A NOVEL TAYLOR EXPANSION-BASED ONLINE MODELING METHOD FOR HIGH-TEMPERATURE FORGING PROCESS

by

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A novel Taylor expansion-based online modeling method is proposed for high-temperature forging process. The main innovation of this study is to propose a derivable index for high-temperature forging process. This derivable index, which can be evaluated by the discrete data points, is developed to determine the derivability of high-temperature forging process at any points. It is found that the proposed method can obviously improve the prediction accuracy comparing with the traditional TE online modeling method.

Key words: *Taylor expansion, online modeling method, high-temperature forging process, data-driven modeling method*

Introduction

The nonlinearity, discontinuity and multi-factor influences of the practical industrial process make the mathematical model too complex to be accurately established [1]. These difficulties bring challenges to simulate the practical industrial process [2]. Especially, the accurate prediction and control of hydraulic press machine (HPM) is a great challenge in high-temperature forging process due to the high complexity and strong nonlinearity.

To meet these challenges, data-driven modeling methods have been extensively studied over the past few decades [3]. Moreover, some data-driven modeling methods have been proposed and improved, such as the neural network (NN) method [4,5], the fuzzy method [6], and the support vector machines (SVM) [7]. However, for NN method, the number of layers and the neuron number at each layer are difficult to be determined [8]. Also, the largest problem of fuzzy method is the curse of dimensionality. In addition, SVM is time-consuming and cannot used for online modeling [9]. Moreover, almost all data-driven modeling methods have the over- or under-fitting problems when their model structures are not suitably selected. All aforementioned issues pose a great challenge to the online modeling for the practical process. Recently, Taylor expansion (TE) has been widely applied in system modeling [10]. According to the TE principle, any nonlinear system can be approximated by a polynomial [11]. However, two challenges should be met for TE method: (1) how to determine the

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suitable model order in real time; (2) how to deal with the problem that the system may not be continuous and derivable at any points.

In this study, a simple, effective and easily understandable online modeling method is proposed to model the practical high-temperature forging process. In the proposed method, the derivability criterion, which is a novel policy to deal with the limitation of traditional TE model, is developed.

Problem statements and modeling basic

In this section, a novel approach for identifying discontinuous and non-differentiable point is proposed to select the suitable model order. Then, the high accurate online model can be established for the practical process. The method is based on the theorem of function continuity and derivative, which is presented as follow.

Theorem 1 Let $y = f(x)$ be defined on $(x_0 - \delta, x_0 + \delta)$. The sufficient and necessary condition for the existence of $f'(x_0)$ is that $f'_+(x_0)$ and $f'_-(x_0)$ are both significative, and

$$f'_+(x_0) = f'_-(x_0) \tag{1}$$

where $f'_+(x_0) = \lim_{\Delta x \to 0^+} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$, $f'_-(x_0) = \lim_{\Delta x \to 0^-} \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$

According to the above-presented theorem, the derivability criterion for the practical process is proposed as follow:

Proposition 1 If the dynamic characteristics of the practical process at time t is represented by $y(t) = f(x(t))$, the sufficient and necessary condition for the derivability of $y(t) = f(x(t))$ is

$$\left| \frac{y(k+1) - y(k)}{\Delta t} - \frac{y(k) - y(k-1)}{\Delta t} \right| < \varepsilon \tag{2}$$

where the derivable index $\varepsilon > 0$.

Novel online modeling method

Taylor expansion-based models

For the practical time-varying system, TE models are used to predict the *k*th output of system according to the first *k*-1 inputs and outputs. Generally, TE models can be classified into several categories: zero-order model, linear (first-order) model, second-order model, three-order model, and high order model. For the practical time-varying high-temperature forging process, the model order only selected as 0, 1, 2, and 3 in this study.

The predict model can be expressed as:

$$\bar{y}(k+1) = f(u(k), y(k)) \tag{3}$$

where $u(k)$ and $y(k)$ are input and output, respectively. $\bar{y}(k+1)$ is the predictive output. Then, TE models can be presented as:

Zero-order model:

$$\bar{y}(k+1) = f(u(k), y(k)) = y(k) \tag{4}$$

Linear model:

$$\bar{y}(k+1) = f(u(k), y(k)) = y(k) + \frac{\partial f}{\partial u}(u(k) - u(k-1)) + \frac{\partial f}{\partial y}(y(k) - y(k-1)) \tag{5}$$

Second-order model:

$$\begin{aligned}\bar{y}(k+1) = f(u(k), y(k)) &= y(k) + \frac{\partial f}{\partial u}(u(k) - u(k-1)) + \frac{\partial f}{\partial y}(y(k) - y(k-1)) \\ &+ \frac{1}{2!}\left(\frac{\partial^2 f}{\partial u^2}(u(k) - u(k-1))^2 + 2\frac{\partial^2 f}{\partial u \partial y}(u(k) - u(k-1))(y(k) - y(k-1)) + \frac{\partial^2 f}{\partial y^2}(y(k) - y(k-1))^2\right)\end{aligned} \tag{6}$$

Third-order model:

$$\begin{aligned}\bar{y}(k+1) = f(u(k), y(k)) &= y(k) + \frac{\partial f}{\partial u}(u(k) - u(k-1)) + \frac{\partial f}{\partial y}(y(k) - y(k-1)) + \frac{1}{2!}\left(\frac{\partial^2 f}{\partial u^2}(u(k) - u(k-1))^2\right. \\ &\left. + 2\frac{\partial^2 f}{\partial u \partial y}(u(k) - u(k-1))(y(k) - y(k-1)) + \frac{\partial^2 f}{\partial y^2}(y(k) - y(k-1))^2\right) + \frac{1}{3!}\left(\frac{\partial^3 f}{\partial u^3}(u(k) - u(k-1))^3\right. \\ &+ 3\frac{\partial^3 f}{\partial u^2 \partial y}(u(k) - u(k-1))^2(y(k) - y(k-1)) + 3\frac{\partial^3 f}{\partial u \partial y^2}(u(k) - u(k-1))(y(k) - y(k-1))^2 \\ &\left. + \frac{\partial^3 f}{\partial y^3}(y(k) - y(k-1))^3\right)\end{aligned} \tag{7}$$

Parameters identification

Due to its simple polynomial structure, the multiple nonlinear regression method is used for parameter identification.The coefficients of zero-order model are known, but those of linear, second-order and third-order models need to be further identified.

The linear model, Eq. (5), can be transformed into:

$$\bar{y}(k+1) = y(k) + a_{11}(u(k) - u(k-1)) + a_{12}(y(k) - y(k-1)) \tag{8}$$

where a_{11} and a_{12} are the unknown parameters. The multiple nonlinear regression model can be expressed as:

$$Y = XB \tag{9}$$

where the matrices are presented as:

$$B = \begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix}, \quad X = \begin{pmatrix} u(k-1) - u(k-2) & y(k-1) - y(k-2) \\ u(k) - u(k-1) & y(k) - y(k-1) \end{pmatrix}, \quad Y = \begin{pmatrix} y(k-1) - y(k-2) \\ y(k) - y(k-1) \end{pmatrix}.$$

Similarly, the parametersof second-order and third-order models can be identified by the multiple nonlinear regression method.

Selectionof Model

The confirmation of non-differentiable points is the key step to determine model order. The sufficient and necessary condition for the first-order derivability of the system at current points is proposed in proposition 1, and can be rewritten as:

$$\left| \frac{y(k+1) - 2y(k) + y(k-1)}{\Delta t} \right| < \varepsilon \tag{10}$$

Similarly, the sufficient and necessary condition for the second-order derivability of the system at current points can be deduced as:

$$\left|\frac{y(k+1)-3y(k)+3y(k-1)-y(k-2)}{(\Delta t)^2}\right|<1.5\varepsilon \tag{11}$$

The sufficient and necessary condition for the third-order derivability of the system at current points can be deduced as:

$$\left|\frac{y(k+1)-4y(k)+6y(k-1)-4y(k-2)+y(k-3)}{(\Delta t)^3}\right|<2\varepsilon \quad , \tag{12}$$

Obviously, three, six and ten groups of data are required to identify the parameters of the linear, second-order and third-order models, respectively. Thus, the online model is also determined by the number of differentiable points. The sign $flag$ is introduced to mark the non-differentiable points. In the local interval, if $k+1-flag<2$, the practical time-varying system can only be approximated by zero-order model, or approximated by linear model at least. If $k+1-flag<5$, the practical time-varying system can be approximated by linear model, or approximated by second-order model at least. If $k+1-flag<9$, the practical time-varying system can be approximated by second-order model, or approximated by third-order model at least.

After the determination of non-differentiable points and the number of differentiable points, the model order can be confirmed. To predict the (k+1)th output of the practical time-varying system, the procedure to determine the model order is demonstrated in the form of a binary tree as shown in Figure 1. According to Eq. (10), the first-order derivability of the (k-1)th output is determined by the kth, (k-1)th and (k-2)th outputs. If Eq. (10) is not satisfied, it means that the (k-1)th output is non-differentiable. Then, $flag=k-1$, and the model order $n=0$. Otherwise, if $k+1-flag<5$, the model order $n=1$. If $k+1-flag>5$, the second-order derivability of the (k-1)th, (k-2)th and (k-3)th outputs are determined by the kth, …, (k-5)th outputs. If Eq. (11) is not satisfied, it means that all or several of the (k-1)th， (k-2)th and (k-3)th outputs are not second-order differentiable, and the model order $n=1$. Otherwise, if $k+1-flag<9$, the model order $n=2$. If $k+1-flag>9$, the third-order derivability of the (k-2)th, …, (k-7)th outputs are determined by the kth,…, (k-9)th outputs. If Eq. (12) is not satisfied, it means that all or several of the (k-2)th, …, (k-7)th outputs are not third-order differentiable, and the model order $n=2$. Otherwise, the model order $n=3$.

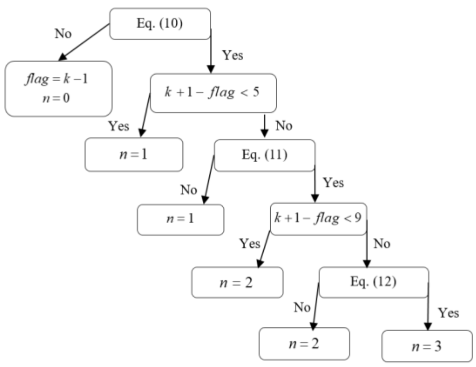


Figure 1. Determination of model order.

The proposed online modeling method

For the practical time-varying process, the proposed online modeling method can be summarized as follows:

Step1: Initialization. The initial inputs and outputs are preset, and $flag = 1$.

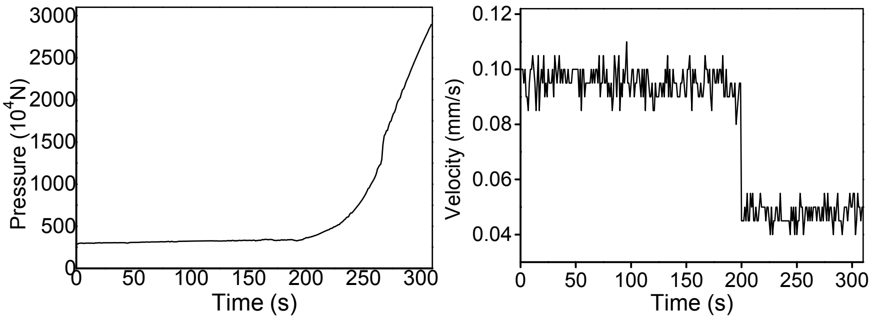
Step2: Model selection. The process of determining model order is presented above.

Step3: Parameter identification. The coefficients of models are identified by the multiple nonlinear regression method.

Step4: Online awareness. The experiment data of the time-varying process are collected online.

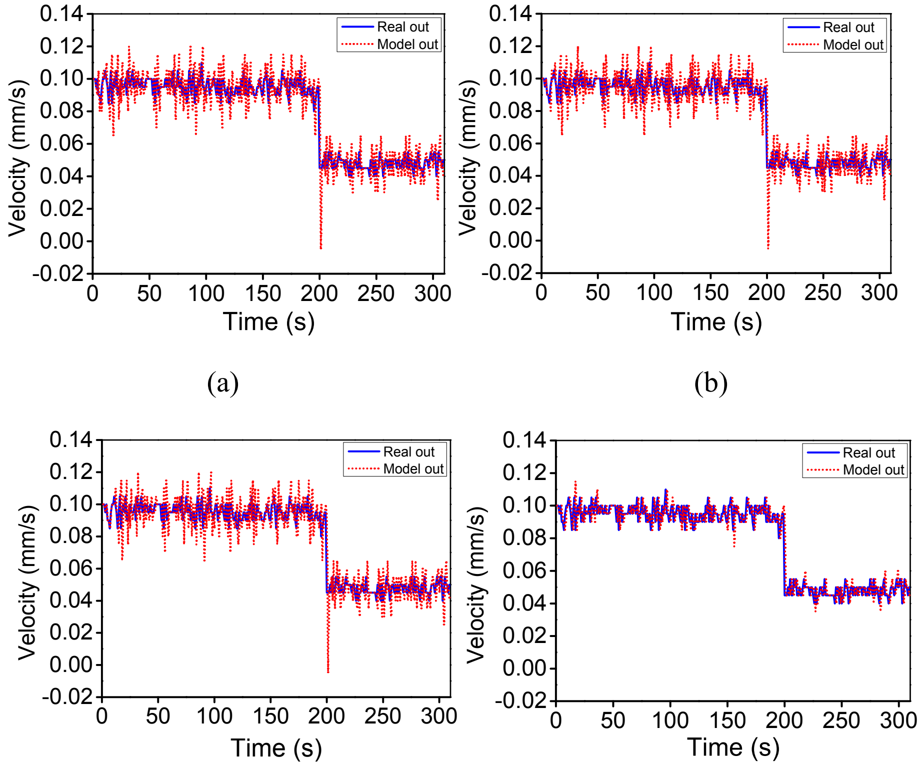
Step5: Re-initialization. When the predicted value $\bar{y}(k+1)$ has been estimated, the truth value $y(k+1)$ is known at a later time.

Step6: Return to Step2 to carry out the next time-step modeling.



(a) (b)

Figure 2.High-temperature forging experiment: (a) input; (b) output.



(a) (b) (c) (d)

Figure3. Model output vs. real output: (a) the linear model; (b) the second-order model; (c)the third-order model; (d)the proposed model

Case study

Practical High-Temperature Forging Case

High-temperature forging is one of metal-forming technologies, and it can effectively decrease the deformation resistance of materials and improve the homogeneity of metal flow [12]. So, it is widely used in the manufacturing the components with complicated geometry and large deformation resistance [13, 14].A high-temperature forging experiment on a large HPM is used to verify the effectiveness of the proposed method. The aluminum alloy work piece is used in this high-temperature forging experiment. The sampling ratio of all sensors is one second, which is sufficient for this experiment. Figure 2showsthe practical pressure of cylinders and the practical velocity, respectively. In this research, the practical pressure of driven cylinders and the velocity of upper die are defined as the input and output, respectively.

With the aid of the developed method, the mathematical model can be written as follows:

$$v(k+1) = f\left(\mathrm{T}(k), p(k), v(k)\right) \tag{13}$$

where v, T and p represent the velocity of upper die, temperature and the pressure of driven cylinders, respectively. Because the high-temperature forging experiment is conducted by isothermal die forging technology, the temperature is constant. So, the model can be simply expressed as follows:

$$v(k+1) = f\left(p(k), v(k)\right) \tag{14}$$

The effectiveness of the proposed modeling method is verified by comparing with the linear, the second-order and the third-order TE models.

From Figure. 3, it is obvious that the proposed model outputs are closer to the real outputs. Thus, the proposed online modeling method could effectively obtain the model for the practical high-temperature forging process. The performances of the linear, the second-order and the third-order models are similar in the practical high-temperature forging case, as shown in Table 1. Clearly, the performance of the proposed model is much better than those of the traditional TE models.

Table 1. Performance comparisons ($\sum_{k=1}^{309} e^2(k)$, e is modeling error)

Method	Performance
Linear model	0.0469
Second-order model	0.0464
Third-order model	0.0461
Proposed model ($\varepsilon = 1\times10^{-5}$)	0.0183

Conclusion

Based on the Taylor expansion, a novel online modeling method is proposed to approximate the practical time-varying system. The derivability criterion of online model is developed according to the Taylor expansion principle, and the derivable index is introduced to achieve the transition from continuous process to discrete process. Finally, the practical high-temperature forging experiment demonstrates that the proposed method can effectively model the time-varying industrial process.

Nomenclature

v –velocity of upper die,[mm/s]

p –pressure of driven cylinders,[N]

T –temperature,[℃]

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