MICROPOLAR MIXED CONVECTIVE FLOW WITH CATTANEO-CHRISTOV HEAT FLUX: NON-FOURIER HEAT CONDUCTION ANALYSIS

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The purpose of this study is to investigate the impact of thermal relaxation time on the mixed convection flow of non-Newtonian micropolar fluid over a continuously stretching sheet of variable thickness in the presence of transverse magnetic field. An innovative and modified form of Fourier's law, namely, Cattaneo-Christov heat flux is employed in the energy equation to study the characteristics of thermal relaxation time. The governing equations are transformed into ordinary differential equations, using similarity transformations. Fourth order Runge-Kutta numerical method is used to solve these equations. The effects of relevant parameters such as a micro-rotation parameter, magnetic parameter, thermal relaxation parameter, Prandtl number, surface thickness parameter and mixed convection parameter on the physical quantities are graphically presented. Results illustrate that fluid temperature enhances with the rise of thermal relaxation parameter, but it reduces with an increase in micro-rotation parameter. The skin friction decreases with a rise in micro-rotation and micro-element parameters. However, variation in the rate of heat transfer is quite significant for small values of thermal relaxation parameter.

Key words: *Micropolar Fluid; Thermal Relaxation Time; Cattaneo-Christov Heat Flux; Slendering Sheet*

1. Introduction

The concept of micropolar theory in fluid mechanics was first introduced by Eringen [1]. This theory belongs to a class of fluids whose particles can rotate around the center of the volume element. The flow motion of these fluids requires a spin vector and micro-inertia tensor in addition to the velocity vector. Hassanien and Gorla [2] studied micropolar fluid heat transfer over a stretching sheet. The effect of thermal radiation on micropolar fluids is considered by Ishak [3]. The effect of Newtonian heating on micropolar fluid with and without thermal radiation is studied by Hussanan *et* *al*. [4, 5]. Khan and Rashad [6] examined the chemical reaction effect on the micropolar fluid with heat and mass transfer stagnation-point flow over a stretching surface. Free convection effects in micropolar fluid flow in the presence of heat source examined by Mishra *et al*. [7]. On the other hand, convection flow problems of non-Newtonian fluid under magnetic field have got much attention because of its wide applications in petro-chemical industry, MHD generators, blood flow measurements and geophysics as well as power generation system. Ashraf and Batool [8] investigated micropolar fluid flow and heat transfer over a stretchable disk under magnetic field. The influence of magnetic flied on micropolar ferrofluid flow caused by stretching sheet is provided by Khan *et al*. [9]. Exact solutions of MHD micropolar fluid with heat transfer for both stretching and shrinking sheet cases was obtained by Khan *et al*. [10]. Hashemi *et al.* [11] computed the numerical solutions of micropolar nanofluid flow inside a radiative porous medium buoyancy and magnetic field effects. Several researchers including [12-16] have investigated micropolar fluid flow over a stretching/shrinking sheet with or without magnetic field.

All these studies are limited to the flow over stretching sheet or surface with the constant thickness [17-20]. However, in many manufacturing processes, the thickness of surface may or may not be constant. Historically, this idea was first presented by Lee [21]. Wang [22] investigated mixed convection over a heated tip vertical needle for both aiding and opposing flows situation. Ahmad *et al*. [23] also studied the same problem along thin vertical needles with variable heat flux. Fang *et al*. [24] discussed viscous flow over a stretching sheet with variable thickness. Subhashini *et al*. [25] extended the problem of Fang *et al*. [24] by adding thermal diffusive effect. Abdel-Wahed *et al*. [26] investigated the impact of hydromagnetic flow and heat transfer characteristic over variable thickness surface using nanofluid. Acharya *et al*. [27] examined the flow of water-based nanofluid over a thin stretching sheet, and Kumar *et al*. [28] analysed radiative heat transfer Williamson nanofluid flow over a Riga plate with variable thickness. Prasad *et al*. [29] considered the magnetic field effects on nanofluid over a thin elastic sheet with variable thickness. Very recently, Liu and Liu [30] investigated the two-dimensional laminar flow of fractional Maxwell fluid over variable thickness stretching sheet.

Classical Fourier's model for thermal conduction heat transfer is being well known for researchers and has been used for a long time, as cited above. The equation stated for the heat conduction describes a parabolic equation. However, one of the most significant drawbacks for parabolic energy equation is that it may not be accurate for specific situations and hence it contradicts with the principle of causality. Maxwell was the first who provided the alteration of the Fourier's model by considering thermal relaxation time between temperature gradient and heat flux. Cattaneo [31] modified Fourier's model by adding a thermal relaxation fixed time term to represent the thermal inertia. This model is recognized Maxwell-Cattaneo's model in the published data. Christov [32] modified the Maxwell-Cattaneo's model to preserve the material invariant formulation. He replaced the ordinary derivative with the Oldroyd upper-convected derivative in Maxwell-Cattaneo's model. The addition of thermal relaxation time causes heat transportation via propagation of thermal waves with finite speed. After the development of this new model, which is known Cattaneo-Christov heat flux model, several attempts have been made in this direction. Tibullo and Zampoli [33] proved the uniqueness of the solution for an incompressible fluid by using Cattaneo-Christov heat flux model. Mustafa [34] discussed the rotating flow of Cattaneo-Christov heat flux model for viscoelastic fluid bounded by a stretching surface. Rubab and Mustafa [35] proposed Maxwell fluid model based on this heat flux and studied MHD flow over a stretching surface. Cattaneo-Christov heat flux considered by Reddy [36] for micropolar fluid flow over a non-linear convective stretching surface with viscous dissipation. Khan *et al.* [17] gave generalized results for Maxwell fluid flow caused by oscillatory surface along using Cattaneo-Christov theory of heat diffusion. Recently, recognizing the interface effect between carbon nanotubes and engine oil, Kundu *et al.* [38] developed a theoretical model for Maxwell nanofluid along stretching sheet based on Cattaneo-Christov heat flux.

Inspired by survey above, the aim of this study is to investigate the impact of thermal relaxation time on the mixed convection flow of non-Newtonian micropolar fluid over a continuously stretching sheet with variable thickness using Cattaneo-Christov heat flux model. Solutions of the formulated governing equations are obtained numerically by using fourth-order Runge-Kutta method, which has been used in solving of many nonlinear transport problems of the fluid dynamics.

2. Mathematical Model

Consider two-dimensional flow of an incompressible micropolar fluid over an impermeable variable thickness stretching sheet. The x-axis is aligned with the stretching sheet and the y-axis is normal to the sheet. The sheet is stretched with $U_w(x) = U_0(x+b)^m$ along x-axis and its thickness varies as $y = A(x+b)^{\frac{1-m}{2}}$, where b is a parameter related to a sheet, U_0 is the reference velocity, constant A being small so that the sheet is sufficiently thin and m is the velocity power index. Thickness of the sheet depends on m due to the acceleration or deceleration of the sheet. The fluid is electrically conducted and the magnetic field is applied perpendicular to the sheet. Figure 1 shows a sketch of momentum and temperature boundary layers in coordinate system. The oriented anticlockwise rotation of particles is driven by a large velocity gradient within the boundary layer and away from the boundary layer are considered at rest. To explore the heat transfer analysis, Cattaneo-Christov heat flux model is found instead of Fourier's law. Under above assumptions, the governing equations of the problem are given by

$$\frac{d\rho}{dt} = \nabla \cdot (\rho \mathbf{V}), \tag{1}$$

$$\rho\left(\frac{d\mathbf{V}}{dt}\right) = -\nabla p + (2\mu+\kappa)\nabla(\nabla\cdot\mathbf{V}) - (\mu+\kappa)\nabla\times(\nabla\times\mathbf{V}) + \kappa(\nabla\times\mathbf{N}) + \mathbf{J}\times\mathbf{B} + \rho\mathbf{g}, \tag{2}$$

$$\rho j\left(\frac{d\mathbf{N}}{dt}\right) = (\varepsilon+\varphi+\gamma_0)\nabla(\nabla\cdot\mathbf{N}) - \gamma_0\nabla\times(\nabla\times\mathbf{N}) + \kappa(\nabla\times\mathbf{V}) - 2\kappa\mathbf{N}, \tag{3}$$

An incompressible flow is considered in present problem, Eq. (1) becomes

$$\nabla\cdot\mathbf{V} = \mathbf{0} \tag{4}$$

Using the vector identity and mass conservation, Eqns. (2) and (3) are simplified to

$$\rho\left(\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right) = -\nabla p + (\mu+\kappa)\nabla^2\mathbf{V} + \kappa(\nabla\times\mathbf{N}) + \mathbf{J}\times\mathbf{B} + \rho\mathbf{g}, \tag{5}$$

$$\rho j\left(\frac{\partial\mathbf{N}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{N}\right) = \gamma_0\nabla^2\mathbf{N} + \kappa(\nabla\times\mathbf{V}) - 2\kappa\mathbf{N}. \tag{6}$$

One of the body force term corresponding to MHD flow, which is defined by Sharma *et al*. [39] as

$$\mathbf{J}\times\mathbf{B} = -\sigma B_0^2\mathbf{V} \tag{7}$$

The velocity, micro-rotation and gravitational fields are defined in present problem as

$$\mathbf{V} = (u, v, 0),\ \mathbf{N} = (0, 0, N) \text{ and } \mathbf{g} = (g, 0, 0). \tag{8}$$

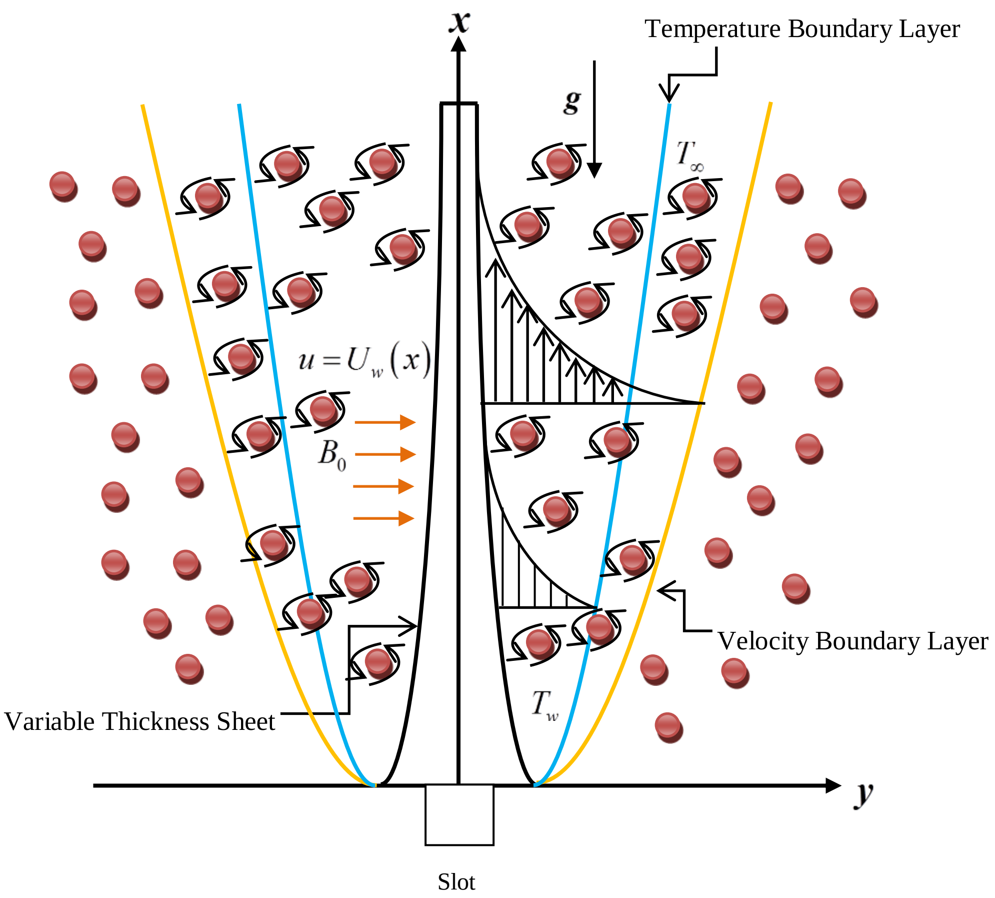

Figure 1: Flow configuration within two dimensional coordinate system.

In view of above Eqns. (7) and (8), Eqns (5) and (6) in case of steady flow are

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\kappa\frac{\partial N}{\partial y}-\sigma B_0^2 u+(\mu+\kappa)\frac{\partial^2 u}{\partial y^2}+\rho g, \tag{9}$$

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma_0\frac{\partial^2 N}{\partial y^2}-\kappa\left(2N+\frac{\partial u}{\partial y}\right), \tag{10}$$

According to Ahmed and Dutta [40] and under Boussinesq approximation, the above equations take the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\kappa}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}u+\left(v+\frac{\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right), \tag{11}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma_0}{\rho j}\frac{\partial^2 N}{\partial y^2}-\kappa\left(2N+\frac{\partial u}{\partial y}\right), \tag{12}$$

Following [32, 41], the energy equation corresponding to Cattaneo-Christov heat flux model is

$$\begin{aligned}&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_0\left(u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}+2uv\frac{\partial^2 T}{\partial x\partial y}\right)\\&+\lambda_0\left(u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}\right)=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}.\end{aligned} \tag{13}$$

The boundary conditions are

$$u = U_0\left(x+b\right)^m,\ v=0,\ N=-\delta\frac{\partial N}{\partial y},\ T=T_w \text{ at } y=A\left(x+b\right)^{\frac{1-m}{2}}, \tag{14}$$

$$u\to 0,\ N\to 0,\ T\to T_\infty \text{ as } y\to\infty. \tag{15}$$

In order to simplify the above equation, we introduce the following similarity variables as [24, 36]

$$\eta = y\sqrt{\frac{(m+1)U_0\left(x+b\right)^{m-1}}{2\nu}},\ \psi=\sqrt{\frac{2\nu U_0\left(x+b\right)^{m+1}}{(m+1)}}\,f(\eta),\ u=U_0\left(x+b\right)^m f'(\eta),$$

$$v=-\sqrt{\frac{(m+1)\nu U_0\left(x+b\right)^{m-1}}{2}}\left[f(\eta)+\left(\frac{m-1}{m+1}\right)\eta f'(\eta)\right], \tag{16}$$

$$N=U_0\left(x+b\right)^m\sqrt{\frac{(m+1)U_0\left(x+b\right)^{m-1}}{2\nu}}h(\eta),\ \phi(\eta)=\frac{T-T_\infty}{T_w-T_\infty}.$$

Moreover, in order to compute the similar solutions, we made the following

- The applied magnetic field is of the form $B(x)=B_0(x+b)^{\frac{m-1}{2}}$, where B_0 is the strength of the magnetic field [42].
- The stretching sheet temperature is $T_w(x)=T_\infty+T_0(x+b)^{\frac{1-n}{2}}$, where T_∞ and T_o represent ambient and reference temperature respectively, with $n=3-4m$ in order to facilitate the similarity transformation.
- The thermal expansion coefficient $\beta_T=\beta_0(x+b)^{4m}$.

Using Eq. (16), Eqns. (11) to (13) can be written as

$$(1+K)f'''(\eta)+f(\eta)f''(\eta)-\left(\frac{2m}{m+1}\right)f'^2(\eta)+Kh'(\eta)-\left(\frac{2}{m+1}\right)\left[M^2f'(\eta)-\lambda\theta(\eta)\right]=0, \tag{17}$$

$$\left(1+\frac{K}{2}\right)h''(\eta)+f(\eta)h'(\eta)-\left(\frac{3m-1}{m+1}\right)f'(\eta)h(\eta)-K\left(\frac{2}{m+1}\right)\left(2h(\eta)+f''(\eta)\right)=0, \tag{18}$$

$$\phi''(\eta)+\Pr f(\eta)\phi'(\eta)+\Pr\gamma\left(\left(\frac{m-3}{2}\right)f(\eta)f'(\eta)\phi'(\eta)-\left(\frac{m+1}{2}\right)f^2(\eta)\phi''(\eta)\right)=0, \tag{19}$$

The transformed boundary conditions in Eqns. (14) and (15) are

$$f(\alpha)=\alpha\left(\frac{1-m}{m+1}\right),\ f'(\alpha)=1,\ h(\alpha)=-\delta f''(\alpha),\ \phi(\alpha)=1, \tag{20}$$

$$f'(\infty)\to 0,\ h(\infty)\to 0,\ \phi(\infty)\to 0. \tag{21}$$

To facilitate the numerical computations, we define $f(\eta)=F(\eta-\alpha)=F(\zeta)$, $h(\eta)=H(\eta-\alpha)=H(\zeta)$, $\phi(\eta)=\theta(\eta-\alpha)=\theta(\zeta)$ so that the above Eqns. (17) to (29) become

$$(1+K)F'''(\zeta)+F(\zeta)F''(\zeta)-\left(\frac{2m}{m+1}\right)F'^2(\zeta)+KH'(\zeta)-\left(\frac{2}{m+1}\right)\left[M^2F'(\zeta)-\lambda\theta(\zeta)\right]=0, \tag{22}$$

$$\left(1+\frac{K}{2}\right)H''(\zeta)+F(\zeta)H'(\zeta)-\left(\frac{3m-1}{m+1}\right)F'(\zeta)H(\zeta)-K\left(\frac{2}{m+1}\right)\left(2H(\zeta)+F''(\zeta)\right)=0, \tag{23}$$

$$\theta''(\zeta)+\Pr F(\zeta)\theta'(\zeta)+\Pr\gamma\left(\left(\frac{m-3}{2}\right)F(\zeta)F'(\zeta)\theta'(\zeta)-\left(\frac{m+1}{2}\right)F^2(\zeta)\theta''(\zeta)\right)=0, \tag{24}$$

where

$$K=\frac{\kappa}{\nu},\ M^2=\frac{\sigma B_0^2}{U_0\rho},\ \gamma=\lambda_0 U_0,\ \Pr=\frac{\mu C_p}{k},$$

$$\alpha=A\sqrt{\frac{U_0(m+1)}{2\nu}},\ \lambda=\frac{Gr_x}{\mathrm{Re}_x^2},\ Gr_x=\frac{g\beta_T(T_w-T_\infty)(x+b)^3}{\nu^2}=\frac{g\beta_0}{\nu^2},$$

K is the micro-rotation parameter, M is the magnetic parameter, γ is the thermal relaxation parameter, $\Pr$ is the Prandtl number, α is the surface thickness parameter, λ is the mixed convection parameter and Gr_x is the Grashof number. The corresponding boundary conditions are presented as

$$F(0)=\alpha\left(\frac{1-m}{m+1}\right),\ F'(0)=1,\ H(0)=-\delta F''(\eta),\ \theta(0)=1, \tag{25}$$

$$F'(\infty)\to 0,\ H(\infty)\to 0,\ \theta(\infty)\to 0. \tag{26}$$

Further, the outer shape of the sheet depends on shape parameter m. The case $m<1$ represents the surface with increasing thickness. The case $m>1$ represents a decrease in the surface thickness and $m=1$ is used for the flat surface problem. Also, this parameter controls the motion such as $m<1$ describes deceleration motion, $m>1$ for acceleration motion and $m=0$ for linear with constant velocity. Skin friction coefficients C_{fx} and local Nusselt number Nu_x are

$$C_{fx}=\frac{1}{\rho u_w^2}\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa N\right]_{y=A(x+b)^{\frac{1-m}{2}}}, \tag{27}$$

$$Nu_x=\frac{-(x+b)}{(T_w-T_\infty)}\frac{\partial T}{\partial y}\bigg|_{y=A(x+b)^{\frac{1-m}{2}}}, \tag{28}$$

and using similarity variables (16) takes

$$\mathrm{Re}_x^{1/2}C_{fx}=\sqrt{\frac{m+1}{2}}\left[1+(1-\delta)K\right]F''(0), \tag{29}$$

$$\mathrm{Re}_x^{-1/2}Nu_x=-\sqrt{\frac{m+1}{2}}\theta'(0), \tag{30}$$

where $\mathrm{Re}_x=\frac{U_0}{\nu}(x+b)^{m+1}$ is the local Reynolds number.

3. Runge Kutta Fehlberg Method

Equations (22)-(24) with boundary conditions (25) and (26) are solved numerically using fourth-fifth order Runge-Kutta-Fehlberg method. This method is more accurate and robust and has been used in several published papers. MAPLE 18 has been used in this study. To enable convergence for all values of governing parameters, the coefficients of the term $F''',\ H''\ \theta''$ are replaced with $(101-100\lambda_1)$ and continuation = λ_1 is used in the dsolve command. Using this modification, MAPLE gives the correct asymptotic values, and the solution converges quickly. The asymptotic boundary conditions given by equation (26) were replaced by using a value of 8 for the similarity variable $\zeta_{\max}$ as follows:

$$\zeta_{\max}=8,\ F'(8)\to 0,\ H(8)\to 0,\ \theta(8)\to 0. \tag{31}$$

The optimal value of $\zeta_{\max}=8$ ensures that numerical solutions approach the asymptotic values correctly. This is a crucial point that is previous ignored in most of papers.

4. Results and Discussion

The effects of governing parameters such as the micro-rotation parameter K, magnetic parameter M, thermal relaxation parameter γ, Prandtl number $\Pr$, surface thickness parameter α, mixed convection parameter λ, micro-element parameter δ and shape parameter or motion parameter m on velocity, micro-rotation, temperature fields, skin friction and Nusselt number are plotted. Figures 2(a) and 2(b) describe the effects of micro-rotation parameter K and mixed convection parameter λ together with variation in shape parameter m and surface thickness parameter α on velocity field in the presence of magnetic field $M \neq 0$. The application of transverse magnetic field always results in a resistive type force also called Lorentz force. This type of resistive force tends to resist the fluid flow and thus reducing the fluid motion. However, it is found that with increasing values of micro-rotation parameter K and shape parameter m, velocity increases as shown in Figure 2(a). It means that micro-rotation parameter K has much influence on the fluid velocity for the case of weak concentration $\delta = 0$. It is also seen that results are similar in both flat surface $(m = 1)$ as well a decrease in the surface thickness $(m > 1)$ cases but change in flat surface problem is slightly smaller when compared with decresing surface thickness.

The velocity behavior remains the same in Figure 2(b), i.e. with increasing mixed convection parameter λ for different values of surface thickness parameter α, velocity increases. An increase in mixed convection parameter λ means a decrease in local Reynolds number and increases the Grashof number Gr. Physically, it is true because the role of Grashof number Gr in heat transfer flow is to increase the strength of the flow. Here $Gr > 0$ represents to the cooling problem. Further, cooling problem is of great importance and encountered in engineering applications, such as in the cooling of electronic components and nuclear reactors. The effects of the same parameters on temperature as in Figure 2 are investigated in Figures 3(a) and 3(b). However, in both of these figures (Figures 3(a) and 3(b)), the temperature has an opposite behavior. More exactly, with increasing values of micro-rotation parameter K and shape parameter m, temperature decreases as shown in Figure 3(a). The variation in temperature as shown in Figure 3(b) is identical with that of velocity as shown in Figure 2 (a), i.e. increases temperature for large values of mixed convection parameter λ and surface thickness parameter α. Increasing the mixed convection parameter λ means flow is dominated by free convection and more heat transfer are carried out of the surface, thus decreasing the thickness of the thermal boundary layers. The results for micro-rotation velocity are plotted in Figures 4(a) and 4(b) for different values of micro-rotation parameter K and mixed convection parameter λ. From Figure 4(a), it is found that micro-rotation velocity shows an oscillatory behavior, i.e. initially increases with increasing of micro-rotation parameter K and shape parameter m until $\zeta \approx 0.9$, and after that the behavior of micro-rotation velocity is reversed, for increasing micro-rotation parameter K and shape parameter m, micro-rotation velocity decreases. Finally, for large ζ, velocity decays to zero. Figure 4(b) indicates that variation in micro-rotation velocity is negligible for different values of mixed convection parameter λ. It is also noticed that sufficient increase is observed in micro-rotation velocity for increasing micro-element parameter δ, which relates to the micro-gyration vector and shear stress. Here $\delta = 0$ considers the situation when micro-elements at the surface are unable to rotate and show weak concentrations, and $\delta = 0.5$ corresponds to the vanishing of antisymmetric part of the stress tensor.

The physical quantities of our interest in this problem are skin friction and heat transfer rate also called Nusselt number. The significances of several physical parameters on skin friction are highlighted in Figures 5(a) and 5(b). In Figure 5(a) skin friction is plotted against magnetic parameter M , for micro-rotation parameter K and two different values of shape parameter m. The skin friction shows an oscillatory behavior. More exactly, for larger values of micro-rotation parameter K, skin friction decreases first and then increases. In Figure 5(b), it is shown that with increasing values of a micro-element parameter δ, skin friction decreases, however, the variation in skin friction is very minor for two different values of thermal relaxation parameter γ. Graphical results for Nusselt number are shown in Figures 6(a) and 6(b). In Figure 6(a), Nusselt number is plotted against magnetic parameter M for micro-rotation parameter K and shape parameter m. It is clear that rate of heat transfer increases with increasing values of micro-rotation parameter K and shape parameter m. An identical behavior is noted as in Figure 6(b), for increasing values of micro-element δ and thermal relaxation parameter γ. However, the variation in rate of heat transfer is quite large for thermal relaxation parameter γ.

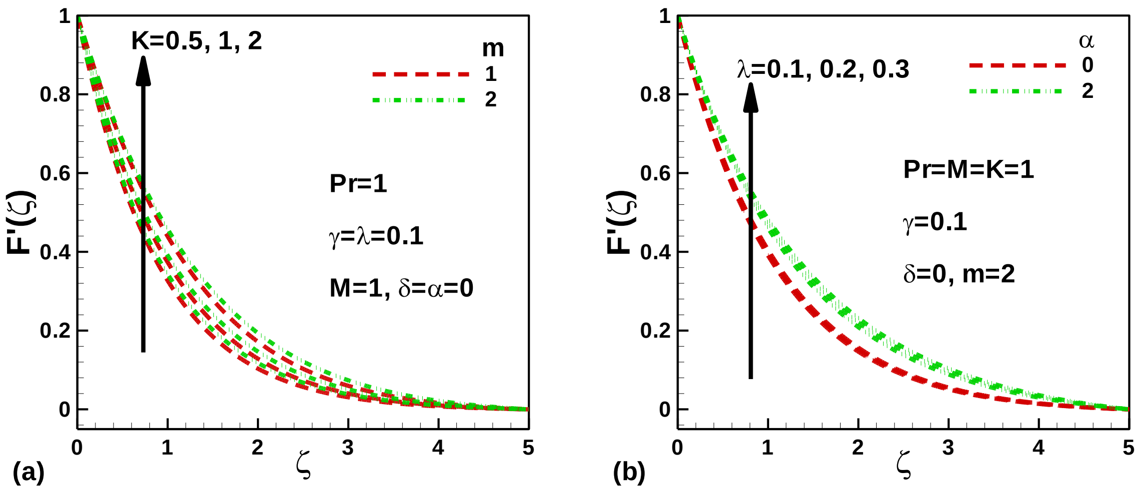

Figure 2: Variation of dimensionless velocity with several governing parameters.

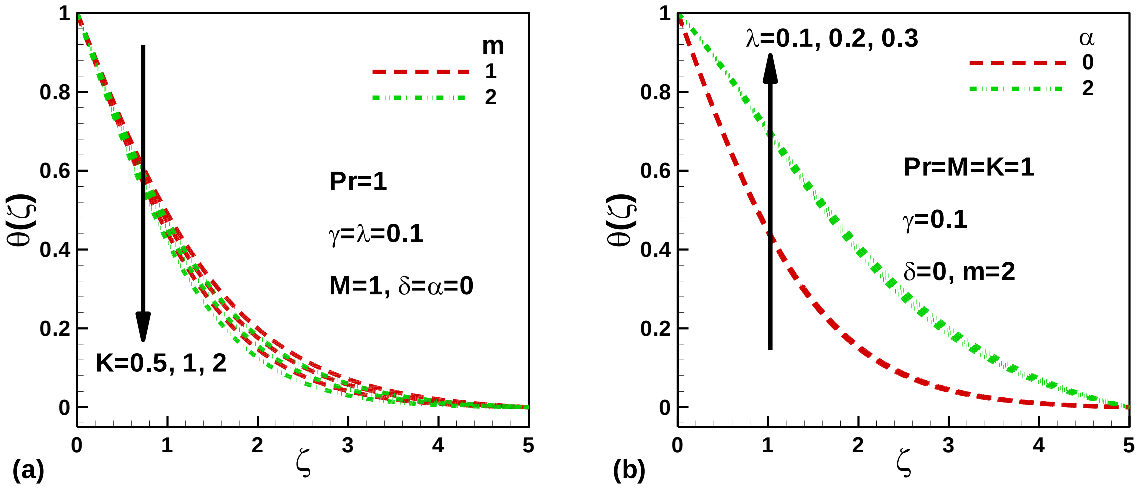

Figure 3: Variation of dimensionless temperature with several governing parameters.

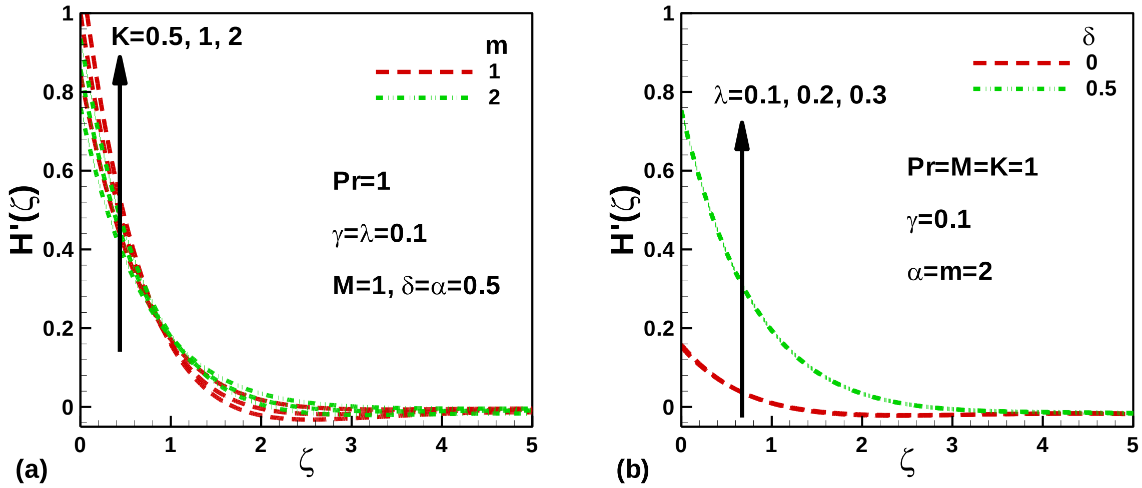

Figure 4: Variation of dimensionless rotational velocity with several governing parameters.

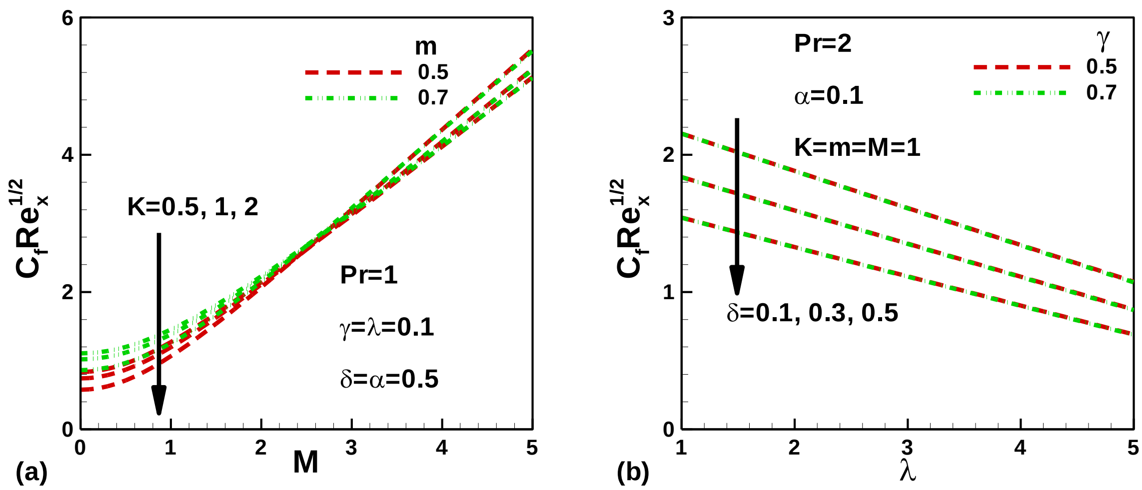

Figure 5: Variation of dimensionless skin friction with several governing parameters.

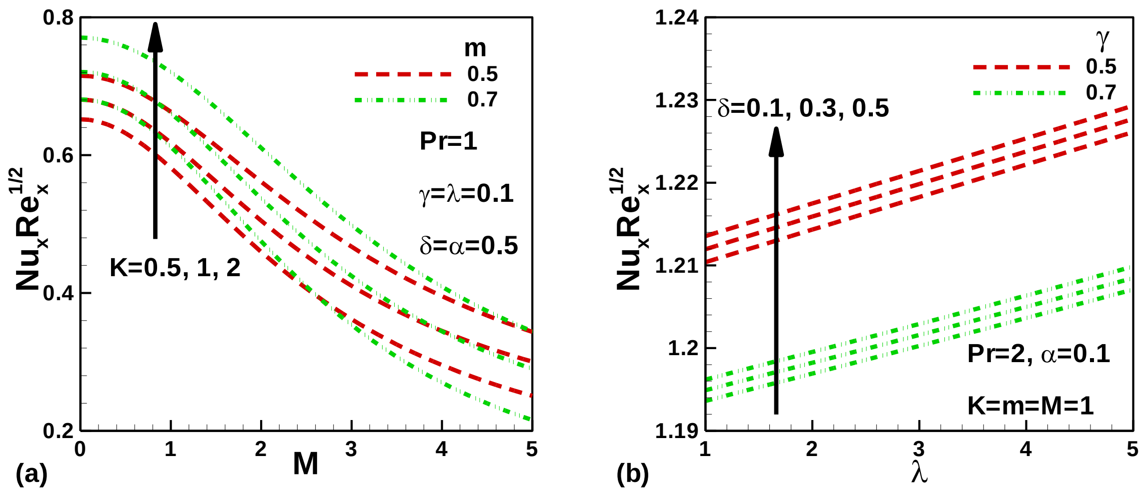

Figure 6: Variation of dimensionless Nusselt number with several governing parameters.

5. Conclusions

Cattaneo-Christov heat flux is applied to investigate the characteristics of thermal relaxation time on the mixed convection flow of non-Newtonian micropolar fluid over a continuously stretching sheet with variable thickness. The results are plotted for micro-rotation parameter, magnetic parameter, thermal relaxation parameter, Prandtl number, surface thickness parameter, mixed convection parameter, micro-element parameter and shape parameter. It is demonstrated that the dimensionless temperature of the fluid enhances with rising of thermal relaxation parameter but reduces with the rise of micro-rotation parameter. Variation in the rate of heat transfer is more effective for small values of thermal relaxation parameter. The results show that magnetic parameter can be controlled the direction and strength of flow. Hence, on the basis of present results it can be concluded that with increment and decrement of different parameters, the temperature for the Cattaneo-Christov heat flux can be improved.

Nomenclature

B_0	Magnetic field intensity
C_p	Heat capacity at constant
F	Dimensionless stream function
$\boldsymbol{g}$	Acceleration due to gravity
$\mathbf{J}$	Current density
j	Micro-inertia density
K	Micro-rotation parameter
k	Thermal conductivity
M	Magnetic parameter
N	Angular velocity
m	Shape parameter
$\Pr$	Prandtl number
T	Temperature of the fluid
T_w	Wall temperature
T_∞	Ambient temperature

Greek symbols

α	Surface thickness parameter
β_T	Volumetric coefficient of thermal expansion
ρ	Fluid density
ν	Kinematic viscosity
σ	Electric conductivity
λ	Mixed convection parameter
λ_0	Thermal relaxation time
γ	Thermal relaxation parameter
γ_0	Spin-gradient viscosity
κ	Vortex viscosity
ε, φ	Spin gradient viscosity coefficients
δ	micro-element parameter
θ	Dimensionless temperature

Acknowledgment

This research is supported by China Postdoctoral Science Foundation (2018M643156) and the National Natural Science Foundation (11571240).

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