SOME ANALYTICAL SOLUTIONS BY MAPPING METHODS FOR NON-LINEAR CONFORMABLE TIME-FRACTIONAL PHI-4 EQUATION

by

Zeliha KORPINAR *

Department of Administration, Faculty of Economic and Administrative Sciences, Mus Alparslan University, Mus, Turkey

Original scientific paper
https://doi.org/10.2298/TSCI190108341K

In this paper, the practice of two types of mapping methods are used to solve the time fractional Phi-4 equation by means of conformable fractional derivative. The solutions are derived using Jacobi's elliptic functions for two different value of the modulus and are obtained the some soliton solutions. The found solutions are identified bright optical soliton, dark soliton, singular soliton, combo soliton solution, and periodic solutions.

Key words: *optical solitons, the time fractional Phi-4 equation, conformable derivative, mapping methods*

Introduction

Homogenous and non-homogenous differential equations have been briefly studied in the literature since they act as a bridge between mathematics and physics [1-12]. Recently, there has been considerable interests and significant theoretical developments in fractional calculus used in many fields and in fractional differential equations and its applications [13-38]. Ekici *et al*. [13] used the first integral method by using conformable fractional derivative for getting the optical soliton solutions. Tchier *et al*. [14] obtained solutions of the time fractional reaction-diffusion equations with residual power series method. Rezazadeh *et al*. [15] found travelling wave solution of conformable fractional generalized reaction Duffing model by generalized projective Riccati equation method. Inc *et al.* [16] investigated approximate solution of some non-linear equations by using Residual power series method. Eslami [17] obtained the exact traveling wave solutions to the fractional coupled non-linear Schrodinger equations and in [18] are studied on numerical solutions for the Caputo-Fabrizio fractional heat-like equation. Many more researches related to fractional derivatives can be saw in [19-38].

In this work, we analyze the time fractional Phi-4 equation by means of conformable fractional derivative operator [19, 20] to form optical solitons using the various types of mapping methods. Recently, there are a lot of important of this equation in particle and nuclear physics [20]. The some wave solutions of the Phi-4 equation with a non-linear variant are analyzed via the Weierstrass elliptic function process. Akter and Akbar [21] is applied the modified simple equation method to obtain the analytical solutions of the Phi-4 equations. Bhrawy *et al*. [22] are investigated the spectral solutions of time dependent non-linear Phi-4 equations by using Jacobi-Gauss-Lobatto collocation process. Triki and Wazwaz [23], are analyzed the envelope solitons for the generalized Phi-4 equations. Exact analytical solutions of where in

* Author's e-mail: zelihakorpinar@gmail.com

said equation are obtained using tanh method with fractional complex transform in [24]. Alquran *et al*. [25] applied the residual power series (RPS) process to find the series solution of this equation. In this work, the found solutions are identified bright optical soliton, dark soliton, singular soliton, combo soliton solution, and periodic solutions. There are several applications for the mapping method and its diversities [26-31]. Two main group of solutions are investigated via mapping methods. These are Jacobian elliptic functions (JEF) and periodic wave solutions (PWS). When modulus $m \to 1$ or $m \to 0$ the JEF re-edited as hyperbolic functions and trigonometric functions.

The time fractional Phi-4 equation is presented [19, 25]:

$$q_t^{(\eta)} - q_{xx} + \varepsilon^2 q + \sigma q^3 = 0,\ t \geq 0,\ 0 < \eta \leq 1 \tag{1}$$

where $q_t^{(\eta)}$ is the conformable derivative operator, ε and σ are real valued constants. In [32] scientists studied conformable type of fractional derivative in 2014 as a new definition of local fractional operator. It is very easier to work with this fractional derivative. Recently, several studies have been done related to conformable type of fractional calculations [33-38].

The definition of conformable fractional derivative of order $\eta \in (0, 1)$ defined [32]:

$$_t\mathrm{D}^\eta f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\eta}) - f(t)}{\varepsilon},\ \ f:(0,\infty) \to \mathrm{R} \tag{2}$$

Some of the features of conformable fractional derivative [32, 33]:

$$_t\mathrm{D}^\eta t^\alpha = \alpha t^{\alpha-\eta},\ \forall \eta \in \mathrm{R} \tag{2a}$$

$$_t\mathrm{D}^\eta (fg) = f_t\mathrm{D}^\eta g + g_t\mathrm{D}^\eta f \tag{2b}$$

$$_t\mathrm{D}^\eta (fog) = t^{1-\eta} g'(t) f'[g(t)] \tag{2c}$$

$$_t\mathrm{D}^\eta \left(\frac{f}{g}\right) = \frac{g_t\mathrm{D}^\eta f - f_t\mathrm{D}^\eta g}{g^2} \tag{2d}$$

Analysis of mapping method

Assume the general non-linear PDE:

$$A[q, q_t^{(\eta)}, q_x, q_{xx}, q_{tt}^{(2\eta)}, ...] = 0 \tag{3}$$

where q is an unknown function depending on x and t, A – the polynomial in $q = q(x, t)$ and the sub-indices represent the partial conformable fractional derivatives.

– Suppose the traveling wave variable:

$$q(x,t) = u(\phi),\quad \phi = x - Q\frac{t^\eta}{\eta} \tag{4}$$

Then, from eq. (4), eq. (3) is turn to an ODE for $u(\phi)$:

$$B(u, u_\phi, u_{\phi\phi}, u_{\phi\phi\phi}, ...) = 0 \tag{5}$$

where the sub-indices represent the ordinary derivatives with respect to ϕ.

– Consider the solution of eq. (5):

$$u(\phi) = \sum_{i=0}^{N} a_i G^i(\phi) \tag{6}$$

where $a_n \neq 0$ and $G(\phi)$ can be expressed:

$$\left[G'(\phi)\right]^2 = fG^2(\phi) + \frac{1}{2}gG^4(\phi) + h \tag{7}$$

where h, g, and f are arbitrary constants.

– N is found by balancing between the non-linear terms and the highest order derivatives in eq. (5).
– Replacing eq. (6) together with eq. (7) into the eq. (5), then equating each coefficient of the polynomials to zero, give a set of algebraic equations for a_i (i = 1, 2,...N), h, g, f, and Q.
– Solving the obtained system, we obtain values for a_i (i = 1, 2,...N) and Q. Then, the solutions of eq. (5) are obtained.

It is clear that if $N \in Z$, we can easily apply the above steps to find the some of JEF and PWS solutions. If $N \notin Z$, the some of solutions are obtained as rational statements including JEF and PWS solutions.

Applications for the time-fractional Phi-4 equation

Mapping method

By placing eq. (4) into eq. (1), are obtained non-linear equation:

$$(Q^2 - 1)u'' + \varepsilon^2 u + \sigma u^3 = 0 \tag{8}$$

Assumed the solution of eq. (8) is demonstrable as a finite series:

$$u(\phi) = \sum_{j=0}^{N} \alpha_j G^j(\phi) \tag{9}$$

where $G(\phi)$ satisfies eq. (7), $\phi = x - Q(t^\eta/\eta)$ and α_j for $j = \overline{1, N}$ are values to be definited.

By balancing with in eq. (8), is obtained N = 1.

We can select the solution of eq. (9):

$$q(\phi) = \alpha_0 + \alpha_1 G(\phi) \tag{10}$$

Substituting (10) into (8), collecting the coefficients of $G(\phi)$, and solving the obtaining system, the following groups of some solutions are found.

One of the four groups of values:

$$\alpha_0 = 0,\ \alpha_1 = \frac{\sqrt{g}\varepsilon}{\sqrt{f}\sqrt{\sigma}},\ Q = \frac{\sqrt{f - \varepsilon^2}}{\sqrt{f}} \tag{11}$$

Type 1. $G(\phi) = sn[\phi; m]$ or $G(\phi) = cd[\phi; m]$. So $f = -(m^2 + 1)$, $g = 2m^2$, and $h = 1$. Then, the PWS of eq. (8):

$$\begin{aligned} u(\phi) &= \frac{\sqrt{2}\varepsilon m}{\sqrt{-(m^2+1)}\sqrt{\sigma}} sn[\phi; m] \\ u(\phi) &= \frac{\sqrt{2}\varepsilon m}{\sqrt{-(m^2+1)}\sqrt{\sigma}} cd[\phi; m] \end{aligned} \tag{12}$$

When $m \to 1$, eq. (12) is rewrote in the form of dark soliton solution of eq. (1):

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{-2\sigma}}\tanh(x-\frac{\sqrt{-2-\varepsilon^2}}{\sqrt{-2}}\frac{t^\eta}{\eta}) \tag{13}$$

and in the form of singular soliton solution:

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{-2\sigma}}\cos\left(x-\frac{\sqrt{-2-\varepsilon^2}}{\sqrt{-2}}\frac{t^\eta}{\eta}\right) \tag{14}$$

Type 2. $G(\phi) = cn[\phi; m]$. So $f = 2m^2 - 1$, $g = -2m^2$, and $h = 1 - m^2$. Then, the PWS of eq. (8):

$$u(\phi)=\frac{\sqrt{-2}\varepsilon m}{\sqrt{2m^2-1}\sqrt{\sigma}}cn[\phi;m] \tag{15}$$

When $m \to 1$, eq. (15) is rewrote in the form of bright optical soliton solution of eq. (1) :

$$q(x,t)=\frac{\sqrt{-2}\varepsilon}{\sqrt{\sigma}}\operatorname{sech}\left(x-\sqrt{1-\varepsilon^2}\frac{t^\eta}{\eta}\right) \tag{16}$$

Type 3. $G(\phi) = dn[\phi; m]$. So $f = 2 - m^2$, $g = -2$, and $h = m^2 - 1$. Then, the PWS of eq. (8):

$$u(\phi)=\frac{\sqrt{2}\varepsilon}{\sqrt{2-m^2}\sqrt{\sigma}}dn[\phi;m] \tag{17}$$

When $m \to 1$, eq. (16) is rewrote in the form of bright optical soliton solution of eq. (1):

$$q(x,t)=\frac{\sqrt{-2}\varepsilon}{\sqrt{\sigma}}\operatorname{sech}\left(x-\sqrt{1-\varepsilon^2}\frac{t^\eta}{\eta}\right)$$

This solution is the same as eq. (16):

Type 4. $G(\phi) = cs[\phi; m]$. So $f = 2 - m^2$, $g = 2$, and $h = 1 - m^2$. Then, the PWS of eq. (8):

$$u(\phi)=\frac{\sqrt{2}\varepsilon}{\sqrt{2-m^2}\sqrt{\sigma}}cs[\phi;m] \tag{18}$$

When $m \to 0$, eq. (18) is rewrote in the form of singular periodic solution of eq. (1) :

$$q(x,t)=\frac{\varepsilon}{\sqrt{\sigma}}\cot\left(x-\frac{\sqrt{2-\varepsilon^2}}{\sqrt{2}}\frac{t^\eta}{\eta}\right) \tag{19}$$

and when $m \to 1$, eq. (18) is rewrote in the form of singular soliton solution of eq. (1) :

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{\sigma}}\operatorname{csch}\left(x-\sqrt{1-\varepsilon^2}\frac{t^\eta}{\eta}\right) \tag{20}$$

Type 5. $G(\phi) = ns[\phi; m]$ or $G(\phi) = dc[\phi; m]$. So $f = -(1 + m^2)$, $g = 2$, and $h = m^2$. Then, the PWS of eq. (8):

$$u(\phi)=\frac{\sqrt{2}\varepsilon}{\sqrt{-(1+m^2)}\sqrt{\sigma}}ns[\phi;m] \tag{21}$$

and

$$u(\phi)=\frac{\sqrt{2}\varepsilon}{\sqrt{-(1+m^2)}\sqrt{\sigma}}dc[\phi;m] \tag{22}$$

When $m \to 0$, eqs. (21) and (22) are rewrote, respectively, in the form of the singular periodic solution of eq. (1):

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{-\sigma}}\csc\left(x-\sqrt{1+\varepsilon^2}\,\frac{t^\eta}{\eta}\right) \tag{23}$$

and

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{-\sigma}}\sec\left(x-\sqrt{1+\varepsilon^2}\,\frac{t^\eta}{\eta}\right) \tag{24}$$

When $m \to 1$, eq. (22) are rewrote in the form of the singular soliton solution of eq. (1):

$$q(x,t)=\frac{\sqrt{2}\varepsilon}{\sqrt{-2\sigma}}\coth\left(x-\sqrt{\frac{2+\varepsilon^2}{2}}\,\frac{t^\eta}{\eta}\right) \tag{25}$$

Modified mapping method

Consider the solution of eq. (9):

$$u(\phi)=\alpha_0+\alpha_1 G(\phi)+\beta_1 G^{-1}(\phi) \tag{26}$$

where $G(\phi)$ satisfies eq. (7).

Substituting (26) into (9), collecting the coefficients of $G(\phi)$, and solving the obtaining system, the following groups of some solutions are found:

One of the sixteen groups of values:

$$\alpha_0=0,\quad \alpha_1=\frac{\sqrt{\dfrac{g(f-3\sqrt{2gh})\varepsilon^2}{f^2-18gh}}}{\sqrt{\sigma}},\quad \beta_1=-\frac{\sqrt{2h}\sqrt{\dfrac{g(f-3\sqrt{2gh})\varepsilon^2}{f^2-18gh}}}{\sqrt{g\sigma}}$$
$$Q=\sqrt{\frac{f^2-18gh-f\varepsilon^2+3\sqrt{2gh}\varepsilon^2}{f^2-18gh}} \tag{27}$$

Type 1. $G(\phi)=sn[\phi;m]$ or $G(\phi)=cd[\phi;m]$. So $f=-(m^2+1)$, $g=2m^2$, and $h=1$. Then, the PWS of eq. (8):

$$u(\phi)=\frac{\sqrt{\dfrac{2[-(m^2+1)-6m]\varepsilon^2}{(m^2+1)^2-36m^2}}\,m}{\sqrt{\sigma}}\left(sn[\phi;m]-\frac{1}{m}ns[\phi;m]\right) \tag{28}$$

and

$$u(\phi)=\frac{\sqrt{\dfrac{2[-(m^2+1)-6m]\varepsilon^2}{(m^2+1)^2-36m^2}}\,m}{\sqrt{\sigma}}\left(cd[\phi;m]-\frac{1}{m}dc[\phi;m]\right) \tag{29}$$

When $m \to 1$, eq. (28) rearranged in the form of the combo periodic singular solution of eq. (1):

$$q(x,t)=\frac{\sqrt{\dfrac{\varepsilon^2}{2}}}{\sqrt{\sigma}}\left[\tanh\left(x-\sqrt{\frac{-4+\varepsilon^2}{-4}}\,\frac{t^\eta}{\eta}\right)-\coth\left(x-\sqrt{\frac{-4+\varepsilon^2}{-4}}\,\frac{t^\eta}{\eta}\right)\right] \tag{30}$$

Type 2. $G(\phi) = dn[\phi; m]$. So $f = 2 - m^2$, $g = -2$, and $h = m^2 - 1$. Then, the PWS of eq. (8):

$$u(\phi) = \frac{\sqrt{\dfrac{-2\left[2-m^2-3\sqrt{-4(m^2-1)}\right]\varepsilon^2}{(2-m^2)^2+36(m^2-1)}}}{\sqrt{\sigma}}\left(dn[\phi;m]-\sqrt{1-m^2}\,nd[\phi;m]\right) \tag{31}$$

When $m \to 1$, eq. (31) rewrote in the form of bright optical soliton solution of eq. (1):

$$q(x,t) = \frac{\sqrt{-2\varepsilon^2}}{\sqrt{\sigma}}\left[\operatorname{sech}\left(x-\sqrt{1-\varepsilon^2}\,\frac{t^\eta}{\eta}\right)\right] \tag{32}$$

Type 3. $G(\phi) = cs[\phi; m]$. So $f = 2 - m^2$, $g = 2$, and $h = 1 - m^2$. Then, the PWS of eq. (8):

$$u(\phi) = \frac{\sqrt{\dfrac{2\left[2-m^2-3\sqrt{4(1-m^2)}\right]\varepsilon^2}{(2-m^2)^2-36(1-m^2)}}}{\sqrt{\sigma}}\left(cs[\phi;m]-\sqrt{(1-m^2)}\,sc[\phi;m]\right) \tag{33}$$

When $m \to 0$, eq. (33) rearranged in the form of the combo periodic singular solution of eq. (1):

$$q(x,t) = \frac{\sqrt{\dfrac{\varepsilon^2}{4}}}{\sqrt{\sigma}}\left[\tanh\left(x-\sqrt{\frac{-8+\varepsilon^2}{-8}}\,\frac{t^\eta}{\eta}\right)-\coth\left(x-\sqrt{\frac{-8+\varepsilon^2}{-8}}\,\frac{t^\eta}{\eta}\right)\right] \tag{34}$$

Graphical representation of the solutions

The surface graphics of the obtained solutions are showed in the figs. 1 and 2 by using MATHEMATICA.

We wrote the some of solutions found for the presented time-fractional Phi-4 equation via conformable fractional derivative operator. Besides we showed 3-D graphics for some of solutions in figs. 1 and 2. The graphics above were drawn for $\varepsilon = 1$, $\sigma = 2$, $\mu = 0.75$.

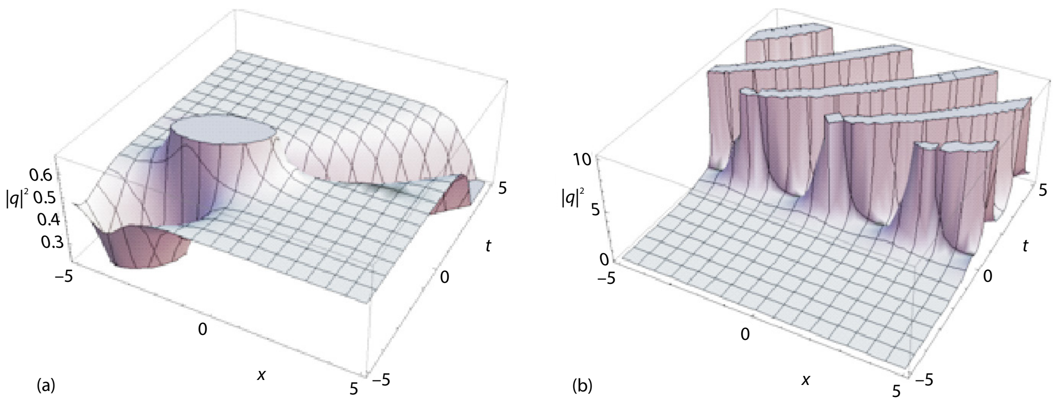

Figure 1. The surface graphics for the $|q(x, t)|^2$ analytical solution of the time-fractional Phi-4 equation obtained with mapping method; (a) eq. (14), (b) eq. (23)

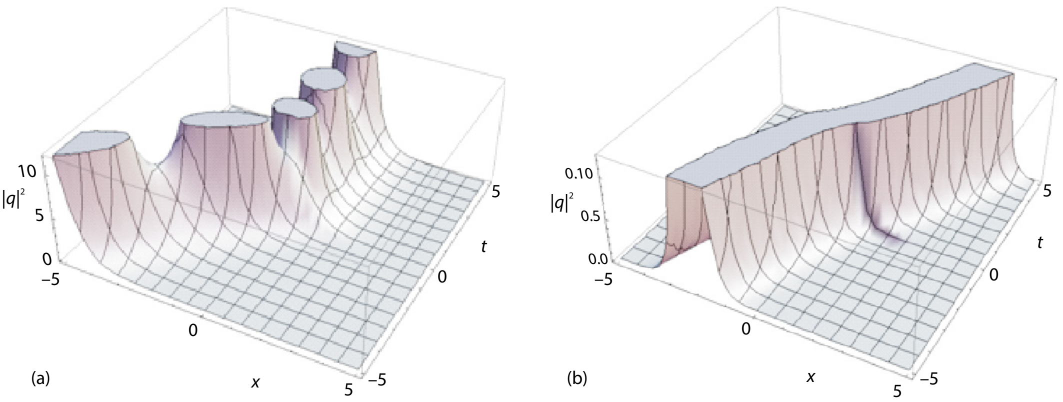

Figure 2. The surface graphics for the $|q(x, t)|^2$ analytical solution of the time-fractional Phi-4 equation obtained with modified mapping method; (a) eq. (32), (b) eq. (34)

Conclusion

In this paper, the mapping methods are used to find new soliton solutions of the time-fractional Phi-4 equation by using conformable fractional derivative. When modulus $m \to 1$ or $m \to 0$ the Jacobian elliptic functions rearranged as trigonometric functions and hyperbolic functions. The existences of solutions derived from these functions are all guaranteed through constraint conditions that are also listed besides the solutions. We say that the presented method is suitable to examine the many problems located in science and engineering.

References

[1] Mohyud-Din, S. T., Travelling Wave Solutions of Seventh-Order Generalized KdV Equations Using He's Polynomials, *International Journal of Non-Linear Sciences and Numerical Simulation*, *10* (2009), 2, pp. 223-229
[2] Mohyud-Din, S. T., *et al*., Numerical Soliton Solution of the Kaup-Kupershmidt Equation, International *Journal of Numerical Methods for Heat and Fluid-Flow*, *21* (2011), 3, pp. 272-281
[3] Khan, U., *et al*., Extracting New Solitary Wave Solutions of Benny-Luke Equation and Phi-Four Equation of Fractional Order by Using (G'/G)-Expansion Method, *Opt Quant Electron*, *49* (2017), Nov., p. 362
[4] Sikander, W., *et al*., Optimal Solutions for Homogeneous and Non-Homogeneous Equations Arising in Physics, *Results in Physics*, *7* (2017), Dec., pp. 216-224
[5] Sikander, W., *et al*., Optimal Solutions for the Evolution of a Social Obesity Epidemic Model, *Eur. Phys. J. Plus*, *132* (2017), June, p. 257
[6] Mohyud-Din, S. T., *et al*., Exact Solutions of (3 + 1)-Dimensional Generalized KP Equation Arising Inphysics, *Results in Physics*, *7* (2017), Oct., pp. 3901-3909
[7] Mirzazadeh, M., *et al*., Soliton Solutions of the Generalized Klein-Gordon Equation by Using G'/G-Expansion Method, *Computational and Applied Mathematics*, *33* (2014), 3, pp. 831-839
[8] Mirzazadeh, M., *et al*., Solitons and Other Solutions to Wu-Zhang System, Non-Linear Analysis: *Modelling and Control*, *22* (2017), 4, pp. 441-458
[9] Mirzazadeh, M., *et al.*, Exact Solutions of the Kudryashov-Sinelshchikov Equation and Non-Linear Telegraph Equation Via the First Integral Method, *Non-Linear Anal. Mode*l, *17* (2012), 4, pp. 481-488
[10] Eslami, M., Trial Solution Technique to Chiral Non-Linear Schrodinger's Equation in (1+2)-Dimensions, *Non-linear Dynamics*, *85* (2016), 2, pp. 813-816
[11] Zhou Q., *et al*., Optical Solitons with Biswas-Milovic Equation by Extended Trial Equation Method, *Non-Linear Dynamics*, *84* (2016), 4, pp. 1883-1900
[12] Eslami, M., *et al*., First Integral Method to Look for Exact Solutions of a Variety of Boussinesq-Like Equations, *Ocean Engineering*, *83* (2014), June, pp. 133-137

[13] Ekici, M., *et al*., Optical Soliton Perturbation with Fractional-Temporal Evolution by First Integral Method with Conformable Fractional Derivatives, *Optik*, *127* (2016), 22, pp. 10659-10669
[14] Tchier, F., *et al*., Solution of the Time Fractional Reaction-Diffusion Equations with Residual Power Series Method, *Advances in Mechanical Engineering*, *8* (2016), 10, pp. 1-10
[15] Rezazadeh, H., *et al*., Traveling Wave Solution of Conformable Fractional Generalized Reaction Duffing Model by Generalized Projective Riccati Equation Method, *Optical and Quantum Electronics*, *50* (2018), 3, p. 150
[16] Inc, M., *et al*., A New Method for Approximate Solution of Some Non-Linear Equations: Residual Power Series Method, *Advances in Mechanical Engineering*, *8* (2016), 4, pp. 1-7
[17] Eslami, M., Exact Traveling Wave Solutions to the Fractional Coupled Non-Linear Schrodinger Equations, *Applied Mathematics and Computation*, *285* (2016), July, pp. 141-148
[18] Korpinar, Z., On Numerical Solutions for the Caputo-Fabrizio Fractional Heat-Like Equation, *Thermal Science*, *22* (2018), 1, pp. 87-95
[19] Rezazadeh, H., *et al*., New Exact Solutions of Non-Linear Conformable Time-Fractional Phi-Four Equation, *Chinese Journal of Physics*, *56* (2018), 6, pp. 2805-2816
[20] Deng, X., *et al*., Travelling Wave Solutions for a Non-Linear variant of the Phi-Four Equation, *Math. Comp. Model*., *49* (2009), 3-4, pp. 617-622
[21] Akter, J., Akbar, M. A., Exact Solutions to the Benney-Luke Equation and the Phi-Four Equations by Using Modified Simple Equation Method, *Results Phys*., *5* (2015), Feb., pp. 125-130
[22] Bhrawy, A. H., *et al*., An Efficient Spectral Collocation Algorithm for Non-Linear Phi-Four Equations, *Bound. Value Probl*., *2013* (2013), 1, pp. 87
[23] Triki, H., Wazwaz, A. M., Envelope Solitons for Generalized Forms of the Phi-Four Equation, *Journal King Saud Univ*., *25* (2013), 2, pp. 129-133
[24] Tariq, H., Akram, G., New Approach for Exact Solutions of Time Fractional Cahn-Allen Equation and Time Fractional Phi-Four Equation, *Phys. A*, *473* (2017), May, pp. 352-362
[25] Alquran, M., *et al*., Analytical Solution of the Time-Fractional Phi-Four Equation by Using Modified Residual Power Series Method, *Non-Linear Dyn*, *90* (2017), 4, pp. 2525-2529
[26] Krishnan, E. V., Mapping Methods to Solve a Modified Korteweg-de Vries Equation, *SQU J. Sci*., *20* (2015), 2, pp. 42-48
[27] Krishnan, E. V., Biswas, A., Solutions to the Zakharov-Kuznetsov Equation with Higher Order Non-Linearity by Mapping and Ansatz Methods, *Phys. Wave Phenom*., *18* (2010), 4, pp. 256-261
[28] Krishnan, E. V., *et al*., Solitons in Optical Metamaterials by Mapping Method, J*ournal Optoelectron. Adv. Mater*., *17* (2015), 3-4, pp. 511-516
[29] Peng, Y., Exact Periodic Wave Solutions to a New Hamiltonian Amplitude Equation, *Journal Phys. Soc. Jpn*., *72* (2003), 6, pp. 1356-1359
[30] Krishnan, E.V., *et al*., Optik, *International Journal for Light and Electron Optics*, *178* (2019), Feb., pp. 104-110
[31] Peng, Y., New Exact Solutions to a New Hamiltonian Amplitude Equation II, *Journal Phys. Soc. Jpn*., *73* (2004), 5, pp. 1156-1158
[32] Khalil, R., *et al*., A New Definition of Fractional Derivative, *Journal Comput. Appl. Math*., *264* (2014), July, pp. 65-70
[33] Eslami, M., Rezazadeh, H., The First Integral Method for Wu-Zhang System with Conformable Time-Fractional Derivative, *Calcolo*, *53* (2016), 3, pp. 475-485
[34] Cenesiz, Y., *et al*., New Exact Solutions of Burgers' Type Equations with Conformable Derivative, *Waves Random Complex Media*, *27* (2017), 1, pp. 103-116
[35] Eslami, M., *et al*., The First Integral Method Applied to the Bogoyavlenskii Equations by Means of Conformable Fractional Derivative, *Opt. Quant. Electron*., *49* (2017), 12, p. 391
[36] Rezazadeh, H., *et al*., Mitigating Internet Bottleneck with Fractional Temporal Evolution of Optical Solitons Having Quadratic-Cubic Non-Linearity, *Optik*, *164* (2018), July, pp. 84-92
[37] Rezazadeh, H., *et al*., Traveling Wave Solutions for Density-Dependent Conformable Fractional Diffusion-Reaction Equation by the Rst Integral Method and the Improved Tan(3c6(3be)/2)-Expansion method, *Opt. Quant. Electron*., *50* (2018), 3, p. 12
[38] Hosseini, K., *et al*., Bright and Singular Soliton Solutions of the Conformable Time-Fractional Klein-Gordon Equations with Different Non-Linearities, *Waves Random Complex Media*, *28* (2018), 3, pp. 426-434

Paper submitted: January 8, 2019
Paper revised: June 20, 2019
Paper accepted: July 10, 2019