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**COMMENTS ON THE PAPER "IMPACT OF TEMPERATURE  
 DEPENDENT HEAT SOURCE AND NON-LINEAR RADIATIVE FLOW  
 OF THIRD GRADE FLUID WITH CHEMICAL ASPECTS"**

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*The current discussion is presented to increase the awareness among the readers of the journal Thermal Science. Comments are presented in particular on the paper by Hayat et al. [1] where the authors investigated effect of non-linear radiative flow of third grade fluid and temperature dependent heat source with chemical aspects. The current discussion concerns some questionable results included in the aforementioned paper.*

In this study, comments in particular on the paper by Hayat *et al.* [1], are presented to increase the awareness among the readers of the journal *Thermal Science*. Details of these comments are given.

Hayat *et al.* [1] investigated effect of non-linear radiative flow of third grade fluid and temperature dependent heat source with chemical aspects. The researchers modeled mathematically the physical problem and obtained non-linear system of PDE. Then, they used transformations in order to obtain non-linear system of ODE. In the aforementioned paper, the transformed equations, which have been solved, eqs. (13)-(16) in Hayat *et al.* [1] are:

$$f''' + ff'' - f'^2 + \beta_1(2f'''f' - ff''') + (3\beta_1 + 2\beta_2)f''^2 + 6\varepsilon_1\varepsilon_2f'''f''^2 - M^2f' = 0 \quad (1)$$

$$\left(1 + \frac{4}{3}Rd\right)\theta'' + \frac{4}{3}Rd\left[(\theta_w - 1)^3(3\theta'^2\theta^2 + \theta^3\theta'') + 3(\theta_w - 1)^2(2\theta'^2\theta + \theta^2\theta'') + 3(\theta_w - 1)^3(\theta'^2 + \theta\theta'')\right] + \text{Pr}f\theta' + \text{Pr}\delta\exp(-\eta) = 0 \quad (2)$$

$$\frac{1}{\text{Sc}}g'' + fg' - K_1gh^2 = 0 \quad (3)$$

$$\frac{\delta}{Sc} h'' + fh' + K_1 gh^2 = 0 \quad (4)$$

In eq. (1), the local Reynolds number,  $\varepsilon_2$ , is defined:

$$\varepsilon_2 = \frac{cx^2}{\nu} \quad (5)$$

Taking into account that the local Reynolds number is a function of co-ordinate  $x$ , it is concluded that the previous eq. (1) is also a function of co-ordinate  $x$  and therefore the problem treated in Hayat *et al.* [1] is non-similar. However, Hayat *et al.* [1] disregarded this fact and dealt with the problem as similar.

In contrast to similar problems, the basic flow quantities in non-similar problems change along the streamwise direction. The following eqs. (6) and (7) have been taken from Minkowycz and Cheng [2] and represent a non-similar problem:

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{f}{2} \frac{\partial \theta}{\partial \eta} = \frac{\xi}{2} \left( \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial f}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right) \quad (6)$$

where the parameter  $\xi$  is a function of  $x$ :

$$\xi = \frac{2\nu_m}{a} \sqrt{\frac{\mu ax}{\rho_\infty g \beta K (T_w - T_\infty)}} \quad (7)$$

In eq. (6), there are derivatives in the streamwise direction ( $\partial f / \partial \xi$ ,  $\partial \theta / \partial \xi$ ) that are not present in the eqs. (1)-(4). The local similarity procedure is incorrect according to the following quotation from Minkowycz and Sparrow [3].

*By deleting the terms involving  $\partial f / \partial \xi$  and  $\partial \theta / \partial \xi$  the computational task is simplified since the resulting equations are, in effect, ODE. In addition, the streamwise coupling is severed so that locally autonomous solutions may be obtained. This approach, which is often designated as local similarity, is computationally attractive but leads to results of uncertain accuracy.*

For the non-similar problems solution, Minkowycz and his co-workers used the local non-similarity procedure in three steps (1<sup>st</sup> truncation, 2<sup>nd</sup> truncation, and 3<sup>rd</sup> truncation). Only the first step was used in the study of Hayat *et al.* [1].

Another way in treating a non-similar problem is the space marching method. In this procedure, the initial equations, after a non-dimensionalization are solved sequentially from upstream to downstream locations starting from the leading edge. This procedure has been used by Pantokratoras [4-7] and Capobianchi and Aziz [8].

It is correct that there are several publications in the literature, especially by mathematicians, where the local similarity procedure is used. However, all these publications are of unconfident accuracy.

Taking into account all the mentioned results presented in the study of Hayat *et al.* [1] is questionable.

In conclusion, Hayat *et al.* [1] treated the problem as similar whereas the problem is non-similar.

## References

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