EFFECTS OF GASEOUS SLIP FLOW AND TEMPERATURE JUMP ON ENTROPY GENERATION RATE IN RECTANGULAR MICRODUCTS

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In this study, the influence of slip flow and temperature jump on the entropy generation rate are investigated in rectangular microducts. The Knudsen numbers are considered in the range between 0.001 and 0.1, and the aspect ratio lies between 0 and 1. The dimensionless governing equations are solved numerically using Chebyshev Spectral collocation method, and the dimensionless velocity and temperature gradients are employed in the entropy generation model. The influences of the dimensionless numbers including Bejan number and irreversibility distribution ratio on the entropy generation rates are investigated and discussed through surface plots and contour diagrams. It is demonstrated that the minimum entropy generation rate exists corresponding to an optimal aspect ratio for each dimensionless number. This minimum entropy generation rate depends upon the nature of dimensionless numbers.

Keywords: *Knudsen number; Bejan number; Entropy generation rate; Irreversibility; Microducts; Slip flow.*

1. Introduction

Enhancement in the overall performance in different mechanical and engineering process always been the topic of interest for the engineers and scientists. Still, no such equipment has launched which convert the input energy into the valuable body of work without any loss. However, these losses can be minimized to get maximum overall performance. Various techniques are available to minimize these losses. However, the best practical method, to get the optimized results in thermodynamically systems, is the use of entropy generation minimization. Entropy generation is the primary concern in different engineering equipment like energy storage systems, geophysical fluid dynamics, heat exchangers and cooling of electronic devices. Gouy-Stodola in his theorem states that the rate of loss of exergy (irreversibility) is proportional to the entropy generation. However, method of entropy generation minimization (EMG) was first discussed by the Bejan [1]. According to him, the energy loss of thermodynamic systems can be controlled if the factors which cause the irreversibility can be controlled, and thermodynamic optimization of the system can be attained. Minimization of entropy generation enhances the efficiency of a thermal system and leads to the reduction of the energy lost. That is for input energy to get maximum work done. The method of entropy generation minimization is well described by Bejan [2-4] for the optimized results in various thermal systems. Carrington and Sun [5] performed the entropy analysis in heat and mass transfer. The established a control volume solution for their model, and they consider in internal and external flows for heat and mass to apply this phenomenon to validate their findings. Nag and Kumar [6] carried out second law analysis through a duct assuming constant heat flux at the wall. They found that entropy generation during the process is directly proportional to lose in available energy and entropy generation can be minimized for an optimal value of the initial temperature difference. Şahin [7], considered various duct geometries to study the irreversibilities by assuming the constant heat flux at the wall. Demirel and Kahraman [8] performed entropy analysis through a rectangular duct filled with spherical particles. They also assumed fluxes at the top (heated) and bottom (cooled) wall and calculated irreversibility distribution and entropy generation. Tasnim et al. [9] performed rational analysis inside two parallel isothermal plates placed in vertical direction saturated in a porous medium. They assumed that magnetic force is acting in transverse direction and expression of irreversibility distribution ratio and entropy generation number is also calculated analytically. Hooman [10] et al. performed first and the second law analysis through a rectangular duct saturated with porous medium and results are found analytically. Further notable studies [11-21] on entropy analysis are also carried in various geometries.

Study of fluid flow and heat transfer rate through microscale has received much importance owing to their vast applications in chemical separation, micro-thermal technology, micro-propulsion, inkjet printheads, cooling of computer chips and in the field of biomedical. At microscopic scale (100 microns etc.) and low pressure, gaseous flow does not follow the law continuum physics. In this situation, fluid velocity and temperature near the solid boundaries is different from an actual temperature of the boundaries. In the literature, this phenomenon is commonly known as velocity slip and thermal slip (temperature jumps), which is the primary source of change in flow characteristics.

The studies in the microchannel with slip flow and temperature jumps have been carried out by most of the researches [22-29], and they carried out useful findings. Hooman [30] performed entropy analysis under various boundary conditions through duct geometry and used a numerical technique. He concluded that heating for the flow of gases through microducts of rectangular cross-section are essential and can change heat transfer. However, the study of entropy generation in rectangular microducts with the gaseous slip and temperature jumps has not been considered yet. Our present investigation majorly emphasizes on entropy generation minimization under velocity slip, and temperature jumps in microducts. A study is performed numerically by using spectral method and results are obtained for different emerging parameters.

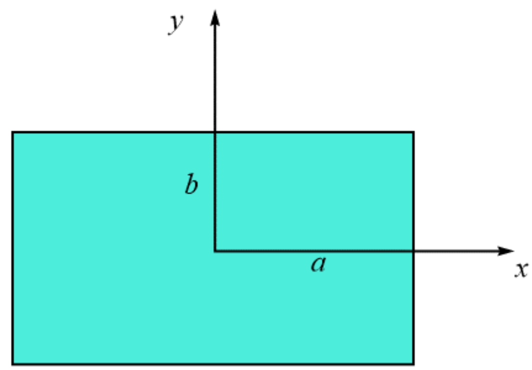


Fig. 1. Schematic diagram

2. Mathematical formulation

Consider a rectangular duct whose semi major and minor axes are taken as a and b along $x-$ and $y-$ axes. The aspect ratio is $\varepsilon = \frac{b}{a}$. The gas is assumed to flow along $z-$the axis. The governing equations for the flow and heat transfer in a microduct can be written as

Momentum equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\left(\frac{\partial p}{\partial z}\right) \tag{1}$$

Energy equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha} u\left(\frac{\partial T}{\partial z}\right) \tag{2}$$

Subjected to fluid flow conditions at the boundary

$$
\left.\begin{aligned}
u &= -\lambda \frac{2-\sigma}{\sigma}\frac{\partial u}{\partial y} \quad at \quad y=b\ , \quad 0 \le x < a, \\
u &= -\lambda \frac{2-\sigma}{\sigma}\frac{\partial u}{\partial x} \quad at \quad x=a\ , \quad 0 \le y < b, \\
\frac{\partial u}{\partial y} &= 0 \quad at \quad y=0\ , \quad 0 \le x \le a, \\
\frac{\partial u}{\partial x} &= 0 \quad at \quad x=0\ , \quad 0 \le y \le b.
\end{aligned}\right. \tag{3a}
$$

And temperature jump conditions are

$$
\begin{aligned}
&\left.\begin{aligned}
T - T_w &= -\frac{\lambda}{\Pr}\frac{2-\sigma_t}{\sigma_t}\frac{2\gamma}{1+\gamma}\frac{\partial T}{\partial y} \quad at \quad y=b\ , \quad 0 \le x < a, \\
T - T_w &= -\frac{\lambda}{\Pr}\frac{2-\sigma_t}{\sigma_t}\frac{2\gamma}{1+\gamma}\frac{\partial T}{\partial x} \quad at \quad x=a\ , \quad 0 \le y < b,
\end{aligned}\right\}\text{Temperature jump conditions} \\
&\left.\begin{aligned}
\frac{\partial T}{\partial y} &= 0 \quad at \quad y=0\ , \quad 0 \le x \le a, \\
\frac{\partial T}{\partial x} &= 0 \quad at \quad x=0\ , \quad 0 \le y \le b.
\end{aligned}\right\}\text{symmetry conditions}
\end{aligned} \tag{3b}
$$

Using following transformations

$$
\xi = \frac{x}{a},\ \eta = \frac{y}{b},\quad w = \frac{u}{u_m p} \ \text{ and } \ T - T_w = \left(\frac{Q\,p}{k_f}\right)\theta \tag{4}
$$

where

$$
u_m = \frac{1}{A}\int_A w\,dA, \quad p = -\frac{b^2}{u_m \mu}\frac{dp}{dz} \ \text{ and } \ \frac{dT}{dz} = \frac{Q}{4ab\rho\, c_p u_m} \tag{5}
$$

we get the following dimensionless forms for both momentum and energy equations

$$
\varepsilon^2 \frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} = -1 \tag{6}
$$

$$
\varepsilon^2 \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} = \frac{1}{4}\varepsilon w \tag{7}
$$

With dimensionless boundary conditions

$$
\begin{aligned}
u &= -\frac{4k_n}{1+\varepsilon}\frac{2-\sigma}{\sigma}\frac{\partial u}{\partial \eta} \quad at \quad \eta = 1\,, \quad 0 \le \xi < 1, \\
u &= -\frac{4\varepsilon}{1+\varepsilon}\frac{2-\sigma}{\sigma}\frac{\partial u}{\partial \xi} \quad at \quad \xi = 1\,, \quad 0 \le \eta < 1, \\
\frac{\partial u}{\partial \eta} &= 0 \quad at \quad \eta = 0\,, \quad 0 \le \xi \le 1, \\
\frac{\partial u}{\partial \xi} &= 0 \quad at \quad \xi = 0\,, \quad 0 \le \eta \le 1.
\end{aligned}
\tag{8a}
$$

The dimensionless temperature at the boundary of the duct is

$$
\begin{aligned}
&\left.\begin{aligned}
\theta &= -\frac{2-\sigma_t}{\sigma_t}\frac{4}{1+\varepsilon}\frac{k_n}{\Pr}\frac{2\gamma}{1+\gamma}\frac{\partial \theta}{\partial \eta} \quad at \quad \eta = 1\,, \quad 0 \le \xi < 1, \\
\theta &= -\frac{2-\sigma_t}{\sigma_t}\frac{4\varepsilon}{1+\varepsilon}\frac{k_n}{\Pr}\frac{2\gamma}{1+\gamma}\frac{\partial \theta}{\partial \xi} \quad at \quad \xi = 1\,, \quad 0 \le \eta < 1,
\end{aligned}\right\}\text{Temperature jump conditions} \\
&\left.\begin{aligned}
\frac{\partial \theta}{\partial \eta} &= 0 \quad at \quad \eta = 0\,, \quad 0 \le \xi \le 1, \\
\frac{\partial \theta}{\partial \xi} &= 0 \quad at \quad \xi = 0\,, \quad 0 \le \eta \le 1.
\end{aligned}\right\}\text{symmetry conditions}
\end{aligned}
\tag{8b}
$$

3. Entropy generation analysis

Following Bejan [1], the entropy generation rate in ducts can be written as

$$
S_G = \overbrace{\frac{k_f}{T_0^{\,2}}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right]}^{\text{Entropy generation due to heat transfer } S_{G,h}} + \overbrace{\frac{\mu}{T_0}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]}^{\text{Entropy generation due to fluid friction } S_{G,f}}
\tag{9}
$$

To develop its dimensionless form, we used Eqns. (3) and (4) and got the following form

$$
\begin{aligned}
S_G = \frac{k_f}{T_0^{\,2}}&\left[\left(\frac{1}{a}\frac{Q\,p}{k_f}\theta_\xi\right)^2 + \left(\frac{1}{b}\frac{Q\,p}{k_f}\theta_\eta\right)^2 + \left(\frac{Q}{4ab\rho c_p u_m}\right)^2\right] + \\
&\frac{\mu}{T_0}\left[\left(\frac{u_m p}{a}w_\xi\right)^2 + \left(\frac{u_m p}{b}w_\eta\right)^2\right]
\end{aligned}
\tag{10}
$$

$$S_G = \frac{Q^2 p^2}{T_0^{\,2} b^2 k_f}\left[\varepsilon^2 \theta_\xi^{\,2} + \theta_\eta^{\,2} + \left(\frac{\varepsilon}{4}\right)^2 \left(\frac{k_f}{bp}\frac{1}{\rho c_p u_m}\right)^2\right] + \frac{\mu}{T_0}\frac{u_m^{\,2} p^2}{b^2}\left[\varepsilon^2 w_\xi^{\,2} + w_\eta^{\,2}\right] \tag{11}$$

$$N_s = \frac{S_G}{S_{G_0}} = \varepsilon^2 \theta_\xi^{\,2} + \theta_\eta^{\,2} + \left(\frac{\varepsilon}{4}\right)^2 \left(\frac{k_f}{bp}\frac{1}{\rho c_p u_m}\right)^2 + \frac{\mu}{T_0}\frac{u_m^{\,2} k_f T_0}{Q^2}\left[\varepsilon^2 w_\xi^{\,2} + w_\eta^{\,2}\right] \tag{12}$$

where $S_{G_0} = \dfrac{Q^2 p^2}{T_0^{\,2} b^2 k_f}$

$$Ns = \overbrace{\varepsilon^2 \theta_\xi^{\,2} + \theta_\eta^{\,2} + \left(\frac{\varepsilon}{4}\right)^2 \left(\frac{1}{Pe}\right)^2}^{Ns_h} + \overbrace{\frac{B_r}{\omega}\left[\varepsilon^2 w_\xi^{\,2} + w_\eta^{\,2}\right]}^{Ns_f} \tag{13}$$

where $Pe = \mathrm{Re}\,\mathrm{Pr} = \dfrac{(\rho u_m) b}{\upsilon}\dfrac{\upsilon}{\alpha} = \dfrac{(\rho u_m) b}{\alpha}$, $B_r = \dfrac{\mu u_m^2}{Q}$, $\omega = \dfrac{\Delta T}{T_0}$ and $\Delta T = \dfrac{Q}{k_f}$

In the above equation, the term Ns_h represents the heat transfer irreversibility and Ns_f represents fluid friction irreversibility. Bejan number is the ratio of heat transfer irreversibility to the total irreversibility and can be written as

$$Be = \frac{Ns_h}{Ns} = \frac{Ns_h}{Ns_h + Ns_f} = \frac{1}{1 + {Ns_f}/{Ns_h}} = \frac{1}{1+\Phi} \tag{13}$$

where $\Phi = \dfrac{Ns_f}{Ns_h}$ is the irreversibility distribution ratio. From Eq. (13), it is inferred that the Bejan number lies between 0 and 1. When *Be* = 1, heat transfer irreversibility dominates over fluid friction irreversibility, whereas *Be=0* confirms the dominance of fluid friction irreversibility over heat transfer irreversibility. When *Be* = 0.5, both heat transfer and friction irreversibilities are equally dominant.

4. Results and Discussion

The obtained system of dimensionless equations (6–8) is simulated through Chebyshev spectral collocation scheme, accurate up to 10^{-6}. The domain $0 \le \xi < 1$ and $0 \le \eta < 1$ is transformed to $-1 \le \xi < 1$ and $-1 \le \eta < 1$ using the formula $\xi = 2\eta/L - 1$. The node points between 1 and -1 are calculated by using the formula $\xi j = \cos(\pi j/N),\ j = 0, 1, 2, \ldots N$ and $\eta i = \cos(\pi i/N),\ i = 0, 1, 2, \ldots N$ along ξ and η direction respectively. The equal number of node points are considered in both directions. These node points are commonly known as Gauss-Lobatto collocation points. The obtained results are presented through the figures 2-9, while the value of the parameter $\sigma_t = \sigma = 1$ is kept fix and others are mentioned corresponding to each figure. The variation of several entropy generation rates with the aspect ratio is displayed in Figs. 2-4 for different dimensionless numbers including Pe, Br, Kn keeping other parameters fixed. In each case, the entropy generation rate due to heat transfer irreversibility is decreasing, whereas, the entropy generation rate due to fluid friction irreversibility is increasing with aspect ratio for each dimensionless number. Also, there is a minimum entropy generation rate corresponding to an optimal aspect ratio for each dimensionless number.

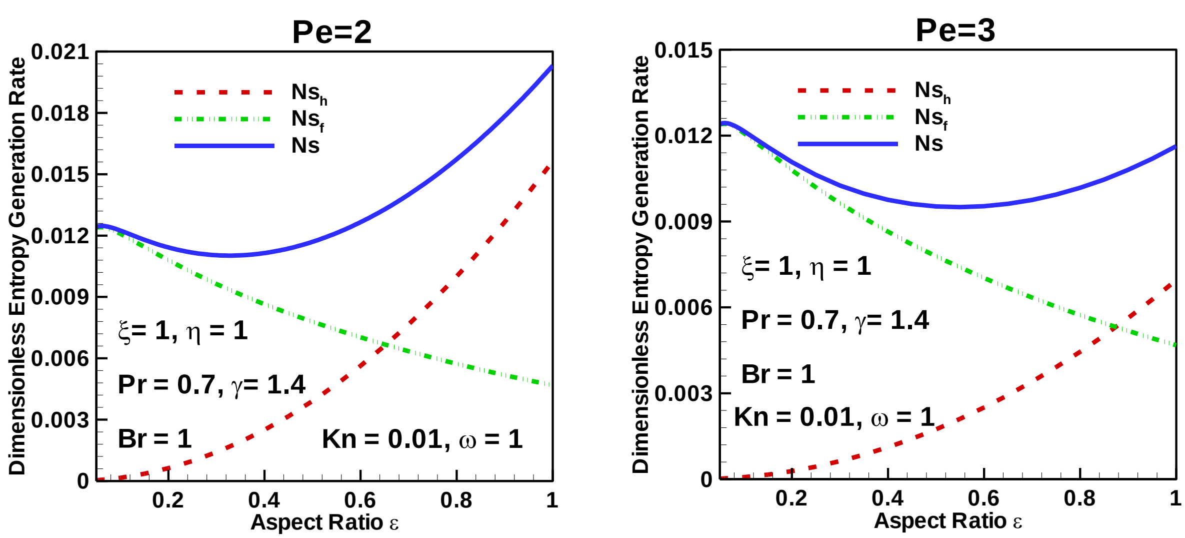


Figure 2: Variation of dimensionless entropy generation rates with aspect ratio when (a) $Pe = 2$ and (b) $Pe = 3$.

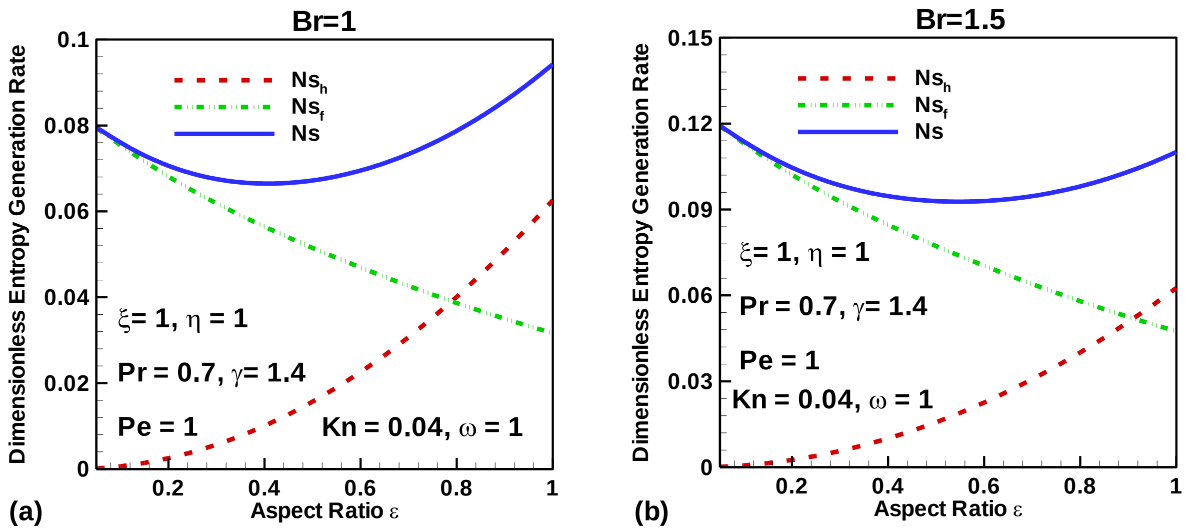


Figure 3: Variation of dimensionless entropy generation rates with aspect ratio when (a) $Br = 1$ and (b) $Br = 1.5$.

This minimum entropy generation rate depends upon the nature of dimensionless numbers. The Péclet number establishes the relation between conducted and convected thermal energy to the fluid. The larger the Péclet number, the larger will be the convected thermal energy to the fluid. This is confirmed in Figs. 2 (a) and (b) for two different Péclet numbers. The Brinkman number demonstrates the relation between viscous heat generation and external heating. It increases with an increase in viscous dissipation and as a result the minimum entropy generation rate increases with Brinkman number. This can be observed in Figs. 3(a) and (b). The same fact can be observed in Figs. 4 (a)-(d) for increasing values of Knudsen numbers in the slip flow regime ($0.01 \le Kn \le 0.1$). The variation of entropy generation rates and Bejan number along axial and transverse directions are shown in surface plots (see Figs. 5 a-d). The isotherms for the same quantities are shown in Figs. 6 and 7 for the fixed parameters. The isotherms of entropy generation rate due to heat and fluid friction are displayed in Figs. 6 (a) and (b) respectively for $Kn = 0.1$. The variation in magnitude with the position exhibits the distribution of irreversibilities due to heat and fluid friction. It is observed that the effect of fluid friction irreversibility is dominant near the left and top surfaces of the duct.

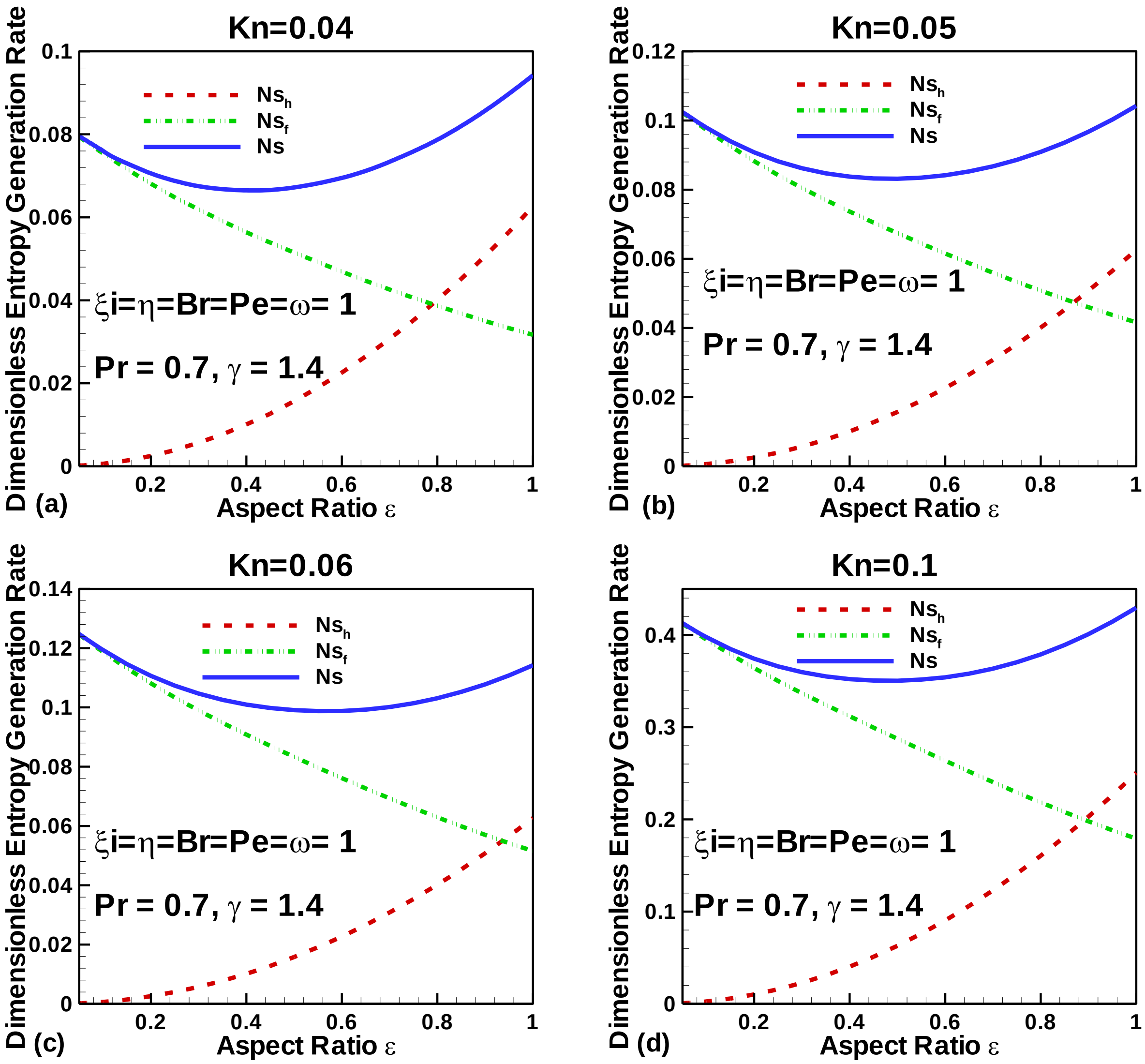
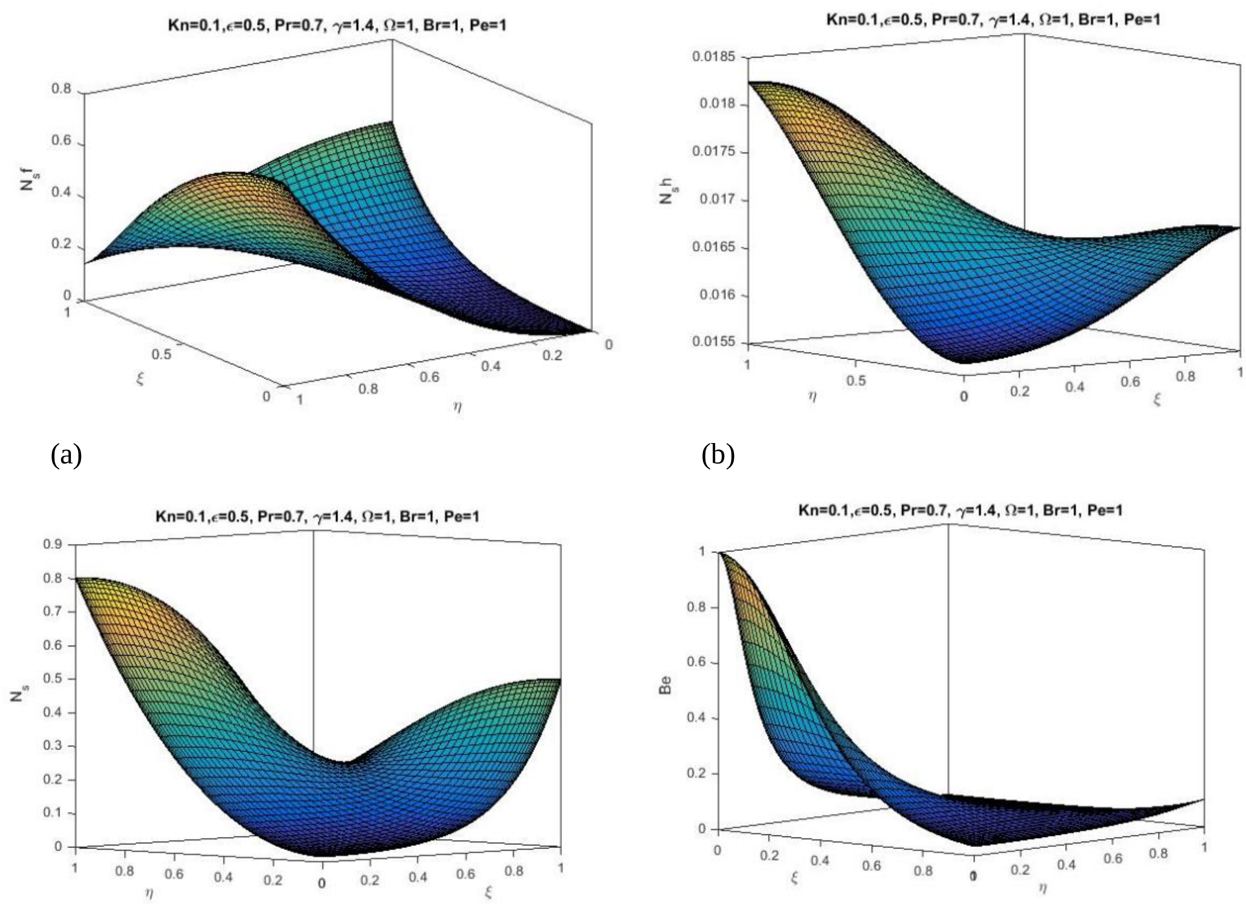


Figure 4: Variation of dimensionless entropy generation rates with aspect ratio when (a) $Kn = 0.04$, (b) $Kn = 0.05$,(c) $Kn = 0.06$, and (d) $Kn = 0.1$.

On the other side, the fluid friction irreversibility shows minimum contribution. Similarly, contours of total entropy generation rate and Bejan numbers are shown in Figs. 7(a) and 7 (b) respectively. Figure 7 (a) demonstrates that the total entropy generation inside the duct is practically the same as the entropy generation due to fluid friction.

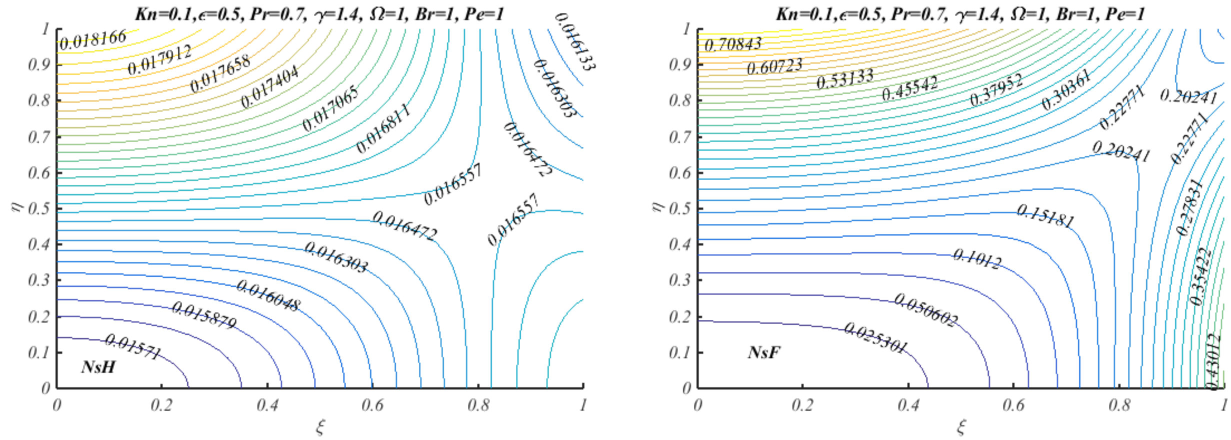
Figures 8 (a) and (b) display the variation of Bejan number with aspect ratios for different values of Kn and Pe numbers respectively. The behavior of Bejan numbers is found to be the same in both cases. In case of parallel plate channel when $\varepsilon \to 0$ the irreversibility due to fluid friction is higher and as the aspect ratio increases to the square duct, the irreversibility due to heat transfer increases and both irreversibilities become comparable depending upon the values of Kn and Pe numbers. In both cases, the Bejan number decreases with increasing Kn or Pe numbers for the square duct. The second law of thermodynamics combines the irreversibility due to fluid friction and heat transfer. In all thermal systems, the entropy changes due to these irreversibilities. The higher the irreversibility due to fluid friction, the higher will be the irreversibility distribution ratio. Figures 9 (a) and (b) demonstrate the variation of irreversibility distribution ratio with aspect ratio for different values of Kn and Pe numbers respectively. In both cases, the irreversibility due to friction is higher for smaller aspect ratios and decreases with increasing Kn or Pe numbers. For square ducts, both irreversibilities due to fluid friction and heat transfer are found to be equivalent.



(a) (b)

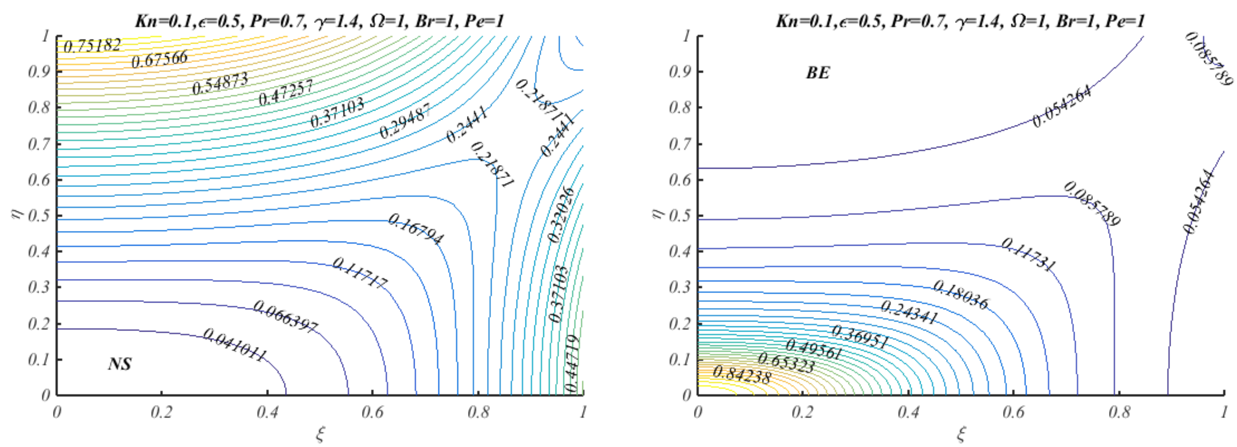
(c) (d)

Figure 5: Surface plots for (a) Entropy generation rate due to fluid friction, (b) Entropy generation rate due to heat, (c) Total entropy generation rate, and (d) Bejan number.



(a) (b)

Figure 6: Entropy generation contours in a microduct due to (a) heat irreversibility and (b) fluid friction reversibility.



(a) (b)

Figure 7: (a) Contours for total entropy generation rate and (b) Bejan number contours in a microduct.

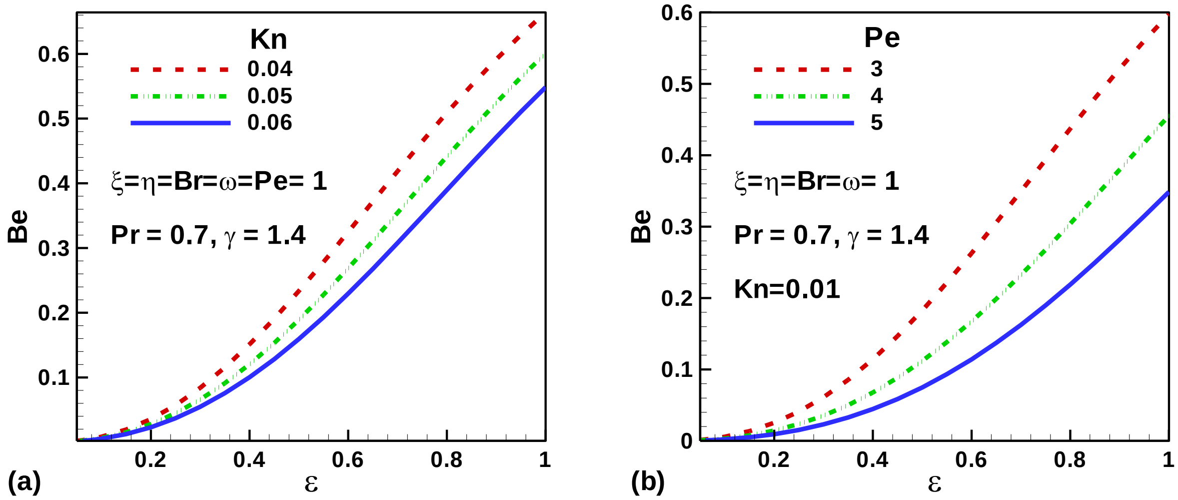


Figure 8: Variation of Bejan number with aspect ratio for different values of (a) Kn, and (b) Pe.

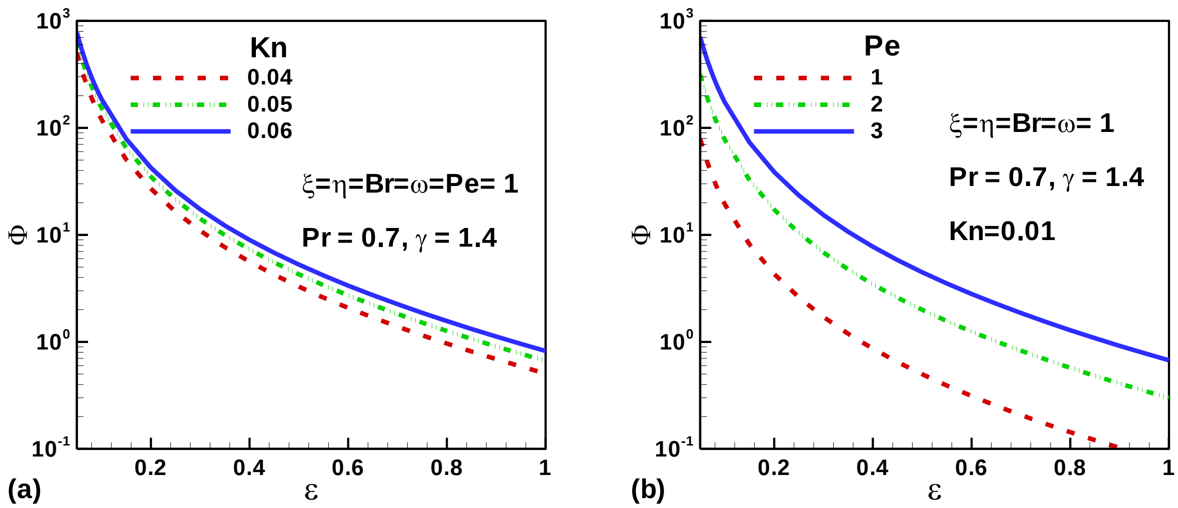


Figure 9: Variation of irreversibility distribution ratio with aspect ratio for different values of (a) Kn, and (b) Pe

5. Conclusions

In this study, a model for the total entropy generation rate is developed for rectangular microducts. The Knudsen numbers are considered in the range between 0.001 and 0.1, and the aspect ratio lies between 0 and 1. The main results are summarized as:

- The entropy generation rate due to heat transfer irreversibility decreases, whereas, the entropy generation rate due to fluid friction irreversibility increases with aspect ratio.
- The minimum entropy generation rate exists corresponding to an optimal aspect ratio in each case.
- The fluid friction irreversibility is dominant near the left and top surfaces of the duct.
- In the case of a parallel plate channel, the irreversibility due to fluid friction is higher.
- The irreversibility distribution ratio decreases with aspect ratio for different dimensionless numbers.

Nomenclature

A	Cross-sectional Area
u_m	Average fluid velocity
w	Dimensionless velocity profile
θ	Dimensionless Temperature profile
p	Pressure
T	Temperature
ε	Aspect ratio
a	Semi-major axis
b	Semi-minor axis
Be	Bejan number
Br	Brinkman number
Pe	Péclet number
Kn	Knudsen number
N_s	Total entropy generation rate
$N_s f$	Entropy generation rate due to fluid friction
$N_s h$	Entropy generation rate due to heat
Φ	the irreversibility distribution ratio
σ	tangential momentum
σ_t	thermal accommodation coefficients
γ	specific heat

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