EFFECT OF FLOW SEPARATION OF TiO$_2$ NANOFUID ON HEAT TRANSFER IN THE ANNULAR SPACE OF TWO CONCENTRIC CYLINDERS

by

Tuqa ABDULRAZZAQ*, Hussein TOGUN$^b$, Mohammad REZA SAFAEI$^{c,d}$, Salim Newaz KAZI$^e$, Mohd Khairol Anuar bin Mohd ARiffin$^f$, and Nor Mariah ADAM$^f$

*a Petroleum and Gas Engineering Department, University of Thi-Qar, Nassiriya, Iraq
$b$ Biomedical Engineering Department, University of Thi-Qar, Nassiriya, Iraq
$c$ Division of Computational Physics, Institute for Computational Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam
$d$ Faculty of Electrical and Electronics Engineering, Ton Duc Thang University, Ho Chi Minh City, Vietnam
$e$ Mechanical Engineering Department, University of Malaya, Kuala Lumpur, Malaysia.
$f$ Mechanical and Manufacturing Engineering Department, Universiti Putra Malaysia, Selangor, Malaysia

Original scientific paper
https://doi.org/10.2298/TSCI180709321A

Introduction

Heat transfer enhancement methods were widely investigated by numerous researchers in the recent past who explored the effect of geometry (such as micro-channel, mini-channel, tube, etc.) on the heat transfer coefficient for the flow in a sudden expansion or contraction of...
backward or forward-facing steps or on ribbed channels. Abu-Mulaweh [1] studied the turbulent heat transfer of air-flow over a vertical forward-facing step experimentally. They also studied the influence of the height of step on the distribution of local Nusselt number and declared that the local Nusselt number enhances with the augmentation of the height of step in the reattachment point. By employing the $k$-$\varepsilon$ turbulence model, Orselli and Lemos [2] numerically simulated the turbulent flow and heat transfer through a sudden contraction with porous insert and indicated that porous insert leads to the reduction of circulation region. Nassab et al. [3] numerically studied the efficacy of step height and slope angle on flow and heat transfer in a single forward-facing step. By using laser Doppler measurement, Armaly et al. [4] numerically and experimentally investigated the efficacy of Reynolds number on reattachment length of flow over a backward-facing step inside a rectangular micro-channel and concluded that, reattachment length of flow increases with the increment of Reynolds number. Lee and Matteescu [5] experimentally and numerically investigated 2-D air-flow at Reynolds numbers less than 3000 over a backward-facing step and evaluated the separation and reattachment lengths at the top and bottom of the tube by employing a heated wire with expansion ratios of 1.17 and 2. They observed results similar to those in the previous experimental studies regarding separation and reattachment length at the top and bottom wall of the tube. Separation and reattachment regions of different fluid-flow over the backward and forward-facing arrangements were investigated by numerous researchers [6-12], and they achieved similar results.

By employing $k$-$\varepsilon$ model, Yilmaz and Oztop [13] numerically investigated the turbulent flow and heat transfer on a double forward-facing step where the bottom of the wall was heated, and the top of wall and steps were insulated. They figured out that, step ratio had a more considerable influence on heat transfer improvement than length ratio. Oztop et al. [14] numerically studied the turbulent flow and heat transfer over a double forward-facing step with a barrier in which the top of the wall was isolated, and the bottom of the wall and steps were heated. They concluded that the aspect ratio of barrier, $Ar$, had a considerable influence on heat transfer augmentation, and the highest Nusselt number was achieved at $Ar = 1$. Neary and Sotiropoulos [15] numerically studied the laminar fluid-flow inside a 90° diversion rectangular cross-section and indicated that the augmentation of the aspect ratio of the tube had a considerable influence on the flow domain. Barbosa et al. [16] numerically studied 3-D mixed and laminar fluid-flow over a backward-facing step with the finite volume method and indicated that, with the increase of Richardson number, the size of primary circulation area decreased.

In the recent past, the nanofluids were widely considered in heat transfer research because of their features. Compared to micron-sized particles, nanoparticles have more surface areas and potential in heat transfer augmentation. Hence, nanofluids can be used in the designing of smaller and lighter heat exchangers.

Abu-Nuda [17] studied the laminar flow of nanofluids with different nanoparticles, including Cu, Ag, Al$_2$O$_3$, CuO, and TiO$_2$ of 0-2% volume fractions at Reynolds numbers of 200-600 in the backward-facing step. He found that, in the circulation areas, the use of comparatively lower thermal conductive TiO$_2$ nanoparticles, the Nusselt number can still be improved. Kherbeet et al. [18] investigated the laminar nanofluid-flow in a micro-scale backward-facing step and observed that, with the decreasing of nanoparticles diameter, Nusselt number enhances significantly.

The purpose of this study is to investigate momentum transfer and heat transfer enhancement in an annular channel with abrupt contraction via SST $k$-$\omega$ turbulence model and compare the obtained results with the results of previous studies in this field.
Numerical methods

Geometrical dimensions

Figure 1 represents the sectional diagram of the test section configuration, flow phenomena, and the heat flux on the geometry. The model pattern and meshing process were performed by ANSYS ICEM, which was exported to ANSYS FLUENT, fig. 2. The diameter and length of the outer cylinder of the central pipe are 100 mm of 1000 mm, respectively. Also, the diameters of the outer cylinder of the exit pipe are 100 mm, 80 mm, 60 mm, and 50 mm with a length of 1500 mm. Both entrance and exit pipes are heated under uniform heat flux. The inner tube has a constant diameter and length of 25 mm and 2500 mm, respectively and it is insulated.

Boundary conditions

In the current numerical simulation, the applied boundary conditions were varied from Reynolds numbers of 10000-40000, contraction ratios of 1-2, heat flux of 6000 W/m² and TiO₂ nanoparticles volume fractions of 0.5%, 1%, 1.5%, and 2% in water as the base fluid. Uniform inlet velocity was considered for the inlet section of the entrance pipe. Outlet pressure boundary condition was applied to the outlet section of the discharge pipe. According to the energy and Reynolds averaged Navier-Stokes equations, an iteration of SST k-ω viscous model was employed in the present simulation, which provided appropriate solutions for incompressible, steady and turbulent flow near-wall treatment. To obtain accurate results, the second-order upwind method was utilized, based on the finite volume cell faces. Further, for coupling velocity and pressure equations, the SIMPLE algorithm was employed.

Governing equations

In the current simulation, the continuity, momentum, energy and turbulence equations, eqs. (1)-(4), are presented sequentially [19]:

\[ \rho \frac{\partial}{\partial x_i} (u_i) = 0 \]  \hspace{1cm} (1)

\[ \rho \frac{\partial}{\partial x_j} (u_i u_j) = - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j} \right) \right] + \frac{\partial}{\partial x_j} \left( -\rho \mu' y_j \right) \]  \hspace{1cm} (2)

\[ \frac{\partial}{\partial x_j} \left[ u_i (\rho E) + P \right] = \frac{\partial}{\partial x_j} \left[ \lambda + \frac{c_p H_r}{Pr_i} \right] \frac{\partial T}{\partial x_j} + u_i (\tau_{ij})_{\text{eff}} \]  \hspace{1cm} (3)

where \( E = T + (u^2/2) \), and \((\tau_{ij})_{\text{eff}}\) are the deviatory stress tensor:

\[ (\tau_{ij})_{\text{eff}} = \mu_{\text{eff}} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \]  \hspace{1cm} (4)
The equations of the SST $k$-$\omega$ model presented by Menter [20] and applied by Klinzing and Sparrow [21], Tseng et al. [22], and Lancial et al. [23], can be presented:

\[
\rho \left[ \frac{\partial}{\partial x_i} \left( k u_i \right) \right] = \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + G_k - Y_k \tag{5}
\]

\[
\rho \left[ \frac{\partial}{\partial x_i} \left( \omega u_i \right) \right] = \frac{\partial}{\partial x_j} \left( \Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + D_\omega \tag{6}
\]

The effective diffusivity $\Gamma_k$ and $\Gamma_\omega$ can be written:

\[
\Gamma_k = \mu + \frac{\mu_t \sigma_k}{\sigma_k} \quad \Gamma_\omega = \mu + \frac{\mu_t \sigma_\omega}{\sigma_\omega} \tag{7}
\]

where $\mu_t$ is given:

\[
\mu_t = \alpha^* \frac{\rho k}{\omega} \tag{8}
\]

and $\alpha^*$ is a coefficient for damping the turbulent viscosity which can be calculated as follow eq. (9):

\[
\alpha^* = \alpha_{\infty}^* \left( \frac{\alpha_0^* + \text{Re}_\tau}{1 + \text{Re}_\tau} \right) \tag{9}
\]

The coefficients in eq. (9) are, $\text{Re}_\tau = \rho k/\nu$, $\alpha_0^* = \beta_1/3$:

\[
\beta_i = F_i \beta_{i,1} + (1 - F_i) \beta_{i,2} \tag{10}
\]

The turbulent Prandtl numbers of $k$ and $\omega$ can be calculated [24]:

\[
\sigma_k = \frac{1}{F_i \sigma_{k,1} + \left[ 1 - F_i \right] \sigma_{k,2}} \tag{11}
\]

\[
\sigma_\omega = \frac{1}{F_i \sigma_{\omega,1} + \left[ 1 - F_i \right] \sigma_{\omega,2}} \tag{12}
\]

The blending function, $F_i$, is calculated:

\[
F_i = \tan h \left( \Phi_i^4 \right) \tag{13}
\]

where

\[
\Phi_i = \min \left[ \max \left( \frac{\sqrt{k}}{0.09 \nu y}, \frac{500 \mu}{\rho y^2 \omega} \right), \frac{4 \rho k}{\sigma_{\omega,2} D_{\omega}^{+} y^2} \right] \tag{14}
\]

and

\[
D_{\omega}^{+} = \max \left[ 2 \rho \frac{1}{\sigma_{\omega,2}} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_j} \right]^{10^{-10}} \tag{15}
\]
where $D_\omega$ is the positive portion of the cross-diffusion term.

The $G_k$ indicates turbulent kinetic energy generation by average velocity gradients and $G_\omega$ suggests the production of $\omega$ and $D_\omega$ in-cross diffusional terms:

$$D_\omega = 2(1 - F_1) \rho \sigma_{\omega,2} \frac{1}{\omega} \frac{\partial \omega}{\partial x_j} \frac{\partial \omega}{\partial x_i}$$

The $Y_k$ and $Y_\omega$ show the dissipations of $k$ and $\omega$ because of turbulence:

$$Y_k = \rho \beta^2 k \omega$$

$$Y_\omega = \rho \beta_k \omega^2$$

The $G_k$ can be written:

$$G_k = \tau_{t,ij} \frac{\partial u_i}{\partial x_j}$$

where

$$\tau_{t,ij} = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \left( \frac{2}{3} \rho k \delta_{ij} \right)$$

The $G_\omega$ is also a function of $G_k$:

$$G_\omega = \frac{\rho \alpha}{\mu_t} G_k$$

where $\alpha$ can be obtained:

$$\alpha = \left( \frac{\alpha_{\omega}}{\alpha} \right) \left( \frac{\alpha_{\omega} + \frac{\text{Re}_i}{\text{Re}_k}}{1 + \frac{\text{Re}_i}{\text{Re}_k}} \right)$$

where $\alpha_{\omega}$ is defined:

$$\alpha_{\omega} = F_1 \alpha_{\omega,1} + (1 - F_1) \alpha_{\omega,2}$$

where

$$\alpha_{\omega,1} = \frac{1}{\infty} = \frac{1}{\infty} \sqrt{\beta_{\omega}}$$

$$\alpha_{\omega,2} = \frac{\beta_{\omega,1}}{\beta_{\omega}} = \frac{k^2}{\sigma_{\omega,2} \sqrt{\beta_{\omega}}}$$

Table 1 presents the constants of the SST $k-\omega$ model

<table>
<thead>
<tr>
<th>$\alpha_{\omega,1}$</th>
<th>$\alpha_{\omega,2}$</th>
<th>$\alpha_{\omega,2}$</th>
<th>$\beta_{\omega,1}$</th>
<th>$\beta_{\omega,2}$</th>
<th>$R_i$</th>
<th>$k$</th>
<th>$\beta^*_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.176</td>
<td>1</td>
<td>2</td>
<td>1.168</td>
<td>0.075</td>
<td>0.0828</td>
<td>6</td>
<td>0.41</td>
</tr>
</tbody>
</table>
Thermophysical properties of nanofluids

The effective density of nanofluid, \( \rho_{nf} \), based on Vajjha and Das [25] equation:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{np}
\]  

(26)

where \( \rho_f \) and \( \rho_{np} \) indicate the density of carrying fluid and nanoparticles, respectively.

The specific heat capacity of nanofluid can be obtained:

\[
(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_{np}
\]  

(27)

where \((\rho c_p)_f\) and \((\rho c_p)_{np}\) are the specific heat capacities of the carrying fluid and the nanoparticles, respectively.

The efficient thermal conductivity of the solid-liquid mixtures presented by Xuan and Roetzel [26] and Wang et al. [27]:

\[
k_{eff} = k_f \left[ \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} \right]
\]  

(28)

According to Brickman [28], the viscosity of the suspension correlates with the concentration of the particles in the suspension:

\[
\mu_{eff} = \frac{\mu_f}{(1 - \phi)^{1.5}}
\]  

(29)

This equation is used for low volume fractions of the particles in the suspension \((\phi < 0.05)\).

Numerical procedure

The ANSYS 14 ICEM software was used to generate the computation mesh while the heat transfer and flow equations were solved by ANSYS-FLUENT 14. For obtaining the precise mesh, the ICEM tools for dealing with complex geometry were utilized. The SST \( k-\omega \) model was employed to analyze the heat transfer and TiO\(_2\) nanofluid-flow in the annular space between two concentric cylinders. To investigate grid independence, the stepwise grid sizes were considered, and the data were compared until the differences became minimal, which could be ignored. An annular channel without step, \( CR = 1 \), \( Re = 10000 \), and pure water was allowed to flow with the varied flow velocity. Five sizes of grids were considered to conduct a grid independence study where the fourth grid represented grid independence because the difference between the fifth and the fourth grids was less than 1, tab. 2.

In this study, the data of turbulent flow and heat transfer over a double forward-facing step, the efficacy of the height of step of Oztop et al. [1] were considered for validation of the generated results. According to fig. 3, the present model is in a reasonable compromise with the numerical data of Oztop et al. [14]. Here the presently obtained values for Nusselt number are compared with the results of Oztop et al. [14] for the previously described specific Case 1, at \( Re = 30000 \) and \( T = 313 \) K. It can be observed that the velocity profile is comparable, as the velocity was overestimated by about 3%. Further, for the turbulent flow, heat transfer to air-flow
in an annular channel with sudden contraction by Togun et al. [29] was simulated, and the numerical results are presented in fig. 4. Here the present results could be compared with the data of Togun et al. [29] for contraction ratio of $CR = 2$ and $q = 4000 \text{ W/m}^2$, and it can be seen that the results of the current simulation are agreed with the data of Togun et al. [29].

**Results and discussion**

Here, the efficacy of Reynolds number, nanoparticles volume fraction, and contraction ratio on surface heat transfer coefficient and pressure drop are analyzed and illustrated.

**Influence of Reynolds number**

Profile of surface heat transfer coefficient for pure water and suspension of water at $2\% \text{ TiO}_2$ nanoparticles at different Reynolds numbers and a contraction ratio of 2 are presented.
Abdulrazzaq, T., et al.: Effect of Flow Separation of TiO$_2$ Nanofluid on Heat ...

Effect of contraction ratio

Figure 7 and 8 present the influence of contraction ratio on the surface heat transfer coefficient for 2% TiO$_2$ nanoparticles volume fraction and pure water at Re = 40000 and heat flux of 6000 W/m$^2$. According to the obtained numerical results in all the cases, the surface heat transfer coefficient profile manifests the same trend at the entrance pipe, whereas the differences between heat transfer coefficient graphs are dependent on the difference in contraction passage of channel. The selected graphical presentations represent the efficacy of the height of the step on the surface heat transfer coefficient. Re-circulation flow, which was created before and after the step, has a significant influence on the surface heat transfer coefficient. In the present investigation, the highest value of heat transfer enhancement was obtained at the contraction ratio of 2.

Influence of volume fraction

Figure 9 represents the distribution of surface heat transfer coefficient at various nanoparticles volume fractions (0.5-2%), Re = 40000, and at the contraction ratios of 2. Generally, the enhancement of nanoparticle volume fraction causes the augmentation of the convective heat transfer coefficient from the surface in annular pipe flow with flow passage contraction. Such increment in the heat transfer coefficient is due to the nanofluid thermal conductivity, viscosity improvement, random and migratory movement of nanoparticles, as well as particle-particle collisions and Brownian diffusion. The results demonstrate that the influence of nanoparticles volume fraction with re-circulation flow results in higher thermal performance,
and the highest value of heat transfer coefficient in the present investigation were obtained at 2% TiO$_2$ volume fraction compared to others.

**Influence of heat flux**

Figure 10 presents the distribution of surface heat transfer coefficients for 2% TiO$_2$ nanoparticles volume fraction at Re = 40000 and contraction ratio of 2. It can be seen that there is no remarkable increase in surface heat transfer coefficients with the enhancement of heat flux. Heat transfer coefficients prevail a similar trend in this case.

![Figure 10. Effect of heat flux on the surface heat transfer coefficient for TiO$_2$ at Re = 40000, and CR = 2](image)

**Pressure drop**

Variations of pressure drop for the suspension of 2% TiO$_2$ volume fraction at Re = 40000 and different contraction ratios are illustrated in fig. 11. Results show that the pressure drop suddenly decreases and increases before and after the step, respectively, due to re-circulation flow created by hydrodynamic fluid-flow through the channel. The effects of Reynolds number on pressure drop at 2% TiO$_2$ volume fraction and contraction ratio of 2 are presented in fig. 12. Pressure drop enhances with the augmentation of Reynolds number, and the highest value of pressure drop is obtained at the highest value of Re = 40000 compared to others in the present range of investigation.

**Velocity distribution**

Figures 13 and 14 show the velocity distribution at five positions before and after the step at Re = 40000 and contraction ratio of 2. The fully developed velocity with two circulation areas is observed before and after the step, where the size of the re-circulation area is appeared to decrease with the increasing distance from the step. The re-circulation flow profiles before the step are close to one side of the wall, while after the step, they are almost axisymmetric on channel walls. The effect of contraction ratio on velocity profile at 99 cm before the step and 101 cm after the step for Re = 40000 is presented in figs. 15 and 16. The velocity distribution looks changed with the increase of contraction ratio owing to the re-circulation areas generated before and after the step, which affects raising the thermal performance. In both the cases of
Abdulrazzaq, T., et al.: Effect of Flow Separation of TiO$_2$ Nanofluid on Heat ...

THERMAL SCIENCE: Year 2020, Vol. 24, No. 2A, pp. 1007-1018

before and after contraction, the maximum re-circulation flow is seen at the contraction ratio of 2, compared with the other cases.

**Conclusio**

In this 3-D numerical investigation, flow and heat transfer in the annular space between two concentric cylinders with abrupt contractions were considered. The influence of Reynolds number and contraction ratio on the surface heat transfer coefficient and the increment of heat transfer coefficient were investigated. The maximum heat transfer improvement was observed at the contraction ratio of 2 in the investigated range. The enhancement of heat transfer occurred due to the development of re-circulation flow before and after the step. The
influence of nanoparticle volume fraction on the improvement of the heat transfer coefficient was investigated and observed that at 2% TiO$_2$ volume fraction, the heat transfer enhancement was the highest compared with water data in the investigation range. It could be inferred that the two circulation areas observed before and after the step was the main reason for the improvement in the heat transfer rate. With the increase of the Reynolds number and contraction ratio, the pressure drop enhances and then drops with the distance before the step and gradually increases after the step in all cases.

**Nomenclature**

- $a$ – entrance pipe length, [m]
- $b$ – exit pipe length, [m]
- $c_p$ – specific heat capacity, [J kg$^{-1}$K$^{-1}$]
- $CR$ – contraction ratio ($= D_e/D_i$)
- $D_e$ – entrance pipe diameter, [m]
- $D_i$ – inner pipe diameter, [m]
- $E$ – total energy, [Jkg$^{-1}$]
- $h_x$ – heat transfer coefficient, [Wm$^{-2}$K$^{-1}$]
- $k$ – turbulent kinetic energy, [m$^2$s$^{-2}$]
- $Nu$ – Nusselt number
- $P$ – pressure, [Pa]
- $Pr$ – Prandtl number
- $q$ – heat flux, [Wm$^{-2}$]
- $Re$ – Reynolds number
- $S$ – step height, [m]
- $T$ – temperature, [K]
- $U$ – velocity component, [m/s]
- $Y$ – upstream length, [m]
- $X$, $Y$, $Z$ – Cartesian co-ordinates, [m]

**References**


© 2020 Society of Thermal Engineers of Serbia

Published by the Vinča Institute of Nuclear Sciences, Belgrade, Serbia.

This is an open access article distributed under the CC BY-NC-ND 4.0 terms and conditions