ON INTERNAL STABILITY LOSS OF A ROW UNIDIRECTED PERIODICALLY LOCATED FIBERS IN THE VISCO-ELASTIC MATRIX

Resat KOSKER

Yildiz Technical University, Faculty of Chemistry and Metallurgy, Dep. Mathematical. Engineering., Davutpasa Campus, 34220, Esenler, Istanbul, Turkey, E-mail: kosker@yildiz.edu.tr

In the present paper, the microbuckling or internal stability loss in the viscoelastic composites containing unidirected fibers under compression along the fibers is studied by use of piecewise homogeneous body model. In this model, it is used the Three-Dimensional Geometrically Nonlinear Exact Equations of Viscoelasticity Theory. The composite material was considered as an infinite viscoelastic body with a row unidirected periodically located elastic fibers that have an initial infinitesimal imperfection. When the initial imperfection starts to increase and becomes indefinitely, this is taken as a stability loss criterion and co-phase microbuckling mode out of plane are taken into account. The numerical results about the influence of the interaction between the fibers on the values of the critical time are abtained and presented.

Key words*: Critical time, Internal stability loss, Microbuckling, Unidirected fibrous composites, Row fibers, Viscoelastic composite.*

1. Introduction

As we can see some of them in [1-19], there are a lot of experimental and theoretical investigations focus on composites including fibers and nanofibers. It is very important to know the mechanisms of the fracture of the composites under uniaxial compression along the reinforcing elements. For this purpose it is needed to investigate the stability loss in the material structure (internal instability or structural). So, the theoretical investigations of the fracture on the unidirectional composites under uniaxial compression along the reinforcing elements are mean to investigate the stability loss in the material structure, and the value of the external critical forces is taken as the failure forces value in compression (see: [14, 15, 19, 20, 21, 22]). The review of investigations carried out in this field is given in [4, 6, 12, 13, 17, 23].

It follows from these review that, with the use of piecewise homogeneous body model, two approaches are used to investigate the fracture and the stability of the composite materials in compression along the reinforcing elements. One of them is certain hypotheses application related to deformation of each components and to the character of interacting between them. The other is application of the Three-Dimensional Linearized Theory of Stability (TDLTS). The approaches related with the first one used in [14, 19, 20, 21, 24, 25] and others, but in the papers given in [5, 10, 13] and others the second approaches were prefered. It is clear that for considered problems, obtained results with the use of the TDLTS are more reliable than those obtained by use of the approximate theories. However, in the studies done by use of the TDLTS listed above were used the time-independent materials.

In the papers [26, 27], an approach is proposed to investigate stability loss in the time-dependent layered composite material using the TDLTS. In the paper [8], the approach [26] is developed for the unidirected fibrous composite material. However in [8], an infinite length fiber embedded into infinite viscoelastic is considered. The filler concentration in the composite is small and the interactions between the fibers is ignored In [9], the approach given in [8] developed to take into account the interaction two fibers and it is assumed that the infinite viscoelestic media contains two neighboring fibers.

In [4], the approaches [8, 9] are developed to analyse the stability loss in the infinite viscoelastic matrix containing a row unidirected periodically located elastic fibers. It is assumed that the midlines of the fibers are located in a plane and they have co-phase curving relative to each other.

In this investigation, the approaches [4] is developed to study the stability loss in the case where the midlines of the fibers are in parallel planes and the fibers have co-phase curving according to each other. This case will be called co-phase out of plane. The stability loss criterion is taken as the imperfection starts to increase and becomes indefinitely. By this way, in the considered problems, it is estimated the values of critical time which occurs as a result of the interaction between the fibers. Below, we will deal with determinations of these critical times values and the co-phase out of plane stability loss mode of row fibers will be investigated.

The investigations are made by the use of the piecewise-homogeneous body model in the framework exact three dimensional geometrical non-linear equations of the linear viscoelasticity theory. Throughout the studies repeated indices are summed over their ranges; but underlined repeated indices are not summed. Furthermore, the tensor notation will be used to simplify the consideration.

2. Formulation of the problem

Periodically located row fibers embedded into an infinite body is considered. The fibers have insignificant initial imperfections. We associate $O_k x_{1k} x_{2k} x_{3k}$ cartesian and cylindrical $O_k r_k \theta_k z_k$ cylindrical coordinates system with the midline of each fibre (Fig. 1). $q = -\infty, ..., -2, -1, 0, 1, 2, ..., +\infty$ denote the fibres number. As can be seen in Fig. 1 between these coordinates we have the following relations:

$$x_{2k} = x_{20},\ x_{3k} = x_{30} = x_3,\ x_{1k} = kR_{12} + x_{10},\ r_0 e^{i\theta_0} = kR_{12} + r_k e^{i\theta_k},\ z_k = z_0 = z \tag{1}$$

It is assumed that the length of the period of the initial infinitesimal curving of the fibers is the same and the middle lines of the fibers are located in the parallel planes with respect to each other, we suppose that the middle lines of the fibers are in the plane $x_{1q} = 0$. The equations of these lines are given as folloes

$$x_{2k} = L \sin\left(\left(2\pi/\ell\right) x_{3k}\right) \tag{2}$$

By this initial imperfections, we will investigate the co-phase stability loss out of plane. Note that the corresponding results about stability loss mode of two fibers in a pure elastic matrix have been obtained in [13, 16].

It is assumed that, the cross-section perpendicular to the middle line of each fiber is a circle with constant radius R and this is invariant along the entire length of the fiber. It is introduced a small parameter $\varepsilon = L/\ell$, $(0 \le \varepsilon << 1)$.where, L is initial curving amplitude of the fiber and ℓ is the length period of the initial curving. (L is smaller than ℓ). The degree of fiber's initial imperfection is characterized by this parameter.

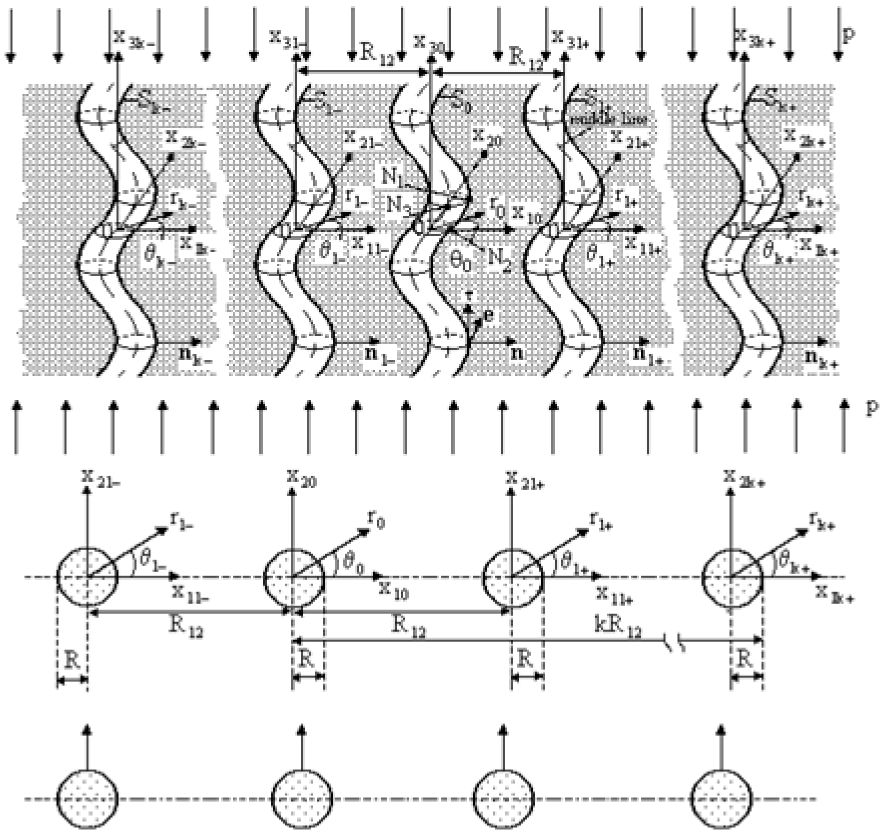

Figure 1: The considered material structure and coordinates.

Below, the values related to the fibers will be denoted by upper indices (2k), the values related to the infinite matrix will be denoted by upper index (1). We sssume that matrix and the fibers materials are homogeneous, linear viscoelastic and isotropic. The development of the infinitesimal initial imperfection of the fibers is investigated when the body is compressed at infinity by uniformly distributed normal forces with an intensity p acting in the direction of fibers. For this purpose, in the cylindrical system of coordinates and in the geometrical nonlinear statement and we write the governing field equations within the infinite matrix and fibers:

$$\nabla_i\left[\sigma^{(q)in}\left(g_n^j+\nabla_n u^{(q)j}\right)\right]=0,\ \ 2\varepsilon_{jm}^{(q)}=\nabla_j u_m^{(q)}+\nabla_m u_j^{(q)}+\nabla_j u^{(q)n}\nabla_m u_n^{(q)},$$

$$\sigma_{(in)}^{(k)}=\lambda^{*(\underline{k})}\left(e^{(\underline{k})}\delta_i^n\right)+2\mu^{*(\underline{k})}\left(\varepsilon_{(in)}^{(\underline{k})}\right),\quad e^{(q)}=\varepsilon_{rr}^{(q)}+\varepsilon_{\theta\theta}^{(q)}+\varepsilon_{zz}^{(q)} \tag{3}$$

where $\mu^{*(k)}$ and $\lambda^{*(k)}$ are the following operators.

$$\lambda^{*(q)}(.)=\lambda_0^{(q)}(.)+\int_0^t\lambda^{(q)}(t-\tau)(.)d\tau,\ \ \mu^{*(k)}(.)=\mu_0^{(k)}(.)+\int_0^t\mu^{(k)}(t-\tau)(.)d\tau \tag{4}$$

We assume that, the completely cohesion conditions are satisfied on the inter-medium surface S_k (Fig. 1):

$$\sigma^{(2\underline{k})in}\left(g_n^j+\nabla_n u^{(2\underline{k})j}\right)\Big|_{S_{\underline{k}}} n_{\underline{k}j}=\sigma^{(1)in}\left(g_n^j+\nabla_n u^{(1)j}\right)\Big|_{S_k} n_{\underline{k}j},\ u_j^{(2\underline{q})}\Big|_{S_{\underline{q}}}=u_j^{(1)}\Big|_{S_{\underline{q}}} \tag{5}$$

where n_{qj} are the the unit normal vector components of the surfaces S_q. In addition, the conditions $\left|\sigma_{(ij)}^{(2k)}\right|<\infty$, $\left|u_{(i)}^{(2k)}\right|<\infty$, $\sigma_{zz}^{(1)}\xrightarrow[r_q\to\infty]{}p$, $\sigma_{(ij)}^{(1)}\xrightarrow[r_q\to\infty]{}0\ \ (ij)\neq zz$ are valid in the considered case. Conventional tensor notation is used and physical components of the corresponding tensors are showed by subscripts in parentheses in Eqs. (3)-(5).

3. Method of Solution

It is used a version of the boundary shape perturbation method [7] to investigate the corresponding problem. Using the fiber cross-section condition and Eq. (2), we easily derive the S_q interfaces equations as follows.

$$r_k = (\varepsilon^2(\delta'_{\underline{k}}(t_3))^2 \sin^2\theta_{\underline{k}} + 1)^{-1}\left\{(\varepsilon^3\delta_{\underline{k}}(t_3)(\delta'_{\underline{k}}(t_3))^2 + \varepsilon\delta_{\underline{k}}(t_3))\sin\theta_{\underline{k}} + \left[R^2 - \varepsilon^2(\delta_{\underline{k}}(t_3))^2 - \varepsilon^4(\delta'_{\underline{k}}(t_3))^2(\delta_{\underline{k}}(t_3))^2(1+\varepsilon^2(\delta'_{\underline{k}}(t_3))^2)\sin^2\theta_{\underline{k}}\right]^{1/2}\right\},$$

$$z_k = t_3 - \varepsilon\delta'_{\underline{k}}(t_3)r_{\underline{k}}(t_3)\sin\theta_{\underline{k}} + \varepsilon^2\delta_{\underline{k}}(t_3)\delta'_{\underline{k}}(t_3),\ \delta'_k(t_3) = \frac{d\delta_k(t_3)}{dt_3} \tag{6}$$

The equation of the middle line of the k-th fiber is denoted by $\varepsilon\delta_k(t_3)$ where $t_3 \in (-\infty, +\infty)$ is a parameter. the boundary shape perturbation method, the unknowns are presented in series form in ε:

$$\left\{\sigma_{(ij)}^{(m)}; \varepsilon_{(ij)}^{(m)}; u_{(i)}^{(m)}\right\} = \sum_{q=1}^{\infty}\varepsilon^q\left\{\sigma_{(ij)}^{(m),q}; \varepsilon_{(ij)}^{(m),q}; u_{(i)}^{(m),q}\right\}, \tag{7}$$

After some calculations, the following expressions are obtained from Eq. (6) for the components of the unit normal vector to S_k:

$$r_q = R + \sum_{k=1}^{\infty}\varepsilon^k a_{\underline{q}k}(\theta_{\underline{q}}, t_3),\ z_q = t_3 + \sum_{k=1}^{\infty}\varepsilon^k b_{\underline{q}k}(\theta_{\underline{q}}, t_3),\ n_{qr} = 1 + \sum_{k=1}^{\infty}\varepsilon^k c_{\underline{q}k}(\theta_{\underline{q}}, t_3),$$

$$n_{q\theta} = \sum_{k=1}^{\infty}\varepsilon^k d_{\underline{q}k}(\theta_{\underline{q}}, t_3),\ n_{qz} = \sum_{k=1}^{\infty}\varepsilon^k g_{\underline{q}k}(\theta_{\underline{q}}, t_3) \tag{8}$$

$a_{\underline{q}k}(\theta_{\underline{q}}, t_3), \ldots, g_{\underline{q}k}(\theta_{\underline{q}}, t_3)$ functions in Eq. (8) can easily be caculated from Eq. (6).

For each approximation in Eq. (7), a set of equations can be obtained by substituting Eq. (7) in Eq. (3). We expand each approximation (7) values in the series form in the vicinity of ($r_q = R$, $z_q = t_3$) by using Eq. (8). Contact conditions satisfied in $r_q = R$, $z_q = t_3$ for each approach in Eq. (7) is obtained by using n_{qr}, $n_{q\theta}$ and n_{qz} given in Eq. (8) and substituting last expressions in Eq. (5).

It is clear that, the Eq. (3) are valid and the conditions (5) are replaced by the same ones satisfied in $r_q = R$, $z_q = t_3$ for the zeroth approximation. It is assumed that $\nabla_n u^{(k)j,0} << 1$ is satisfied and therefore $g_m^j + \nabla_m u^{(k)j,0}$ can be replaced by δ_n^j where δ_n^j are Kronecker symbols. From that, we obtain the following system of equations and and contact conditions, respectively for the zeroth approximation

$$\nabla_i\sigma^{(q)ij,0} = 0,\ 2\varepsilon_{ij}^{(q),0} = \nabla_j u_i^{(q),0} + \nabla_i u_j^{(q),0}, \tag{9}$$

$$\left.\sigma_{(ij)}^{(2q),0}\right|_{r_q=R} = \left.\sigma_{(ij)}^{(1),0}\right|_{r_q=R},\ \left.u_{(i)}^{(2q),0}\right|_{r_q=R} = \left.u_{(i)}^{(1),0}\right|_{r_q=R};\ (ij) = rr, r\theta, rz,\ (i) = r, \theta, z \tag{10}$$

If the last assumption is taken into account, the following system of equations can be obtained for the first approximation

$$\nabla_i\left[\sigma^{(q)ij,1} + \sigma^{(q)in,0}\nabla_n u^{(q)j,1}\right] = 0,\ 2\varepsilon_{ij}^{(q),1} = \nabla_j u_i^{(q),1} + \nabla_i u_j^{(q),1}. \tag{11}$$

Additionally, the constitutive relations are written as follows:

$$\sigma_{(in)}^{(q),1} = \lambda^{*(\underline{q})}e^{(\underline{q}),1}\delta_i^n + 2\mu^{*(\underline{q})}\varepsilon_{(in)}^{(\underline{q}),1}, \tag{12}$$

The contact conditions can be written as follows using physical components of the displacement vector and stress tensor for the first approximation.

$$\left[\sigma_{(i)r}\right]_{1,1}^{2q,1} + f_{1\underline{q}}\left[\frac{\partial\sigma_{(i)r}}{\partial r}\right]_{1,0}^{2\underline{q},0} + \phi_{1\underline{q}}\left[\frac{\partial\sigma_{(i)r}}{\partial z}\right]_{1,0}^{2\underline{q},0} + \gamma_{r\underline{q}}\left[\sigma_{(i)r}\right]_{1,0}^{2\underline{q},0} + \gamma_{\theta\underline{q}}\left[\sigma_{(i)\theta}\right]_{1,0}^{2\underline{q},0} + \gamma_{z\underline{q}}\left[\sigma_{(i)z}\right]_{1,0}^{2\underline{q},0} = 0$$

$$\left[u_{(i)}\right]_{1,1}^{2q,1}+f_{1\underline{q}}\left[\frac{\partial u_{(i)}}{\partial r}\right]_{1,0}^{2\underline{q},0}+\phi_{1\underline{q}}\left[\frac{\partial u_{(i)}}{\partial z}\right]_{1,0}^{2\underline{q},0}=0 \tag{13}$$

where $(i)=r,\theta,z$. The rest of the contact conditions are obtained from (13) by means of cyclic permutation of the indices r,θ and z only in the components stress tensor (the first index is permuted) and displacement vector. In (13) the following notation is used.

$$[\phi]_{1,s}^{2q,s}=\phi^{(2q),s}-\phi^{(1),s};\ f_{1q}=\delta_q(t_3)\cos\theta_q;\ \phi_{1q}=-R\delta_q'(t_3)\cos\theta_q,\ \gamma_{rq}=\left(\frac{\delta_q(t_3)}{R}-\delta_q''(t_3)R\right)\cos\theta_q;$$

$$\gamma_{\theta q}=-\frac{\delta_q(t_3)}{R}\sin\theta_q;\ \gamma_{zq}=-\delta_q'(t_3)\cos\theta_q;\ \ \delta_q(t_3)=\ell\sin(2\pi t_3/\ell)=\ell\sin(\alpha t_3),\ \ \alpha=2\pi/\ell \tag{14}$$

We can obtain similar contact conditions for the subsequent approximations.
Let's determine the unknown values of these approximations. Assume that the materials of each fiber are the same and the material of the fibers is pure elastic with elastic constants $E^{(2)}$ (Young's moduli), $\nu^{(2)}$ (Poisson coefficient), material of the matrix is viscoelastic with operators (6). We will suppose $\nu^{(2)}=\nu_0^{(1)}$, where $\nu_0^{(1)}=\nu^{(1)}\big|_{t=0}$ (t is time) and the stresses arising in the zeroth approximation as a result of $\nu^{(2)}\neq\nu^{(1)}$ for $t>0$ will be ignored, because these stresses have an order $O(\nu^{(2)}-\nu^{(1)})$ and according to [12], do not have only considerable effect on numerical results. Thus, taking the above stated into account consider the determination of each approximation separately.

Let us determine the zeroth approximation. This approximation has an exact analytical solution in the pure elastic case. Here, we obtain a solution to the corresponding quasistatic problem by replacing the elastic constants by corresponding operators. The followings are obtained:

$$\sigma_{zz}^{(1),0}=p;\ \sigma_{zz}^{(2k),0}=\frac{E^{(2)}}{E^{*(1)}}p;\ \ \varepsilon_{zz}^{(2k),0}=\varepsilon_{zz}^{(1),0}=\frac{p}{E^{*(1)}};\ u_z^{(2k),0}=u_z^{(1),0}=\varepsilon_{zz}^{(1),0}z;\ u_r^{(1),0}=-\nu^{(1)}\varepsilon_{zz}^{(1),0}r;$$

$$u_r^{(2k),0}=-\nu^{(2)}\varepsilon_{zz}^{(2k),0}r\ \ u_\theta^{(1),0}=u_\theta^{(2q),0}=0,\ \ \sigma_{(ij)}^{(2k),0}=\sigma_{(ij)}^{(1),0}=0\ ;\ z_1=z_2=z;\ E^{(2)}=E^{(2k)},$$

$$k=-\infty,...,-2,-1,0,1,2,...,+\infty;\ (ij)=rr,\theta\theta,r\theta,\theta z,rz \tag{15}$$

where $E^{*(1)}$ and $\nu^{*(1)}$ are the following operators

$$E^{*(1)}=E_0^{(1)}+\int_0^t E^{(1)}(t-\tau)d\tau:\ \ \nu^{*(1)}=\nu_0^{(1)}+\int_0^t \nu^{(1)}(t-\tau)d\tau \tag{16}$$

It is time to get the first approximation. Using the solution of zeroth approximations, we derive the following for Eq. (11).

$$\frac{\partial\sigma_{rr}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{rz}^{(k),1}}{\partial z}+\frac{1}{r}\left(\sigma_{rr}^{(k),1}-\sigma_{\theta\theta}^{(k),1}\right)+\sigma_{zz}^{(\underline{k}),0}\frac{\partial^2u_r^{(\underline{k}),1}}{\partial z^2}=0,$$

$$\frac{\partial\sigma_{r\theta}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{\theta z}^{(k),1}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(k),1}+\sigma_{zz}^{(\underline{k}),0}\frac{\partial^2u_\theta^{(\underline{k}),1}}{\partial z^2}=0,$$

$$\frac{\partial\sigma_{rz}^{(k),1}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}^{(k),1}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(k),1}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(k),1}+\sigma_{zz}^{(\underline{k}),0}\frac{\partial^2u_z^{(\underline{k}),1}}{\partial z^2}=0. \tag{17}$$

Eq. (12) will not change, but, because of pure elasticity of the fiber material, Eq. (12) for the fibers are replaced by folowing

$$\sigma_{(in)}^{(2k),1}=\lambda^{(2)}e^{(2k),1}\delta_i^n+2\mu^{(2)}\varepsilon_{(in)}^{(2k),1}. \tag{18}$$

Moreover, the geometrical relations for both the matrix and fibers have the following form.

$$\varepsilon_{rr}^{(k),1}=\frac{\partial u_r^{(k),1}}{\partial r},\ \varepsilon_{\theta\theta}^{(k),1}=\frac{\partial u_\theta^{(k),1}}{r\partial\theta}+\frac{u_r^{(k),1}}{r},\ \varepsilon_{zz}^{(k),1}=\frac{\partial u_z^{(k),1}}{\partial z},\varepsilon_{r\theta}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_r^{(k),1}}{r\partial\theta}+\frac{\partial u_\theta^{(k),1}}{\partial r}-\frac{u_\theta^{(k),1}}{r}\right),$$

$$\varepsilon_{\theta z}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_{\theta}^{(k),1}}{\partial z}+\frac{\partial u_{z}^{(k),1}}{r\partial\theta}\right),\varepsilon_{zr}^{(k),1}=\frac{1}{2}\left(\frac{\partial u_{z}^{(k),1}}{\partial r}+\frac{\partial u_{r}^{(k),1}}{\partial z}\right). \tag{19}$$

From Eq. (15), the contact conditions of the first approximation become as follows:.

$$\left[\sigma_{rr}\right]_{1,1}^{2k,1}=0,\ \left[\sigma_{r\theta}\right]_{1,1}^{2k,1}=0,\ \left[\sigma_{rz}\right]_{1,1}^{2k,1}=\delta_{\underline{k}}'(t_3)\left(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}\right)\cos\theta_{\underline{k}},$$

$$\left[u_r\right]_{1,1}^{2k,1}=0,\ \left[u_{\theta}\right]_{1,1}^{2k,1}=0,\ \left[u_z\right]_{1,1}^{2k,1}=0. \tag{20}$$

t (time) is a parameter in the all equations written for the fiber but an independent variable in the equations written for the matrix. Here, Laplace transform is applied

$$\bar{\phi}(s)=\int_0^{\infty}\phi(t)e^{-st}dt, \tag{21}$$

with $s>0$, to all relations and equations (except the contact relations (20)) belonging to the matrix material. From the procedure, Eqs. (17), (19) are valid for the Laplace transforms of the corresponding sought-for quantities and the constitutive relations (12) are transformed to the following ones:

$$\bar{\sigma}_{(in)}^{(1),1}=\bar{\lambda}^{(1)}\bar{e}^{(1),1}\delta_i^n+2\bar{\mu}^{(1)}\bar{\varepsilon}_{(in)}^{(1),1},\ \ \bar{\lambda}^{(1)}=\frac{\bar{E}^{(1)}\bar{\nu}^{(1)}}{(1+\bar{\nu}^{(1)})(1-2\bar{\nu}^{(1)})},\bar{\mu}^{(1)}=\frac{\bar{E}^{(1)}}{2(1+\bar{\nu}^{(1)})}. \tag{22}$$

As it has been noted above, the Eqs. (17)-(19) coincide with the corresponding equations of the TDLTS, therefore to solve the obtained equation systems we can use the following representations in the cylindrical system of coordinates (see: [16]).

$$u_r=\frac{1}{r}\frac{\partial}{\partial\theta}\psi-\frac{\partial^2}{\partial r\partial z}\chi,\ u_{\theta}=-\frac{\partial}{\partial r}\psi-\frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}\chi,$$

$$u_3=(\lambda+\mu)^{-1}\left((\lambda+2\mu)\Delta_1+(\mu+\sigma_{zz}^0)\frac{\partial^2}{\partial z^2}\right)\chi,\ \ \Delta_1=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}. \tag{23}$$

The functions ψ and χ are determined from the equations

$$\left(\Delta_1+\xi_1^{\ 2}\frac{\partial^2}{\partial z^2}\right)\psi=0,\ \left(\Delta_1+\xi_2^{\ 2}\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1+\xi_3^{\ 2}\frac{\partial^2}{\partial z^2}\right)\chi=0 \tag{24}$$

where

$$\xi_1=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}},\ \xi_2=\sqrt{\frac{\mu+\sigma_{zz}^0}{\mu}},\ \xi_3=\sqrt{\frac{\lambda+2\mu+\sigma_{zz}^0}{\lambda+2\mu}}. \tag{25}$$

Eqs. (23)-(25), are used for fibers and matrix. The quantities $u_{(i)}$, λ, μ and σ_{zz}^0 are replaced by $\bar{u}_{(i)}^{(1),1}$, $\bar{\lambda}^{(1)}$, $\bar{\mu}^{(1)}$, $\sigma_{zz}^{(1),0}$ respectively for the matrix and by $u_{(i)}^{(2k),1}$, $\lambda^{(2)}$, $\mu^{(2)}$, $\sigma_{zz}^{(2),0}$ respectively for the fibers. If we take the contact conditions (20) into account, the Eqs. (24) can be solved as follows:

$$\psi^{(2\underline{k}),1}=\alpha\sin\alpha z\sum_{n=-\infty}^{\infty}A_n^{(2\underline{k})}(t)I_n(\xi_1^{(2)}(t)\alpha r_{\underline{k}})\exp(in\theta_{\underline{k}}),$$

$$\chi^{(2\underline{k}),1}=\cos\alpha z\sum_{n=-\infty}^{\infty}\left[B_n^{(2\underline{k})}(t)I_n(\xi_2^{(2)}(t)\alpha r_{\underline{k}})+C_n^{(2\underline{k})}(t)I_n(\xi_3^{(2)}(t)\alpha r_{\underline{k}})\right]\exp(in\theta_{\underline{k}}) \tag{26}$$

$$\bar{\psi}^{(1),1}=\alpha\sin\alpha z\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\bar{A}_n^{(1)k}(s)K_n(\xi_1^{(1)}(s)\alpha r_k)\exp(in\theta_k),$$

$$\bar{\chi}^{(1)}=\cos\alpha z\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\left[\bar{B}_n^{(1)k}(s)K_n(\xi_2^{(1)}(s)\alpha r_k)+\bar{C}_n^{(1)k}(s)K_n(\xi_3^{(1)}(s)\alpha r_k)\right]\exp(in\theta_k), \tag{27}$$

where $\alpha=2\pi/\ell$ and $I_n(x),\ K_n(x)$ are Bessel functions of a purely imaginary argument and Macdonald functions, in turn. Moreover, the unknowns $A_n^{(2k)}(t),\ldots,C_n^{(2k)}(t)$, $\bar{A}_n^{(1)k}(s),\ldots,\bar{C}_n^{(1)}(s)$ are the complex numbers and satisfy the relations

$$A_n^{(2k)}(t)=\overline{A_{-n}^{(2k)}}(t),\ B_n^{(2k)}(t)=\overline{B_{-n}^{(2k)}}(t),\quad C_n^{(2k)}(t)=\overline{C_{-n}^{(2k)}}(t),\ \mathrm{Im}\ A_0^{(2k)}=\mathrm{Im}\ B_0^{(2k)}=\mathrm{Im}\ C_0^{(2k)}=0,$$

$$\bar{A}_n^{(1)k}(s)=-\overline{\bar{A}_{-n}^{(1)k}}(s),\ \bar{B}_n^{(1)k}(s)=-\overline{\bar{B}_{-n}^{(1)}}(s),\quad \bar{C}_n^{(1)k}(s)=-\overline{\bar{C}_{-n}^{(1)k}}(s),$$

$$\mathrm{Im}\ \bar{A}_0^{(1)k}(s)=\mathrm{Im}\ \bar{B}_0^{(1)k}(s)=\mathrm{Im}\ \bar{C}_0^{(1)k}(s)=0. \qquad (28)$$

We want to obtain the expressions of the the first approximation values of by satisfying the contact condition (20). To make these operations, the expressions (26) and (27) must be represented in the q-th ($q=-\infty,...,-2,-1,0,1,2,...,+\infty$) cylindrical coordinate system. We already present the expressions (26) in the q-th cylindrical system of coordinates and It be used the summation theorem (Watson ([28]) for the $K_n(x)$ function in expressions (27). The theorem can be written for the case at hand as follows

$$r_{\underline{m}}\exp i\theta_{\underline{m}}=(n-m)R_{12}\exp i\phi_{\underline{mn}}+r_{\underline{n}}\exp i\theta_{\underline{n}}$$

$$K_{\underline{\nu}}(cr_{\underline{n}})\exp i\underline{\nu}\theta_{\underline{n}}=\sum_{k=-\infty}^{\infty}(-1)^{\underline{\nu}}I_k(cr_{\underline{m}})K_{\underline{\nu}-k}(c|\underline{n}-\underline{m}|R_{12})\exp\left[i(\underline{\nu}-k)\phi_{\underline{mn}}\right]\exp ik\theta_{\underline{m}}$$

$$\phi_{mn}=0 \text{ for } n>m,\ \phi_{mn}=\pi \text{ for } m>n,\ c=const.\ r<R_{12}; \qquad (29)$$

Using Eqs. (23)-(25), (27)-(29) and Eq. (22) the expressions Laplace transform of the values of the first approximation related to the matrix are obtained.

Now, we consider the determination of the inverse Laplace transform. For this purpose, we will use the Schapery [29] method. This technique is described in [30].

Obtained infinite system must be replaced with the corresponding finite system of equations to get numerical results.. It can be proven the validity of these replacements. Note that such a proof was also performed in [7]. Consequently, we can replace algebraic equations infinite system by the following one for numerical investigations:

$$Y_n^{(1)0}+\sum_{\nu=0}^{N_\nu}Y_\nu^{(1)0}\left(\sum_{q=1}^{N_q}2F_{n\nu}^{(1)q}\right)+F_n^{(2)0}Y_n^{(2)0}=2\pi\delta_n^3(\sigma_{zz}^{(1),0}-\sigma_{zz}^{(2),0}),\ n=1,2,...,N_\nu \qquad (30)$$

where $n=0,1,2,...,\infty$, $\delta_3^3=1$ and $\delta_n^3=0$, if $n\neq 3$. The values of N_ν and N_q in Eq. (30) are determined from the convergence requirement of numerical results. In this way we determine the unknowns for any selected t and within the framework of the first approximation find the critical time from the criterion

$$\max_{z\in(0,\ell);\theta\in[0,\pi/2]} u_r^{(2k),1}\rightarrow\infty \qquad (31)$$

The critical time (or critical compressive force for pure elastic problem) we denote as $t_{cr.}$ ($p_{cr.}$).

4. Numerical Results and Discussions

We assume that the constitutive relations for the matrix material are described by the operators

$$E^{*(1)}=E_0^{(1)}\left[1-\omega_0R_{\alpha'}^*(-\omega_0-\omega_\infty)\right],\quad \nu^{*(1)}=\nu_0^{(1)}\left[1+\frac{1-2\nu_0^{(1)}}{2\nu_0^{(1)}}\omega_0R_{\alpha'}^*(-\omega_0-\omega_\infty)\right],$$

$$\lambda^{(1)*}=\lambda_0^{(1)}\left[1+\frac{1-2\nu_0^{(1)}}{2\nu_0^{(1)}(1+\nu_0^{(1)})}\omega_0R_{\alpha'}^*\left(-\frac{3}{2(1+\nu_0^{(1)})}\omega_0-\omega_\infty\right)\right],$$

$$\mu^{(1)*}=\mu_0^{(1)}\left[1-\frac{3\omega_0}{2\nu_0^{(1)}(1+\nu_0^{(1)})}R_{\alpha'}^*\left(-\frac{3}{2(1+\nu_0^{(1)})}\omega_0-\omega_\infty\right)\right], \qquad (32)$$

where $E_0^{(1)}$, $\nu_0^{(1)}$ are the instantaneous values of Young's modulus and of the Poisson coefficient, respectively; $\lambda_0^{(1)}$, $\mu_0^{(1)}$ are the instantaneous values of Lamé's constants, α', ω_0, ω_∞ are the rheological parameters of the matrix material, $R_{\alpha'}^{*}$ is the fractional-exponential Rabotnov operator [4].

The Rabotnov operators be allowed to describe with required accuracy, the initial parts of the experimentally and theoretically constructed creep and relaxation graphs and to determine the asymptotic values of these graphs with very high accuracy. Moreover, these operators have many simple rules for various complicated mathematical transformations. So, these operators are employed successfully to describe various polymer materials and epoxy-based composites with continuous fibers or layers..

We introduce the dimensionless rheological parameter $\omega\left(=\omega_\infty/\omega_0\right)$ and the dimensionless time $t'\left(=\omega_0^{1/(1+\alpha')}t\right)$ and assume that $\nu_0^{(1)}=\nu^{(2k)}=0.3$, $E^{(2k)}=E^{(2)}$. Moreover, we introduce the parameters $\in=p/E_0^{(1)}$, $\kappa=2\pi R/\ell$ and investigate the internal stability loss of the infinite viscoelastic matrix containing a row fibers. Under internal stability loss we will understand the stability loss in the material structure, which arises for certain relations of the stiffness and geometrical parameters of the matrix and fibers and does not depend on boundary surfaces, sizes and forms of members of constructions. It is well known that under investigation of stability loss problems for viscoelastic materials the external compressive force p must satisfy the following inequalities

$$\in_{cr.\infty}\left(=p_{cr.\infty}/E_0^{(1)}\right)\le\in\left(=p/E_0^{(1)}\right)\le\in_{cr.0}\left(=p_{cr.0}/E_0^{(1)}\right) \qquad (33)$$

where $\in_{cr.0}$ is a $\in_{cr.}$ obtained at $t'=0$, $\in_{cr.\infty}$ is a $\in_{cr.}$ obtained at $t'=\infty$. A lot of investigations reviewed in [4] show that, the internal stability loss phenomenon for unidirected fibrous composites occurs under $E^{(2)}>E_0^{(1)}$.

Thus, taking the above-stated consideration into account we analyze the numerical results related to the internal stability loss of the considered viscoelastic matrix containing a row of fibers.

First, we consider numerical results related to the internal stability loss in the above described sense for the pure elastic deformation state under $t'=0$ and $t'=\infty$. Consider the case where $E^{(2)}/E_0^{(1)}=50$ (for all numerical investigation we will assume that $E^{(2)}/E_0^{(1)}=50$, $\kappa=2\pi R/\ell=0.3$) and introduce the parameter $\rho=R_{12}/R$ through which we will characterize the interaction between the fibers under their stability loss. The corresponding results obtained for $\in_{cr.0}$ and $\in_{cr.\infty}$ for various values of ρ and ω are given in Tab. 1. Note that these results relay to pure elastic problem and coincide with corresponding obtained in [10]. Tab. 1 shows that the dependence between $\in_{cr.0}$, $\in_{cr.\infty}$ and ρ is monotonic, i.e. the values of $\in_{cr.0}$ (or $\in_{cr.\infty}$) increase monotonically with ρ and approach asymptotically the corresponding values of $\in_{cr.0}$ (or $\in_{cr.\infty}$) obtained for a single fiber [11].

Table 1 : The values of $\in_{cr.\infty}$ $(for\, t'=\infty)$ and of $\in_{cr.0}$ (for $t'=0$) obtained for $E^{(2)}/E_0^{(1)}$ =50, $\kappa=0.3$, $\alpha=-0.5$ with various R_{12}/R

R_{12}/R	$\in_{cr.0}$	$\in_{cr.\infty}$			
		$\omega=0.5$	$\omega=1.0$	$\omega=2.0$	$\omega=3.0$
2.1	-0.0762	-0.0394	-0.0485	-0.0577	-0.0623
2.2	-0.0785	-0.0402	-0.0497	-0.0592	-0.0639
2.5	-0.0849	-0.0426	-0.0531	-0.0636	-0.0689
3.0	-0.0947	-0.0463	-0.0585	-0.0705	-0.0766
4.0	-0.1110	-0.0525	-0.0674	-0.0820	-0.0893
5.0	-0.1233	-0.0572	-0.0742	-0.0908	-0.0990
6.0	-0.1326	-0.0607	-0.0793	-0.0974	-0.1062
7.0	-0.1397	-0.0633	-0.0831	-0.1023	-0.1117
9.0	-0.1491	-0.0666	-0.0879	-0.1087	-0.1189
10.0	-0.1523	-0.0676	-0.0895	-0.1108	-0.1212
15.0	-0.1599	-0.0698	-0.0929	-0.1156	-0.1267
20.0	-0.1621	-0.0703	-0.0938	-0.1168	-0.1282
∞	-0.1630	-0.0704	-0.0940	-0.1172	-0.1287

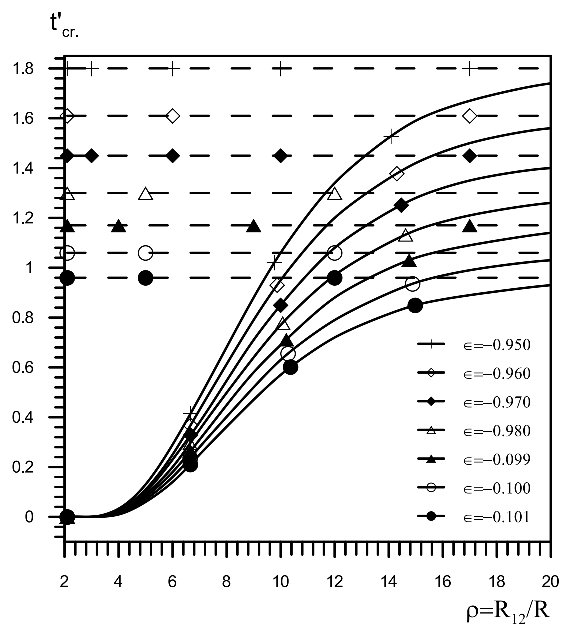

Figure 2 : The graphs of the dependencies between $t'_{cr.}$ and $\rho=R_{12}/R$ for various values of $\in$ under $\alpha'=-0.5$, $\omega=0.5$.

In Fig. 2 the graphs of the dependencies between $t'_{cr.}$ and ρ are given for $\omega=0.5$ under $\alpha'=-0.5$ and for various $\in$. It follows from these graphs that the values of $t'_{cr.}$ approach the corresponding values of $t'_{cr.}$ obtained in [8] for a single fiber in a viscoelastic matrix. These graphs show that dependencies between $t'_{cr.}$ and ρ are monotonic. Moreover, these graphs show that $t'_{cr.}\to 0$ ($t'_{cr.}\to\infty$) as $|\in|\to|\in_{cr.0}|$ ($|\in|\to|\in_{cr.\infty}|$) and as a result of the interaction between the fibers the values of $t'_{cr.}$ decrease significantly. It follows from the above-discussed numerical results that interaction between the fibers in the viscoelastic matrix is more considerable than that in the pure elastic matrix. It follows from the comparison of the graphs constructed in the various figures that, for the same $\in$ the values of $t'_{cr.}$ increase with ω.

In Tab. 2 the values of $t'_{cr.}$ are given for various values of α' which shows the order of the singularity of the operator (33). These results are obtained for the case where $\omega=0.5$ with various $\in$

and ρ. The Tab. 2 shows that under $t'_{cr.} > 0.5$ ($t'_{cr.} < 0.5$) the values of $t'_{cr.}$ increase (decrease) monotonically with $|\alpha'|$. Moreover, this table shows that the interaction between the fibers becomes more significantly with $|\alpha'|$.

As it has been noted above, the numerical results analyzed here are obtained within the framework of the first approximation. Under obtaining these results the infinite system of equations are replaced by the corresponding finite one. For the illustration of the numerical results with respect to the number of the equations in this finite system in Tab. 3 the values of $t'_{cr.}$ obtained for various number of the equations are given under $\omega = 0.5$, $\rho = 2.1$, $\alpha' = -0.5$ for various $\in$. It follows from these table that the convergence of the solution method employed is highly effective.

Table 2 : The values of $t'_{cr.}$ obtained for $\omega = 0.5$ with various $\rho = R_{12}/R$, α' and $\in$

$\in$	ρ	α'		
		-0.3	-0.5	-0.7
-0.0830	2.1	0.0000	0.0000	0.0000
	2.5	0.0035	0.0004	0.0000
	3.0	0.0549	0.0227	0.0028
	5.0	0.5600	0.5800	0.6500
	7.0	1.3600	2.0300	5.1800
	15.0	3.6800	8.2000	52.9700
	∞	4.1300	9.6200	69.1100
-0.0890	2.1	0.0000	0.0000	0.0000
	2.5	0.0000	0.0000	0.0000
	3.0	0.0159	0.0040	0.0010
	5.0	0.3200	0.2700	0.1800
	7.0	0.7900	0.9500	1.4500
	15.0	1.9100	3.2800	11.5000
	∞	2.1000	3.7500	14.3800

Table 3 : The values of $t'_{cr.}$ obtained for various values of N_q and N_ν in equation (30) under $\rho = 2.1$, $\alpha' = -0.5$, $\omega = 0.5$ with various $\in$

$\in$	N_q ($N_\upsilon = 80$)							
	3	5	7	9	11	13	15	17
-0.0500	2.2500	1.5300	1.3900	1.3600	1.3500	1.3400	1.3400	1.3400
	N_υ ($N_q = 17$)							
	14	20	32	44	56	68	74	80
-0.0500	1.2790	1.3350	1.3430	1.3450	1.3460	1.3460	1.3460	1.3460

5. Conclusions

In the present paper, the internal stability loss (microbuckling) in the structure of the viscoelastic unidirected fibrous composites under compression along the fibers is studied within the framework of the piecewise homogeneous body model with the use of the Three-Dimensional Geometrically Nonlinear Exact Equations of the Theory of Viscoelasticity. This study concerns mainly the cases where the interaction between the fibers is taken into account and all investigations are carried out for the infinite viscoelastic matrix containing a row of fibers.

As a microbuckling criterion the initial imperfection criterion is used and two cases of the location of the initial imperfected fibers (co-phase periodical curving of the fibers out of plane) with respect to each other are considered. For the stability of the rising of the initial imperfection with the time under fixed external compressed forces the Three-Dimensional Geometrically Nonlinear Exact Equations of the Theory of Viscoelasticity is employed. Introducing the dimensionless small parameter characterizing the degree of the insignificant initial imperfection for the solution to the corresponding nonlinear boundary value problem, the perturbation of the boundary-shape method is employed. It is proven that the equations and relations related to the first approximation are the corresponding equations and relations of the TDLTS. For first approximation the corresponding closed system of linearized equations and contact conditions are obtained and for the solution of these equations the Laplace transformation with respect to time and method of separation of variables are employed. For determination of inverse Laplace transform the Schapery method is used. It is proven that the values of the critical parameters can be determined in the framework of the zeroth and first approximations only (see: [9]). So, these parameters have been determined in the framework of the zeroth and first approximations.

The numerical results related to the critical time are also analyzed. According to these results it is established that, for the considered microbuckling mode of the fibers as a result of the interaction between the fibers the values of the critical time decrease. Further, this interaction is more significant than that in the pure elastic buckling. The numerical results obtained in the present work in the particular cases coincide with the corresponding ones obtained in the other investigations. This situation guaranties the correctness of the developed approach.

References

[1] Liu, Y.-Q., et al., Air Permeability of Nanofiber Membrane With Hierarchical Structure, *Thermal Science*, *22*, 2018, 4, pp. 1637-1643.

[2] Wang, P., et al., Energy Absorption in Friction-Based Stab-Proof Fabrics and the Puncture Resis tance of Nanofiber Membrane, *Thermal Science, 22*, 2018, 1A, pp. 39-41.

[3] Liu, P. and He, J., Geometric Potential An Explanation of Nanofiber's Wettability, *Thermal Science*, *22*, 2018, 1A, pp. 33-38..

[4] Akbarov, S.D., *Stability Loss and Buckling Delamination: Three-Dimensional Linearized Approach for Elastic and Viscoelastic Composites*, Springer, Berlin, Germany, 2012:

[5] 5 Akbarov, S. D., Guz, A. N., Stability of Two Fibers in an Elastic Matrix with Small Strains, *Soviet Appl. Mech., 21*, 1985, 1, pp. 1-7.

[6] Akbarov, S. D. And Guz, A.N., Statics of Laminated and Fibrous Composites With Curved *Structures. Appl. Mech. Rev.*, *45*, 1992, 2, pp. 17-35.

[7] Akbarov, S. D., Guz, A. N., *Mechanics of curved composites,* Kluwer Academic Pubishers, Dortrecht/ Boston/ London, 2000.

[8] Akbarov, S. D., Kosker, R., Fiber Buckling in a Viscoelastic Matrix, *Mechanics of Composite Materials, 37*, 2001, 4, pp. 299-306.

[9] Akbarov, S. D., Kosker, R., Internal Stability Loss of Two Neighbouring Fibers in a Viscoelastic Matrix, *Int. J. Eng. Scien., .42*, 2004, 17/18, pp.1847-1873.

[10] Babaev, M.S., et al., Stability of the Row of Fibers in the Elastic Matrix at Small Precritical Deformations., *Soviet Appl. Mech., 21*, 1985, 5, pp. 27-34.

[11] Babich, I., Stab. of Fiber in a Matrx with Small Str., *Soviet Appl. Mech.*, *9*, 1973, 4, pp. 29-35.

[12] Babich, I.Yu., Guz, A.N., Stability of Fibrous Com., *Appl. Mech. Rev., 45*, 1992, 2, pp. 60-80.

[13] Babich, I.Yu., et al., The Three-Dimensional Theory of Stability of Fibrous and Laminated Materials, *Int. Appl. Mech., 37,* 2001, 9, pp. 1103-1141.

[14] Budianski, B. and Fleck, N.A., Compressive Failure of Fibre Composites, *J. Mech. Phys. Solids, 41*,1993, 1, pp. 183-21.

[15] Greszczuk, I.B., Fracture of Composite Reinforced by Circular Fibers From Loss of Stability Of Fibers, *J. AIAA*, *13*, 1975, 10, pp. 67-75.

[16] Guz, A.N., *Fundamentals of the Three-Dimensional Theory of Stability of Deformable Bodies*, Springer-Verlag, Berlin, Germany, 1999.

[17] Guz, A. N.; Lapusta, Yu. N., Three-Dimensional Problems of the Near-Surface Instability of Fiber Composites in Compression,. *Inter. Appl. Mech., 35*, 1999, 7, pp. 641-671.

[18] Guz, A. N., Rushchitskii, Ya. Ya., Nanomaterials: on the Mechanics of Nanomaterials, *Int. App. Mech., 39*, 2003, 11, pp. 1271-1293.

[19] Rosen, B.W., *Fiber composite materials*. Amer. Soc. For. Metals, Met. P., Ohio, USA, 1965.

[20] Dow, N.F., Grunfest, I.J. *Determination of Most Needed Potentially Possible Improvements in Materials for Ballistic and Space Vehicles*, General Electric Co. Space Sci Lab., USA, 1960.

[21] Drapier S., et al., A structural approach of plastic microbuckling in long fibre composites: comparison with theoretical and exp.results. *Int. J. Solids and Str., 38*, 2001, 28, pp. 3877-3904.

[22] Chamis, C.C. *Micromechanics strength theories. [for unidirectional composites]*, NASA Lewis Research Center, Cleveland, Ohio, USA, 1974.

[23] Akbarov, S.D., Three-dimensional stability loss problems of the viscoelastic composite materials and structural members. *Int. Appl. Mech, .43*, 2007, 10, pp. 3-27.

[24] Sadovsky, M.A., et al., Buckling of Microfibers, *Trans ASME, 34*, 1967, 4, pp. 295-302.

[25] Schuerch, H., Prediction of Compressive Strength in Uniaxial Boron Fibermetal Matrix *Composite Materials, J. AIAA 4*, 1966, 1, pp. 102-106.

[26] Akbarov, S.D. et al., On The Fracture of the Unidirectional Composites in Compression. *Int.J.Eng. Sci. 35*, 1997, 12/13, pp. 1115-1136.

[27] Akbarov, S.D. et al., The Theoritical Strenght Limit in Compression of Viscoelastic Layered Composite Materials. *Composites Part B: Engineering, 30*, 1999, 5, pp. 465-472.

[28] Watson, G.N., *A Treatise on the Theory of Bessel functions*, Cambridge University Press, England., 1944.

[29] Schapery, R.A., Approximate Methods of Transform Inversion for Viscoelastic Stress Analysis, *Proceedings 4th US Nat. Congress on App. Mech.*, Berkely, USA, 1962, pp. 1075-1085.

[30] Guz, A.N. . et al., Plane Problems of Stability of Composite Materials with a Finite Size Filler. *Mechan. Comp. Materials., 36,* 2000, 1, pp 77-86.