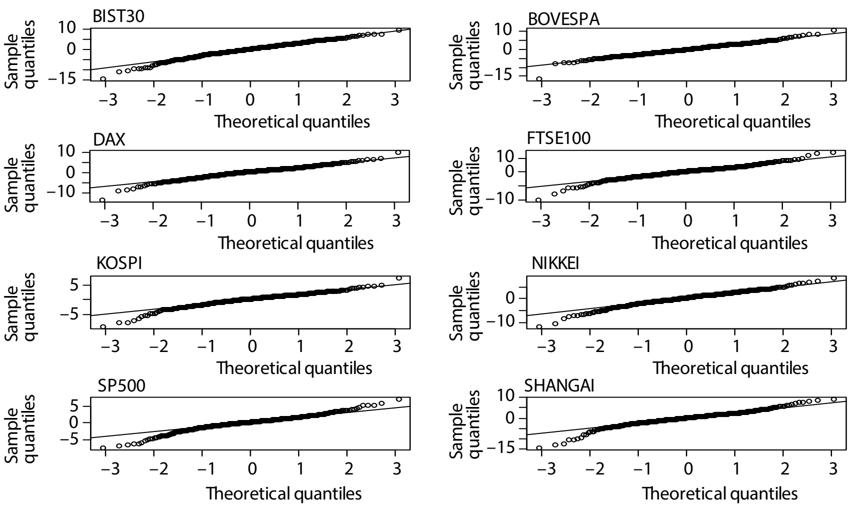

Figure 1. Quantile graphs for data of weekly world indices

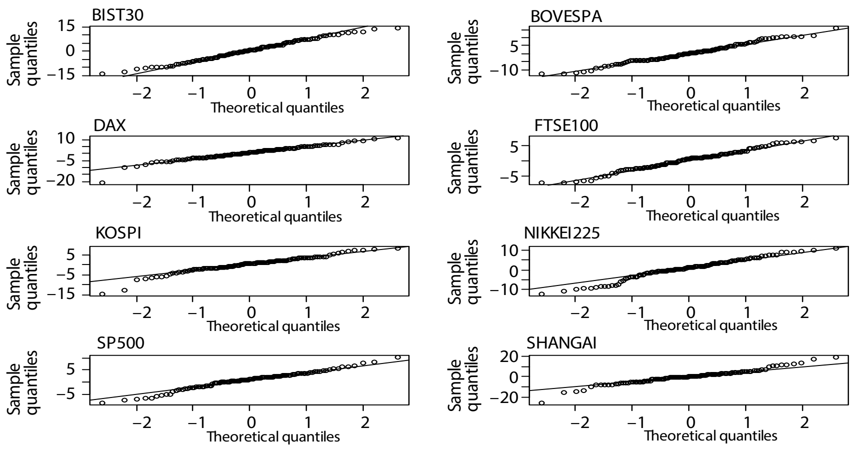

Figure 2. Quantile graphs for data of monthly world indices

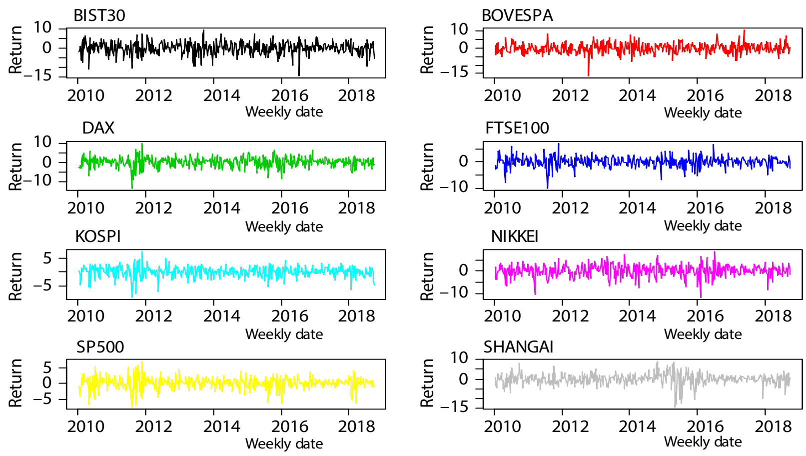

Figure 3. Returns of weekly World indices graphs

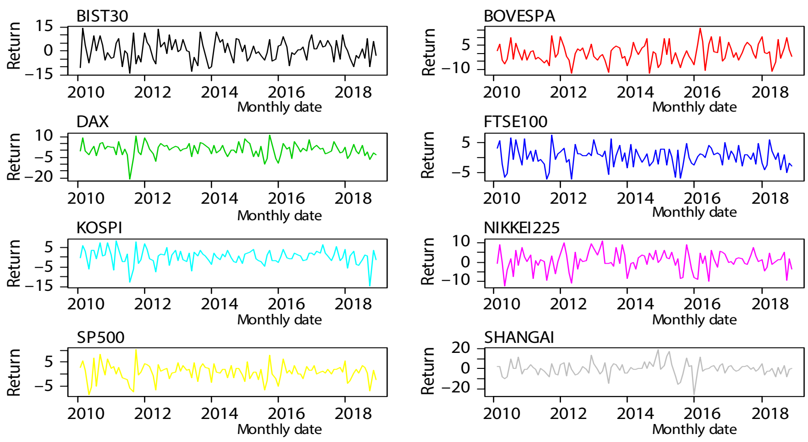

Figure 4. Returns of weekly world indices graphs

Entropy approach

We use the entropy method for volatility of Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500, and Shangai. For this, we calculate to Shannon, Tsallis, Renyi, and approximate entropies. In tabs. 5-12, firstly, we have obtained estimators for the Shannon entropy. Later, we have obtained the Tsallis for α parameter and Renyi for β parameter, calculated approximate entropy. If all likely events are same probability, the entropy takes maximum value. In our empirical results, volatility does not show differentness; this model indicates linear and non-linear dynamics. We obtain from the results that entropies are positive so, characters of our data series are non-linear. In the daily data series, we obtain that Kospi, Ftse 100, Shangai, SP 500, Dax, Nikkei 225, Bovespa, and Bist 30 series have great value of approximate entropy, respectively. In conclusion, Kospi data series are higher volatility than other data series. For the Shannon entropy estimators, it is clear that Kospi series have larger values, similarly for the Tsallis and Renyi entropy, if α and β are close to 1, we get the Shannon entropy. Volatility for Kospi, Ftse 100, Shangai, SP 500, Dax, Nikkei 225, Bovespa, and Bist 30 series is connect with α and β.

Table 5. Weekly and monthly results: BIST 30

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.103737	0	456.0000000	0	6.124683	0.0002459086
MM	6.103743	0.2	166.0114952	0.25	6.119475	
Jefferys	6.103738	0.4	63.7403415	0.5	6.114245	
Laplace	6.103738	0.6	26.3223210	1	6.103737	
SG	6.103737	0.8	11.9628782	2	6.082640	
Minimax	6.103744	1	6.1037374	4	6.041096	
CS	6.103737	1.2	3.5237079	8	5.967595	
Shrink	6.103748	1.4	2.2816880	16	5.874881	
		1.6	0.6235490	32	5.800713	
		1.8	1.2404034	64	5.751964	
		2	0.9977179	Infinite	5.676757	
Monthly						
ML	4.66099	0	107.0000000	0	4.682131	
MM	4.660995	0.2	51.4954742	0.25	4.676863	0.0002403245
Jefferys	4.66099	0.4	25.8569039	0.5	4.671581	
Laplace	4.660991	0.6	13.6848672	1	4.660990	
SG	4.66099	0.8	7.7112032	2	4.639824	
Minimax	4.661003	1	4.6609903	4	4.598506	
CS	4.66099	1.2	3.0298930	8	4.526028	
Shrink	4.661	1.4	2.1112092	16	4.432964	
		1.6	1.5641920	32	4.355918	
		1.8	1.2195629	64	4.307517	
		2	0.9903406	Infinite	4.252376	

Table 6. Weekly and monthly results: BOVESPA

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.10976	0	456.0000000	0	6.124683	0.000539021
MM	6.109768	0.2	166.1731573	0.25	6.120985	
Jefferys	6.10976	0.4	63.8351777	0.5	6.117263	
Laplace	6.10976	0.6	26.3640917	1	6.109760	
SG	6.10976	0.8	11.9792500	2	6.094556	
Minimax	6.109765	1	6.1097598	4	6.063859	
CS	6.10976	1.2	3.5258370	8	6.005903	
Shrink	6.109776	1.4	2.2824206	16	5.923613	
		1.6	1.6237962	32	5.851882	
		1.8	1.2404856	64	5.805064	
		2	0.9977449	Infinite	5.733371	
Monthly						
ML	4.66648	0	107.000000	0	4.682131	
MM	4.666488	0.2	51.542595	0.25	4.678257	0.0005063682
Jefferys	4.66648	0.4	25.893665	0.5	4.674356	
Laplace	4.66648	0.6	13.706399	1	4.666480	
SG	4.66648	0.8	7.722426	2	4.650486	
Minimax	4.666492	1	4.666480	4	4.618088	
CS	4.66648	1.2	3.032474	8	4.556780	
Shrink	4.666496	1.4	2.112390	16	4.470816	
		1.6	1.564722	32	4.399297	
		1.8	1.219797	64	4.355629	
		2	0.990443	Infinite	4.296051	

Table 7. Weekly and monthly results: DAX

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.092127	0	456.0000000	0	6.124683	
MM	6.092181	0.2	165.6744494	0.25	6.116337	0.001683361
Jefferys	6.092131	0.4	63.5463430	0.5	6.108117	
Laplace	6.092134	0.6	26.2385133	1	6.092127	
SG	6.092127	0.8	11.9306731	2	6.062346	
Minimax	6.092159	1	6.0921273	4	6.012956	
CS	6.092127	1.2	3.5196871	8	5.948968	
Shrink	6.092239	1.4	2.2803333	16	5.888417	
		1.6	1.6231016	32	5.841889	
		1.8	1.2402578	64	5.807570	
		2	0.9976711	Infinite	5.744302	

→

Table 7. Continuous

Monthly						
ML	4.650723	0	107.0000000	0	4.682131	
MM	4.650776	0.2	51.3999269	0.25	4.674045	0.001727683
Jefferys	4.650727	0.4	25.7837949	0.5	4.666104	
Laplace	4.65073	0.6	13.6428895	1	4.650723	
SG	4.650723	0.8	7.6897674	2	4.622307	
Minimax	4.650787	1	4.6507232	4	4.575793	
CS	4.650787	1.2	3.0251699	8	4.516262	
Shrink	4.650834	1.4	2.1090959	16	4.459363	
		1.6	1.5632654	32	4.414168	
		1.8	1.2191628	64	4.381040	
		2	0.9901699	Infinite	4.329294	

Table 8. Weekly and monthly results: FTSE 100

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.118595	0	456.0000000	0	6.124683	0.01281646
MM	6.118673	0.2	166.4052413	0.25	6.123152	
Jefferys	6.118596	0.4	63.9721419	0.5	6.121626	
Laplace	6.118597	0.6	26.4247577	1	6.118595	
SG	6.118595	0.8	12.0031525	2	6.112625	
Minimax	6.118602	1	6.1185951	4	6.101146	
CS	6.118595	1.2	3.5289745	8	6.080524	
Shrink	6.118751	1.4	2.2835046	16	6.049283	
		1.6	1.6241633	32	6.014148	
		1.8	1.2406081	64	5.985013	
		2	0.9977853	Infinite	5.932302	
Monthly						
ML	4.676177	0	107.0000000	0	4.682131	
MM	4.676254	0.2	51.6227225	0.25	4.680630	0.01301788
Jefferys	4.676178	0.4	25.9568067	0.5	4.679138	
Laplace	4.676179	0.6	13.7437439	1	4.676177	
SG	4.676177	0.8	7.7420730	2	4.670370	
Minimax	4.676192	1	4.6761773	4	4.659284	
CS	4.676177	1.2	3.0370720	8	4.639582	
Shrink	4.676332	1.4	2.1145113	16	4.609917	
		1.6	1.5656812	32	4.575951	
		1.8	1.2202246	64	4.546882	
		2	0.9906312	Infinite	4.495915	

Table 9. Weekly and monthly results: KOSPI

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.11985	0	456.0000000	0	6.124683	0.04998745
MM	6.120096	0.2	166.4413856	0.25	6.123487	
Jefferys	6.119852	0.4	63.9930171	0.5	6.122283	
Laplace	6.119855	0.6	26.4337980	1	6.119850	
SG	6.11985	0.8	12.0066316	2	6.114884	
Minimax	6.11986	1	6.1198501	4	6.104565	
CS	6.11985	1.2	3.5294089	8	6.082783	
Shrink	6.120318	1.4	2.2836507	16	6.041033	
		1.6	1.6242115	32	5.989933	
		1.8	1.2406237	64	5.951743	
		2	0.9977903	Infinite	5.885510	
Monthly						
ML	4.677429	0	107.0000000	0	4.682131	
MM	4.677673	0.2	51.6341211	0.25	4.680966	0.05089337
Jefferys	4.677432	0.4	25.9655869	0.5	4.679794	
Laplace	4.677434	0.6	13.7488150	1	4.677429	
SG	4.677429	0.8	7.7446758	2	4.672616	
Minimax	4.677449	1	4.6774293	4	4.662652	
CS	4.677429	1.2	3.0376499	8	4.641664	
Shrink	4.677893	1.4	2.1147706	16	4.601036	
		1.6	1.5657951	32	4.549927	
		1.8	1.2202738	64	4.510640	
		2	0.9906522	Infinite	4.450596	

Table 10. Weekly and monthly results: NIKKEI 225

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.074772	0	10.0000000	0	2.397895	
MM	6.074805	0.2	7.2613731	0.25	2.397807	0.0006786497
Jefferys	6.074775	0.4	5.3583540	0.5	2.397719	
Laplace	6.074779	0.6	4.0231951	1	2.397544	
SG	6.074772	0.8	3.0765167	2	2.397197	
Minimax	6.074811	1	2.3975438	4	2.396515	
CS	6.074772	1.2	1.9045196	8	2.395206	
Shrink	6.074841	1.4	1.5417734	16	2.392819	
		1.6	1.2711551	32	2.388973	
		1.8	1.0663400	64	2.384169	

→

Table 10. Continuous

Monthly		2	0.9090274	Infinite	2.372129	
ML	4.63172	0	107.0000000	0	4.682131	
MM	4.631753	0.2	51.2300312	0.25	4.669011	0.0006602917
Jefferys	4.631724	0.4	25.6527336	0.5	4.656204	
Laplace	4.631727	0.6	13.5669274	1	4.631720	
SG	4.63172	0.8	7.6505627	2	4.588089	
Minimax	4.631801	1	4.6317202	4	4.522583	
CS	4.63172	1.2	3.0163117	8	4.448562	
Shrink	4.631788	1.4	2.1050742	16	4.381831	
		1.6	1.5614736	32	4.327930	
		1.8	1.2183756	64	4.284722	
		2	0.9898277	Infinite	4.220564	

Table 11. Weekly and monthly results: SP 500

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.086088	0	456.0000000	0	6.124683	
MM	6.086359	0.2	165.5130332	0.25	6.114825	0.007051094
Jefferys	6.086109	0.4	63.4516884	0.5	6.105091	
Laplace	6.08613	0.6	26.1967973	1	6.086088	
SG	6.086088	0.8	11.9142969	2	6.050465	
Minimax	6.086173	1	6.0860878	4	5.990556	
CS	6.086088	1.2	3.5175445	8	5.909588	
Shrink	6.086637	1.4	2.2795928	16	5.826955	
		1.6	1.6228504	32	5.763557	
		1.8	1.2401738	64	5.721544	
		2	0.9976432	Infinite	5.657663	
Monthly						
ML	4.642473	0	107.0000000	0	4.682131	
MM	4.642739	0.2	51.3299457	0.25	4.671967	0.006736359
Jefferys	4.642495	0.4	25.7291183	0.5	4.661953	
Laplace	4.642516	0.6	13.6107839	1	4.642473	
SG	4.642473	0.8	7.6729756	2	4.606224	
Minimax	4.642652	1	4.6424731	4	4.546177	
CS	4.642473	1.2	3.0212709	8	4.467321	
Shrink	4.643014	1.4	2.1073009	16	4.390072	
		1.6	1.5624543	32	4.331403	
		1.8	1.2188013	64	4.290574	
		2	0.9900105	Infinite	4.235104	

Table 12. Weekly and monthly results: SHANGAI

Shannon		Tsallis		Renyi		Approximate entropy
Method		α		β		
Weekly						
ML	6.105707	0	456.0000000	0	6.124683	
MM	6.105886	0.2	166.0652167	0.25	6.119977	0.009177048
Jefferys	6.105714	0.4	63.7717519	0.5	6.115244	
Laplace	6.10572	0.6	26.3361044	1	6.105707	
SG	6.105707	0.8	11.9682578	2	6.086375	
Minimax	6.10574	1	6.1057067	4	6.046680	
CS	6.105707	1.2	3.5244003	8	5.962134	
Shrink	6.106052	1.4	2.2819247	16	5.808390	
		1.6	1.6236283	32	5.670889	
		1.8	1.2404295	64	5.591353	
		2	0.9977264	Infinite	5.506409	
Monthly						
ML	4.663645	0	107.0000000	0	4.682131	
MM	4.663824	0.2	51.5182143	0.25	4.677536	0.009435007
Jefferys	4.663652	0.4	25.8746592	0.5	4.672922	
Laplace	4.663659	0.6	13.6952742	1	4.663645	
SG	4.663645	0.8	7.7166300	2	4.644937	
Minimax	4.663712	1	4.6636453	4	4.607080	
CS	4.663645	1.2	3.0311410	8	4.530336	
Shrink	4.663991	1.4	2.1117799	16	4.402599	
		1.6	1.5644478	32	4.296305	
		1.8	1.2196758	64	4.237145	
		2	0.9903899	Infinite	4.172405	

Conclusion

In this article we have considered the entropy approach to explain volatility of Bist 30, Bovespa, Dax, Ftse 100, Kospi, Nikkei 225, SP 500, and Shangai indices variables. For the analysis of the results of this study, firstly we give descriptive statistics of time series and return series of this world indices, tabs. 1-4 and figs. 1-4. Later, we have used the entropy approach in order to evaluate the volatility for the world indices. Our results indicate that Kospi is more volatile than other world indices in the period 2010-2018.

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