Haar wavelets scheme for solving the unsteady gas flow in four-dimensional

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Abstract

The system of unsteady gas flow of four-dimensions is solved successfully by alter the possibility of an algorithm based on collocation points and four-dimensional Haar wavelet method (HWM). Empirical rates of convergence of the HWM are calculated which agree with theoretical results. To exhibit the efficiency of the strategy, the numerical solutions which are acquired utilizing the recommended strategy demonstrate that numerical solutions are in a decent fortuitous event with the exact solutions.

Keywords: Haar wavelets, a system of four-dimensions unsteady gas flow, Stability analysis, Error analysis.

1. Introduction

Haar wavelets are the most basic ones which are defined by an investigative articulation. Because of their straightforwardness, the HWM is extremely powerful instruments for approximating arrangements of partial differential equations. The wavelets are utilized as a scientific device for taking care of many classes of equations in biology, physics, fluid mechanics and chemical reactions. This technique comprises of diminishing the issue to a lot of arithmetical conditions by growing the term which has the greatest subsidiary, given in the condition as Haar wavelets with obscure coefficients.

The solution of the system of four-dimensional unsteady gas flow problem under the reasonable initial condition is an essential field of study. The solutions of the unsteady gas flow are contemplated in writing and are explained by the different strategies [1-6]. A standout amongst the most amazing strategies to decide solutions for nonlinear PDEs is the HWM [7– 12]. Hence, utilizing this technique over and over and with the assistance of closeness factors, we can lessen the arrangement of PDEs to an arrangement of ODEs, which is by and large nonlinear. Now and again, it is conceivable to fathom these ODEs to decide the estimate arrangements; nonetheless, much of the time the ODEs must be illuminated numerically. Utilizations of this technique for temperamental one-dimensional issues might be found in [13]. In [14] they built up a new homotopy perturbation strategy (NHPM) to get arrangements of the frameworks of nonlinear partial differential equations (NPDE). In [15] they proposed another homotopy analysis scheme to obtain solutions of the systems of NPDE. In [16] scheme of (RDTM) is used to the systems of NPDE and there are many methods for solving systems of partial differential equations [17-24].

In this paper, we extend the Haar wavelets scheme to solve the unsteady gas flow in four-dimensions then, analysis of the increase or decrease velocity components throughout the increment of the adiabatic index.

The paper is masterminded as pursues; Partition 2 is dedicated to describing an algorithm for the unsteady gas flow in four-dimensions. Partition 3 is dedicated to the Formulas for calculating the four-dimensional Haar wavelets scheme. In partition 4; The proposed scheme is presented for solving the unsteady gas flow in four-dimensions. In partition 5; we apply the new scheme for solving a four-dimensional system of the unsteady gas flow.

2. Model formulation of the problem

The governing equations describing the unsteady gas flow in four-dimensions are formulated from the general Navier–Stokes equations and Raja et al. work [25] on the following form;

$$\begin{aligned}
&\frac{\partial \rho}{\partial t}+\rho\left(\frac{\partial L}{\partial x}+\frac{\partial M}{\partial y}+\frac{\partial N}{\partial z}\right)+L\frac{\partial \rho}{\partial x}+M\frac{\partial \rho}{\partial y}+N\frac{\partial \rho}{\partial z}=0\\
&\frac{\partial L}{\partial t}+L\frac{\partial L}{\partial x}+M\frac{\partial L}{\partial y}+N\frac{\partial L}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial x}=0\\
&\frac{\partial M}{\partial t}+L\frac{\partial M}{\partial x}+M\frac{\partial M}{\partial y}+N\frac{\partial M}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial y}=0\\
&\frac{\partial N}{\partial t}+M\frac{\partial N}{\partial x}+N\frac{\partial N}{\partial y}+N\frac{\partial N}{\partial z}+\frac{1}{\rho}\frac{\partial P}{\partial z}=0\\
&\frac{\partial P}{\partial t}+\gamma P\left(\frac{\partial L}{\partial x}+\frac{\partial M}{\partial y}+\frac{\partial N}{\partial z}\right)+L\frac{\partial P}{\partial x}+M\frac{\partial P}{\partial y}+N\frac{\partial P}{\partial z}=0
\end{aligned} \tag{3}$$

where x, y, z are the space coordinates and t is the time, P the pressure, ρ is the density, γ is the adiabatic index and L, M and N the velocity components in the x, y and z directions, respectively.

There are a few endeavors to explain frameworks of nonlinear partial differential equations.

3. Haar wavelet method

Haar wavelet is a successful instrument to tackle many issues emerging in numerous regions of sciences. Usually, the Haar wavelets are defined for the interval $x \in [0,1]$ however in general case $x \in [a, b]$, one can divide the interval into m equal subintervals each of width $\Delta x = \frac{(b-a)}{m}$. The Haar wavelets family $\{h_i(x)\}$ is defined as a gathering of symmetrical square waves with greatness ± 1 in some intervals and zero elsewhere as follows:

$$h_i(x) = \begin{cases} 1, & \alpha < x < \beta \\ -1, & \beta < x < \chi \\ o & elsewhere \end{cases} \tag{3}$$

Where

$$\alpha = \frac{k}{m}, \beta = \frac{k+0.5}{m}, \text{ and } \chi = \frac{k+1}{m} \tag{4}$$

The integer $m = 2^j \ where\ j = 0,1,2, \dots, J,$ denote the wavelet level, and $k = 0,1,2, \dots, m-1$ is denote the translation parameter. Resolution level is known as the integer J. The index i is established according to the formula $i = m + k + 1$. In case of the values $m = 1$, $k = 0$. we own $i = 2$; The value of i in is $i = 2M = 2^{J+1}$. So, the integrable function $f(x)$ characterized on $[0,1)$ as a finite sum as: $f(x) = \sum_{i=1}^{N} a_i h_i(x)$.

To solve partial differential equations of any order, we need the following integrals

$$P_{i,1}(t) = \int_0^t h_i(T)\, dT \quad (5)$$

$$P_{i,n+1}(t) = \int_0^t P_{i,n}(T)\, dT, \quad n = 1,2, \ldots. \quad (6)$$

Using Eqs. (3), (5) and (6) we have

$$P_{i,1}(t) = \begin{cases} t-\alpha, & t\epsilon[\alpha,\beta) \\ \chi - t, & t\epsilon[\beta,\chi) \\ o & elsewhere \end{cases} \quad (7)$$

$$P_{i,2}(t) = \begin{cases} \frac{1}{2}(t-\alpha)^2, & t\epsilon[\alpha,\beta), \\ \frac{1}{4m^2} - \frac{1}{2}(\chi - t)^2, & t\epsilon[\beta,\chi), \\ \frac{1}{4m^2}, & t\epsilon[\chi,1), \\ o, & elsewhere. \end{cases} \quad (8)$$

$$P_{i,3}(t) = \begin{cases} \frac{1}{6}(t-\alpha)^3, & t\epsilon[\alpha,\beta), \\ \frac{1}{4m^2}(t-\beta) - \frac{1}{6}(\chi - t)^3, & t\epsilon[\beta,\chi), \\ \frac{1}{4m^2}(t-\beta), & t\epsilon[\chi,1), \\ o, & elsewhere\,. \end{cases} \quad (9)$$

$$P_{i,4}(t) = \begin{cases} \frac{1}{24}(t-\alpha)^4, & t\epsilon[\alpha,\beta), \\ \frac{1}{8m^2}(t-\beta)^2 - \frac{1}{24}(\chi - x)^4 + \frac{1}{192m^4}, & t\epsilon[\beta,\chi), \\ \frac{1}{8m^2}(t-\beta)^2 + \frac{1}{192m^4}, & t\epsilon[\chi,1), \\ o, & elsewhere. \end{cases} \quad (10)$$

$$P_{i,n}(t) = \begin{cases} \frac{1}{n!}(t-\alpha)^n, & t\epsilon[\alpha,\beta), \\ \frac{1}{n!}(t-\alpha)^n - \frac{2}{n!}(t-\beta)^n, & t\epsilon[\beta,\chi), \\ \frac{1}{n!}(t-\alpha)^n - \frac{2}{n!}(t-\beta)^n - \frac{1}{n!}(t-\chi)^n, & t\epsilon[\chi,1), \\ o, & elsewhere. \end{cases} \quad (11)$$

The quasilinearization process [26] is a popularize Newton–Raphson method for functional equations which converges quadratically to the exact solution.

Consider the nonlinear mth order differential equation

$$L^m v(x) = f(v(x), v'(x), \ldots, v^{(m-1)}(x), x)$$

where n is the order of the differentiation,stratifying the quasilinearization technique to the above equation yields

$$L^m v_{i+1}(x) = f(v_i(x), v_i'(x), \dots, v_i^{(m-1)}(x), x) + \sum_{j=0}^{m-1} (v_{i+1}^{(j)}(x) - v_i^{(j)}(x)) f_{v(j)}(v_i(x), v_i'(x), \dots, v_i^{(m-1)}(x), x)$$

a linear differential equation and can be solved periodically, where $v_i(x)$ is known and one can utilize it to gain $v_i(x)$ for i = $0, 1, \dots$ ..with the initial conditions and boundary conditions at $(i + 1)$ th iteration.

4. Modification of Haar Wavelet scheme

We describe a new modification of the HWM for solving systems of nonlinear partial differential equations with help of the initial and boundary conditions and the results are displayed graphically for different value of the adiabatic index. It is known that any integrable function

$$L(x, y, z, t) \in L^4([0,1) \times [0,1) \times [0,1) \times [0,1)),$$
$$M(x, y, z, t) \in L^4([0,1) \times [0,1) \times [0,1) \times [0,1)),$$
$$N(x, y, z, t) \in L^4\big([0,1) \times [0,1) \times [0,1) \times [0,1)\big),$$
$$Q(x, y, z, t) \in L^4([0,1) \times [0,1) \times [0,1) \times [0,1)) \text{ and}$$
$$P(x, y, z, t) \in L^4([0,1) \times [0,1) \times [0,1) \times [0,1)).$$

can be expanded by a Haar series with an infinite number of terms

$$L(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=1}^{\infty} a_{i,j,k,s}\, h_i(x) h_j(y) h_k(z)\, h_s(t),$$
$$M(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=1}^{\infty} b_{i,j,k,s}\, h_i(x) h_j(y) h_k(z)\, h_s(t),$$
$$N(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=1}^{\infty} c_{i,j,k,s}\, h_i(x) h_j(y) h_k(z) h_s(t),$$
$$Q(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=1}^{\infty} d_{i,j,k,s}\, h_i(x) h_j(y) h_k(z) h_s(t),$$
$$P(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \sum_{s=1}^{\infty} e_{i,j,k,s}\, h_i(x) h_j(y) h_k(z)\, h_s(t).$$

We can be expressed as $W_{2M\times 2M}(yz)$=$H_{2M}(y)\, H_{2M}^T(z)$, The above series terminate at finite terms if $L(x, y, z, t)$, $M(x, y, z, t)$, $N(x, y, z, t)$ and $Q(x, y, z, t)$are piecewise constant functions or can be approximated as piecewise constant functions during each subinterval, then $L(x, y, z, t)$, $M(x, y, z, t)$, $N(x, y, z, t)$ and $Q(x, y, z, t)$will be terminated at finite terms, i.e.,

$$L(x, y, z, t) = \sum_{i=1}^{2M} \sum_{j=1}^{2M} \sum_{k=1}^{2M} \sum_{s=1}^{2M} a_{i,j,k,s}\, h_i(x) h_j(y) h_k(z)\, h_s(t)$$

$$= H_{2M}^T(x)\, A_{2M\times 2M} W_{2M\times 2M}(yz)\, H_{2M}(t) \quad (12)$$

$$M(x,y,z,t) = \sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M} b_{i,j,k,s}\, h_i(x)h_j(y)h_k(z)\, h_s(t)$$

$$= H_{2M}^T(x)\, B_{2M\times 2M} W_{2M\times 2M}(yz)\, H_{2M}(t) \quad (13)$$

$$N(x,y,z,t) = \sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M} c_{i,j,k,s}\, h_i(x)h_j(y)h_k(z)h_s(t)$$

$$= H_{2M}^T(x)\, C_{2M\times 2M} W_{2M\times 2M}(yz)\, H_{2M}(t) \quad (14)$$

$$\rho(x,y,z,t) = \sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M} d_{i,j,k,s}\, h_i(x)h_j(y)h_k(z)h_s(t)$$

$$= H_{2M}^T(x)\, D_{2M\times 2M} W_{2M\times 2M}(yz)\, H_{2M}(t) \quad (15)$$

$$P(x,y,z,t) = \sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M} e_{i,j,k,s}\, h_i(x)h_j(y)h_k(z)h_s(t)$$

$$= H_{2M}^T(x)\, E_{2M\times 2M} W_{2M\times 2M}(yz)\, H_{2M}(t) \quad (16)$$

where the coefficients $A_{2M\times 2M}$, $B_{2M\times 2M}$, $C_{2M\times 2M}$, $D_{2M\times 2M}$ and the Haar function vectors $H_{2M}^T(x)$, $H_{2M}(y)$ are defined as:

$H_{2M}^T(x) = [h_1(x), h_2(x), \dots, h_{2M}(x)]$, $H_{(2M)}(y) = [h_1(y), h_2(y), \dots, h_{2M}(y)]^T$

$H_{2M}^T(z) = [h_1(z), h_2(z), \dots, h_{2M}(z)]$, $H_{(2M)}(t) = [h_1(t), h_2(t), \dots, h_{2M}(t)]^T$

$W_{2M\times 2M}(yz)=H_{2M}(y)\, H_{2M}^T(z)$, $A_{(2M\times 2M)} = (a_{i,j,k,s})_{2M\times 2M}$,

$B_{(2M\times 2M)} = (b_{i,j,k,s})_{2M\times 2M}$, $C_{(2M\times 2M)} = (c_{i,j,k,s})_{2M\times 2M}$,

and $D_{(2M\times 2M)} = (d_{i,j,k,s})_{2M\times 2M}$.

We apply the modified HWM to solve the four-dimensional system of nonlinear partial differential Eq. (1) with the initial and boundary conditions

$L(x,y,z,0) = f_1(x,y,z)$, $L(x,y,0,t) = s_1(x,y,t)$, $L(x,0,z,t) = w_1(x,z,t)$,

$L(0,y,z,t) = r_1(y,z,t)$, $L(0,0,0,t) = e_1(t)$. (17)

$M(x,y,z,0) = f_2(x,y,z)$, $M(x,y,0,t) = s_2(x,y,t)$, $M(x,0,z,t) = w_2(x,z,t)$,

$M(0,y,z,t) = r_2(y,z,t)$ $M(0,0,0,t) = e_2(t)$ (18)

$N(x,y,z,0) = f_3(x,y,z)$, $N(x,y,0,t) = s_3(x,y,t)$, $N(x,0,z,t) = w_3(x,z,t)$,

$N(0,y,z,t) = r_3(y,z,t)$ $N(0,0,0,t) = e_3(t)$. (19)

$\rho(x,y,z,0) = f_4(x,y,z)$, $\rho(x,y,0,t) = s_4(x,y,t)$, $\rho(x,0,z,t) = w_4(x,z,t)$,

$\rho(0,y,z,t) = r_4(y,z,t)$ $\rho(0,0,0,t) = e_4(t)$. (20)

$P(x,y,z,0) = f_5(x,y,z)$, $P(x,y,0,t) = s_5(x,y,t)$, $P(x,0,z,t) = w_5(x,z,t)$,

$P(0,y,z,t)=r_5(y,z,t) \qquad\qquad P(0,0,0,t)=e_5(t).$ (21)

where the above functions are got from the exact solution in [2] and $0\le x\le 1,\ 0\le y\le 1,\ 0\le z\le 1$, and $0\le t\le 1$.

First, we consider the first equation of system (1), with the initial and boundary conditions (17). We assume that $\dot{L}'^{*\bullet}(x,y,z,t)$ can be expanded in terms of Haar wavelets as:

$$\dot{L}'^{*\bullet}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,h_i(x)h_j(y)h_k(z)\,h_s(t) \tag{22}$$

where dot, prime, star and closed circle mean differentiation with respect to t, x, y and z, respectively. We integrate (22) one time with respect to t on $[0,t]$, then one time with respect to x on $[0,x]$, one time with respect to y on $[0,y]$, and finally one time with respect to z on $[0,z]$. These, respectively, yield

$$L'^{*\bullet}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)h_i(x)h_j(y)h_k(z)+L'^{*\bullet}(x,y,z,0)- \\ -L'^{*\bullet}(x,y,z,t) \tag{23}$$

$$L^{*\bullet}(x,y,z,t)= \\ (t-t_s)\ \sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)\,p_{i,1}(x)h_j(y)h_k(z)+ \\ L^{*\bullet}(x,y,z,0)-L^{*\bullet}(x,y,z,t)+L^{*\bullet}(0,y,z,0)-L^{*\bullet}(0,y,z,t), \tag{24}$$

$$L^{\bullet}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)p_{i,1}(x)\,p_{j,1}(y)h_k(z)+L^{\bullet}(x,y,z,0)- \\ L^{\bullet}(x,y,z,t)+L^{\bullet}(x,0,z,0)-L^{\bullet}(x,0,z,t)+L^{\bullet}(0,y,z,0)-L^{\bullet}(0,y,z,t)+ \\ L^{\bullet}(0,0,z,0)-L^{\bullet}(0,0,z,t). \tag{25}$$

$$L(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)\,p_{i,1}(x)p_{j,1}(y)p_{k,1}(z)+L(x,y,z,0)-L(x,y,z,t) \\ +L(0,y,z,0)-L(0,y,z,t)+L(x,0,z,0)-L(x,0,z,t)+L(x,y,0,0) \\ -L(x,y,0,t)+L(0,0,0,0)-L(0,0,0,t) \tag{26}$$

Now, individual differentiation of (26) with respect to t, x, y and z, separately, yield

$$\dot{L}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,h_i(t)\,p_{i,1}(x)p_{j,1}(y)p_{k,1}(z)-\dot{L}(x,y,z,t)- \\ \dot{L}(0,y,z,t)-\dot{L}(x,0,z,t)-\dot{L}(x,y,0,t)-\dot{L}(0,0,0,t) \tag{27}$$

$$L'(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)\,h_{i,1}(x)p_{j,1}(y)p_{k,1}(z)+L'(x,y,z,0)- \\ L'(x,y,z,t)+L'(x,0,z,0)-L'(x,0,z,t)+L'(x,y,0,0)-L'(x,y,0,t) \tag{28}$$

$$L^{*}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)\,p_{i,1}(x)h_i(y)p_{k,1}(z)+L^{*}(x,y,z,0)- \\ L^{*}(x,y,z,t)+L^{*}(0,y,z,0)-L^{*}(0,y,z,t)+L^{*}(x,y,0,0)-L^{*}(x,y,0,t) \tag{29}$$

$$L^{\bullet}(x,y,z,t)=\sum_{i=1}^{2M}\sum_{j=1}^{2M}\sum_{k=1}^{2M}\sum_{s=1}^{2M}a_{i,j,k,s}\,p_{s,1}(t)\,p_{i,1}(x)p_{j,1}(y)h_i(z)+L^{\bullet}(x,y,z,0)- \\ L^{\bullet}(x,y,z,t)+L^{\bullet}(0,y,z,0)-L^{\bullet}(0,y,z,t)+L^{\bullet}(x,0,z,0)-L^{\bullet}(x,0,z,t) \tag{30}$$

There are a few conceivable outcomes for treating the nonlinearity in Eq. (1). But, here the quasi-linearization procedure [26] is utilized to handle the nonlinearity in Eq. (1). The system of (1) trailed by the quasi-linearization prompts to,

$$\dot{\rho}(x,y,z,t)+\rho(x,y,z,t)\big(L'(x,y,z,t)+M^{*}(x,y,z,t)+L^{\bullet}(x,y,z,t)\big)+$$
$$L(x,y,z,t)\,\rho'(x,y,z,t)+M(x,y,z,t)\,\rho^{*}(x,y,z,t)+N(x,y,z,t)\,\rho^{\bullet}(x,y,z,t)=0$$

$$\dot{L}(x,y,z,t)+L(x,y,z,t)L'(x,y,z,t)+M(x,y,z,t)L^{*}(x,y,z,t)+$$
$$N(x,y,z,t)L^{\bullet}(x,y,z,t)+\frac{1}{\rho}p'(x,y,z,t)=0$$
$$\dot{M}(x,y,z,t)+L(x,y,z,t)\,M'(x,y,z,t)+M(x,y,z,t)\,M^{*}(x,y,z,t)+$$
$$N(x,y,z,t)\,M^{\bullet}(x,y,z,t)+\frac{1}{\rho(x,y,z,t)}\,P^{*}(x,y,z,t)=0$$
$$\dot{N}(x,y,z,t)+M(x,y,z,t)\,N'(x,y,z,t)+N(x,y,z,t)\,N^{*}(x,y,z,t)+$$
$$N(x,y,z,t)N^{\bullet}(x,y,z,t)+\frac{1}{\rho(x,y,z,t)}P^{\bullet}(x,y,z,t)=0$$
$$\dot{P}(x,y,z,t)+\gamma P(x,y,z,t)\big(L'(x,y,z,t)+M^{*}(x,y,z,t)+N^{\bullet}(x,y,z,t)\big)+$$
$$L(x,y,z,t)\,P'(x,y,z,t)+M(x,y,z,t)P^{*}(x,y,z,t)+N(x,y,z,t)P^{\bullet}(x,y,z,t)=0 \quad (31)$$

Now, discretizing the result (31) by $x\to x_l, y\to y_l, z\to z_l$, $t\to t_l$ and using Eqs. (27)–(30) and discretizing using the collocation points $x_l=y_l=z_l=t_l=\frac{l-0.5}{2M}, l=1..2M$ yield a nonlinear system of algebraic equations, with the initial and boundary conditions and the wavelets coefficients $a_{i,j,k,s}$, $b_{i,j,k,s}, c_{i,j,k,s}$, $d_{i,j,k,s}$ and $e_{i,j,k,s}$ can be successively calculated for all i,j,k and s. Further, putting the computed wavelets coefficients $a_{i,j,k,s}$, $b_{i,j,k,s}, c_{i,j,k,s}$, $d_{i,j,k,s}$ and $e_{i,j,k,s}$ into Eqs. (12-16), we can successively calculate the approximate solutions at different times.

5. Discussion of the results

we present the numerical solutions of the system of unsteady gas flow in four-dimensional (1), The analytical solutions are taken from [2] as:

$$L(x,y,z,t)=\frac{-\sqrt{2c_3-c_4^2}y+c_4x+c_3t\,x}{c_3t^2+2\,c_4\,t+2},\qquad M(x,y,z,t)=\frac{\sqrt{2c_3-c_4^2}x+c_4y+c_3t\,y}{c_3t^2+2\,c_4\,t+2}$$

$$N(x,y,z,t)=\frac{z+c_2+tan^{-1}\left(\frac{y}{x}\right)-tan^{-1}\left(\frac{c_3\,t+c_4}{\sqrt{2c_3-c_4^2}}\right)}{t+1},\qquad Q(x,y,z,t)=\frac{c_5}{(t+1)\,(\,y^2+x^2)}$$

$$P(x,y,z,t)=\frac{C_1}{(t+1)^{\gamma}(C_3\,t^2+2C_4t+2)^{\gamma}}$$

In figs. 1, 2 and 3 show representations of velocity component profiles $(L, M\ and\ N)$ respectively indicating a decay in velocity during time increment and an increase with a spatial direction x, $y\ and$ increment respectively. The largest velocity component was achieved by $M\ and\ N$ while the largest increase with a spatial variable was achieved by L.

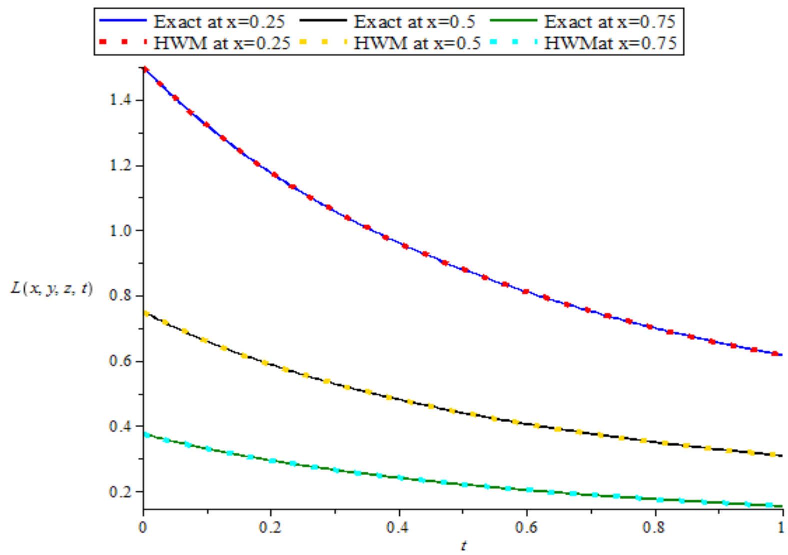

Fig. 1. Comparison of the velocity component (L) at $C_3 = C_4 = 1\ at\ y = 0$ by HWM and the analytical solutions in [2].

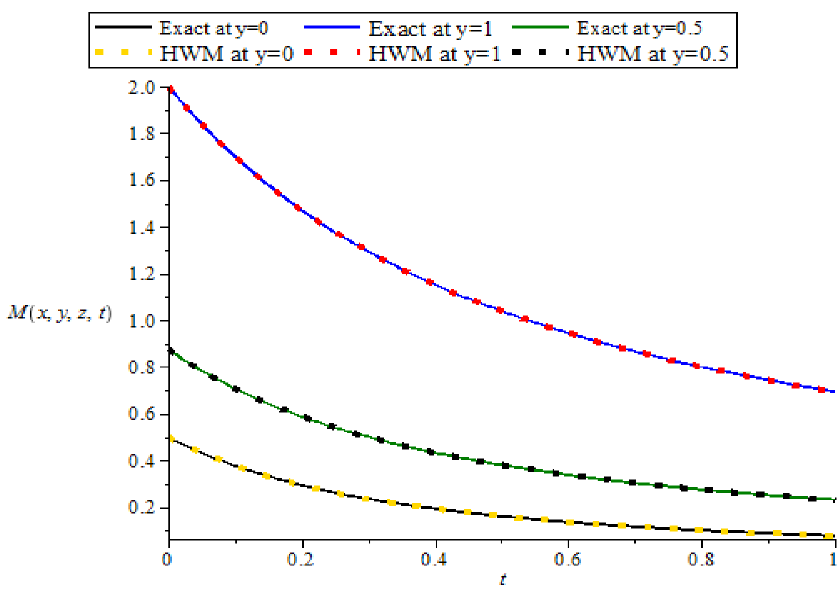

Fig. 2. Comparison of the velocity component (M) plot at $C_3 = C_4 = 1\ at\ y = 0$ by HWM and the analytical solutions in [2].

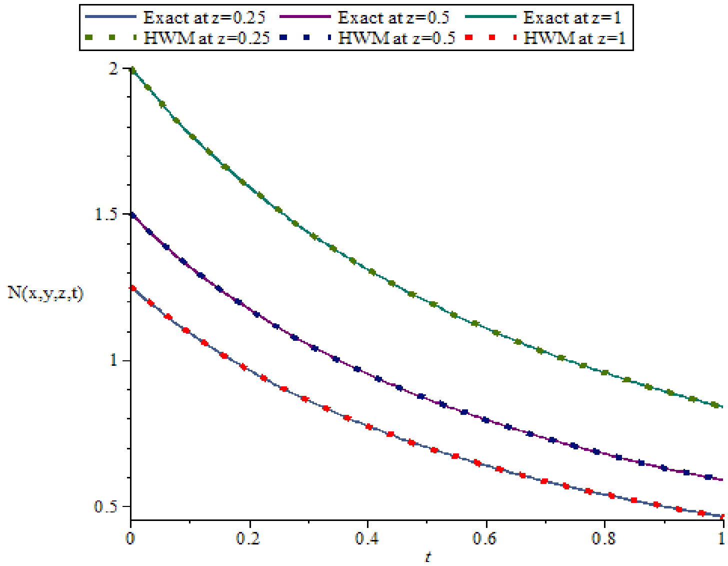

Fig. 3. Comparison of the velocity component (N) plot at $C_3 = C_4 = 1$ at $y = 0$ by HWM and the analytical solutions in [2].

Fig. 3 illustrates velocity profile N showing effect of velocity value with increasing the time. The effect of an adiabatic index is illustrated in Fig. 5. In Fig. 4, showing effect of with increasing x. Fig. 5 showing the effect adiabatic index and this is just as it exists in reference [2].

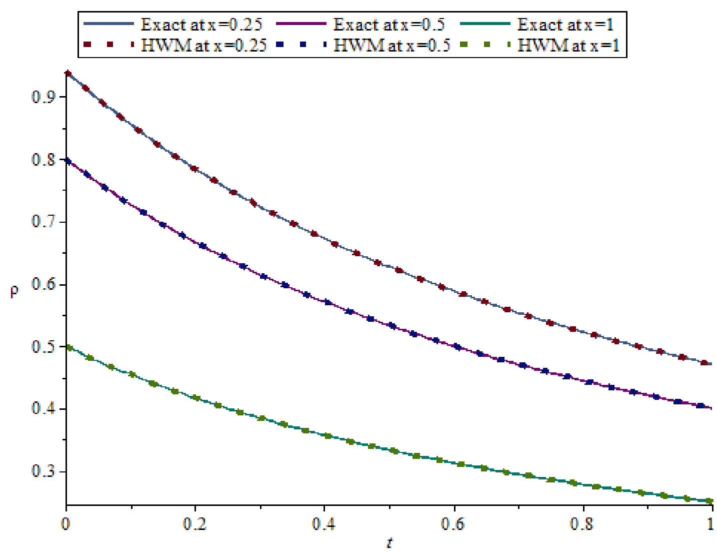

Fig. 4. Comparison of the density compilation at $C_5 = 1$ by HWM and the dual vector combination analytical solutions in [2].

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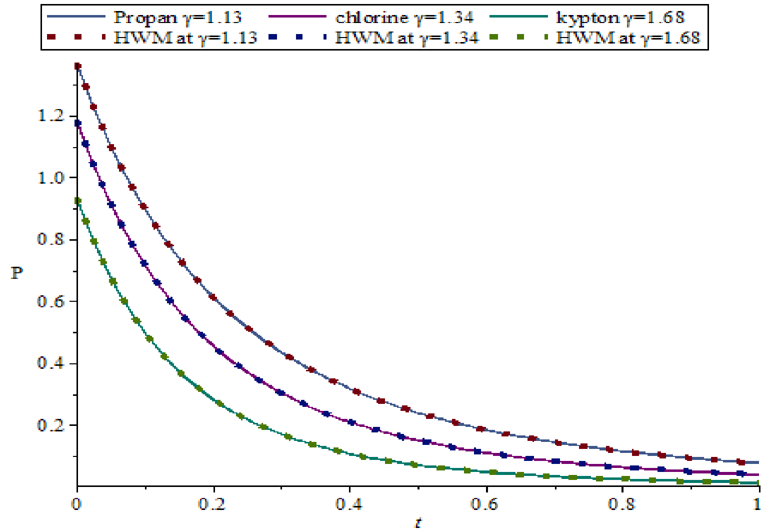

Fig. 5. Comparison of the Pressure compilation at $C_1 = C_2 = C_3 = C_4 = C_5 = 1$ at $\gamma = 1.13$, $\gamma = 1.34$, $\gamma = 1.68$ by HWM and the analytical solutions in [2].

In table1 show the comparison of approximate solutions of the four-dimensional system of the unsteady gas flow (1) obtained by using the modified Haar wavelet method with the analytical solutions in [2] at $(x, y, z) = (1,1,1)$.

Table1 Comparison between the approximate solutions using HWM at $M = 4$and the analytical solutions in [2].

t	$Absolute$ $Errors\ of\ \ L$	$Absolute$ $Errors\ of\ M.$	$Absolute$ $Errors\ of\ N.$	$Absolute$ $Errors\ of\ P.$	$Absolute$ $Errors\ of\ \rho.$
0.1	2.16×10^{-6}	6.322×10^{-6}	1.201×10^{-6}	3.025×10^{-6}	3.321×10^{-6}
0.2	1.065×10^{-6}	5.102×10^{-6}	4.302×10^{-6}	4.156×10^{-6}	6.254×10^{-6}
0.3	3.102×10^{-6}	4.021×10^{-6}	4.142×10^{-6}	2.216×10^{-6}	4.358×10^{-6}
0.4	4.015×10^{-6}	3.250×10^{-6}	3.306×10^{-6}	1.541×10^{-6}	6.024×10^{-6}
0.5	3.025×10^{-6}	4.203×10^{-6}	1.512×10^{-6}	1.487×10^{-6}	4.254×10^{-6}
0.6	2.96×10^{-6}	3.021×10^{-6}	6.325×10^{-6}	6.241×10^{-6}	3.652×10^{-6}
0.7	3.21×10^{-6}	2.302×10^{-6}	1.241×10^{-6}	4.212×10^{-6}	3.201×10^{-6}
0.8	1.36×10^{-5}	6.215×10^{-5}	2.021×10^{-5}	5.275×10^{-5}	4.021×10^{-5}
0.9	2.25×10^{-5}	4.302×10^{-5}	6.045×10^{-5}	3.214×10^{-5}	2.541×10^{-5}

6. Conclusion

In the perspective on above numerical precedents, it is presumed that four-dimensional Haar wavelet technique are progressively solid and precise scientific device for settling the unsteady gas flow in four-dimensional. For getting the vital accuracy, the quantity of estimation focuses might be expanded. This technique is totally another plan to unravel the unsteady gas flow in four-dimensional. It is a joined methodology and a novel intermingling hypothesis is introduced. It is a stable numerical technique and its strength has been appeared. Setting up the calculation is simple and straight forward. The merit of this strategy is that the maximum absolute errors are diminished by expanding the quantity of collocation points. The proposed plan yields better exactness in examination with the other numerical techniques which are exhibited in [27,28] accessible in the writing. Calculation can be stretched out to comprehend other frameworks of higher dimensional issues in various regions of physical and numerical sciences.

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