COMMENTS ON "IMPACT OF TEMPERATURE DEPENDENT HEAT SOURCE AND NONLINEAR RADIATIVE FLOW OF THIRD GRADE FLUID WITH CHEMICAL ASPECTS"

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Abstract: The current discussion is presented to increase the awareness among the readers of Thermal Science. Comments are presented in particular on the paper by Hayat et al. [1] where the authors investigated effect of nonlinear radiative flow of third grade fluid and temperature dependent heat source with chemical aspects. The current discussion concerns some questionable results included in the above paper.

In this study, comments in particular on the paper by Hayat et al. [1], are presented to increase the awareness among the readers of Thermal Science. Details of these comments are given below.

Hayat et al. [1] investigated effect of nonlinear radiative flow of third grade fluid and temperature dependent heat source with chemical aspects. The researchers modeled mathematically the physical problem and obtained nonlinear system of partial differential equations (PDFs). Then, they used transformations in order to obtain nonlinear system of ordinary differential equations (ODFs). In the above paper, the transformed equations, which have been solved (Eqs. (13)-(16) in Hayat et al. [1]) are as follows:

$$f''' + ff'' - f'^2 + \beta_1\left(2f'''f' - ff''''\right) + \left(3\beta_1 + 2\beta_2\right)f''^2 + 6\varepsilon_1\varepsilon_2 f''' f''^2 - M^2 f' = 0 \tag{1}$$

$$\left(1+\frac{4}{3}Rd\right)\theta'' + \frac{4}{3}Rd\left[(\theta_w-1)^3\left(3\theta'^2\theta^2+\theta^3\theta''\right)+3(\theta_w-1)^2\left(2\theta'^2\theta+\theta^2\theta''\right)+3(\theta_w-1)^3\left(\theta'^2+\theta\theta''\right)\right] \\ +\Pr f\theta' + \Pr\delta\exp(-\eta) = 0 \tag{2}$$

$$\frac{1}{Sc}g'' + fg' - K_1 gh^2 = 0 \tag{3}$$

$$\frac{\delta}{Sc}h'' + fh' + K_1 gh^2 = 0 \tag{4}$$

In Eq. (1), the local Reynolds number (ε_2) is defined as follows:

$$\varepsilon_2 = \frac{cx^2}{\nu} \tag{5}$$

Taking into account that the local Reynolds number (ε_2) is a function of coordinate x, it is concluded that the above Eq. (1) is also a function of coordinate x and therefore the problem treated in Hayat et al. [1] is non-similar. However, Hayat et al. [1] disregarded this fact and dealt with the problem as similar.

In contrast to similar problems, the basic flow quantities in non-similar problems change along the streamwise direction. The following Equations (6) and (7) have been taken from Minkowycz and Cheng [2] and represent a non-similar problem:

$$\frac{\partial^2\theta}{\partial\eta^2} + \frac{f}{2}\frac{\partial\theta}{\partial\eta} = \frac{\xi}{2}\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right) \tag{6}$$

where the parameter ξ is a function of x:

$$\xi = \frac{2\nu_m}{a}\left[\frac{\mu a x}{\rho_\infty g\beta K\left(T_w - T_\infty\right)}\right]^{1/2} \tag{7}$$

In the above Eq. (6), there are derivatives in the streamwise direction ($\partial f/\partial\xi$; $\partial\theta/\partial\xi$) that are not present in the Eqs. (1)-(4). The local similarity procedure is incorrect according to the following quotation from Minkowycz and Sparrow [3].

“By deleting the terms involving $\partial f/\partial \xi$ and $\partial \theta/\partial \xi$ the computational task is simplified since the resulting equations are, in effect, ordinary differential equations. In addition, the streamwise coupling is severed so that locally autonomous solutions may be obtained. This approach, which is often designated as local similarity, is computationally attractive but leads to results of uncertain accuracy”.

For the non-similar problems solution, Mincowycz and his co-workers used the local nonsimilarity procedure in 3 steps (1^{st} truncation, 2^{nd} truncation and 3^{rd} truncation). Only the first step was used in the study of Hayat et al. [1].

Another way in treating a non-similar problem is the space marching method. In this procedure, the initial equations, after a non-dimensionalization are solved sequentially from upstream to downstream locations starting from the leading edge. This procedure has been used by Pantokratoras [4-7] and Capobianchi and Aziz.[8].

It is correct that there are several publications in the literature, especially by mathematicians, where the local similarity procedure is used. However, all these publications are of unconfident accuracy.

Taking into account all the above, the results presented in the study of Hayat et al. [1] are questionable.

In conclusion, Hayat et al. [1] treated the problem as similar whereas the problem is non-similar.

References

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