APPLICATION OF A NEW SINGLE STAGGERED GRID METHOD TO THE HEAT-TRANSFER PROBLEMS

by

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In this paper, a new single staggered grid method is proposed to solve the fluid dynamic problems numerically. The advantages of the new grid method are analyzed in comparison with the classical grid algorithms such as the staggered grids, collocated grids and semi-staggered grids. The discretization of the basic equations for the fluid dynamics on the new single staggered grids is derived and the corresponding SIMPLE algorithm is introduced. As an example, the heat-transfer problem of fluid flow at a right angle is solved to prove the validity of the new single staggered grid method.

Key words: *single staggered grid method, SIMPLE algorithm, fluid dynamics, heat-transfer problem*

Introduction

The heat-transfer problems are widely concerned as one of the key scientific problems in nature, and occur in many engineering branches such as the unconventional gas exploitation, geothermal mining and geological storage of CO_2 [1-2]. For the steady heat-transfer problems, analytical solutions are obtained with the aid of the integral transform methods easily (see [3]). However, in practical engineering, the heat-transfer problems are difficult to be solved analytically due to the irregular and complex boundary conditions of different models. The finite volume method (FVM) as an effective numerical method has been widely applied to obtain numerical solutions of complex heat-transfer problems [4-5]. For the FVM, it is important to choose a reasonable grid system for improving the accuracy of numerical solution and reducing the computation time.

Originally, to solve the incompressible fluid flow problem and heat-transfer problems numerically, the staggered grid system was used to release the influence of oscillating pressure field [6-9]. However, when the fluid dynamic equations are discretized on the staggered grids, different variables are stored in three different control volumes, which causes the huge amount of computation,

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especially for three dimensional problems. In view of this problem, Rhie and Chow [10] proposed a momentum interpolation method to the collocated grids. Since all variables of the fluid dynamics are stored in a set of control volume in the collocated grids, it has advantages in reducing complexity of the programming and computational time. It is worth noting that the use of the collocated grid system greatly depends on the momentum interpolation method to coupling the pressure and velocities [10]. In details, the momentum interpolation technique adds a term to the discrete fluid continuity equation for ensuring their integrability and regularity. For example, Armfield [11-12] and Russell [13] added a pressure's fourth-order derivative term and sixth-order derivative item to the continuous equation to avoid the loss of even-odd respectively. However, the collocated grid method has lower precision resulting from the momentum interpolation. In particular, the pressure oscillation phenomenon still appears in the range of small time steps when the above interpolation methods were applied to the unsteady problems [14]. Recently, combining the advantages of staggered grid method and collocated grid method, Ye and Zhang [15] proposed a semi-staggered grid method storing different variables in two different control volumes to solve the Navier-Stokes equations. Liu [16] gave the theoretical derivation of a new single staggered grid algorithm. Aiming to verify the validity of the single staggered grid algorithm, we applied this algorithm to solve the heat-transfer problem of fluid flow at a right angle in this paper.

The remainder of this paper is organized as follow. In Section 2, the new single staggered grid system is introduced graphically. The discretization forms of the fluid dynamic equations on the single staggered grids are presented. The corresponding SIMPLE algorithm is updated. In Section 3, with the aid of the single staggered grid method, the heat-transfer problem of fluid flow at a right angle is solved. In Section 4, we summarize the conclusions.

The single staggered grid system and the SIMPLE algorithm

The single staggered grid system

To illustrate the characteristics of single staggered grid system, four typical grid systems for FVM are shown in Fig. 1. The green regions represent the control volumes of different grid system storing different variables. For the staggered grid system with three sets of control volumes, the scalar variables P,T,ρ (pressure, temperature, mass) are stored on the central node of main control volume and the velocity components u and v are stored on the boundary of their control volumes with half grid difference to the main control volume respectively (see Fig. 1(a)) [17]. For the collocated grid system with a set of control volume, all variables are stored on the central node of main control volume (see Fig. 1(b)) [18]. For the semi-staggered grid system with two sets of control volumes, the scalar variables P,T,ρ are stored on the central node of main control volume and velocity components u and v are stored on the corner node of main control volume (see Fig. 1(c)) [15]. For the single staggered grid system, the scalar variables P,T,ρ and the velocity component v are stored on the central node of main control volume while the velocity component u is stored on the boundary of its control volumes with half grid difference to the main control volume (see Fig. 1(d)).

Obviously, compared with the staggered grids, the single staggered grid system is of less control volumes and it is benefit to design program and reduce the computation time. Moreover, due to the retention of the part staggered grid system, the single staggered grid system is of higher accuracy than the collocated grid system.

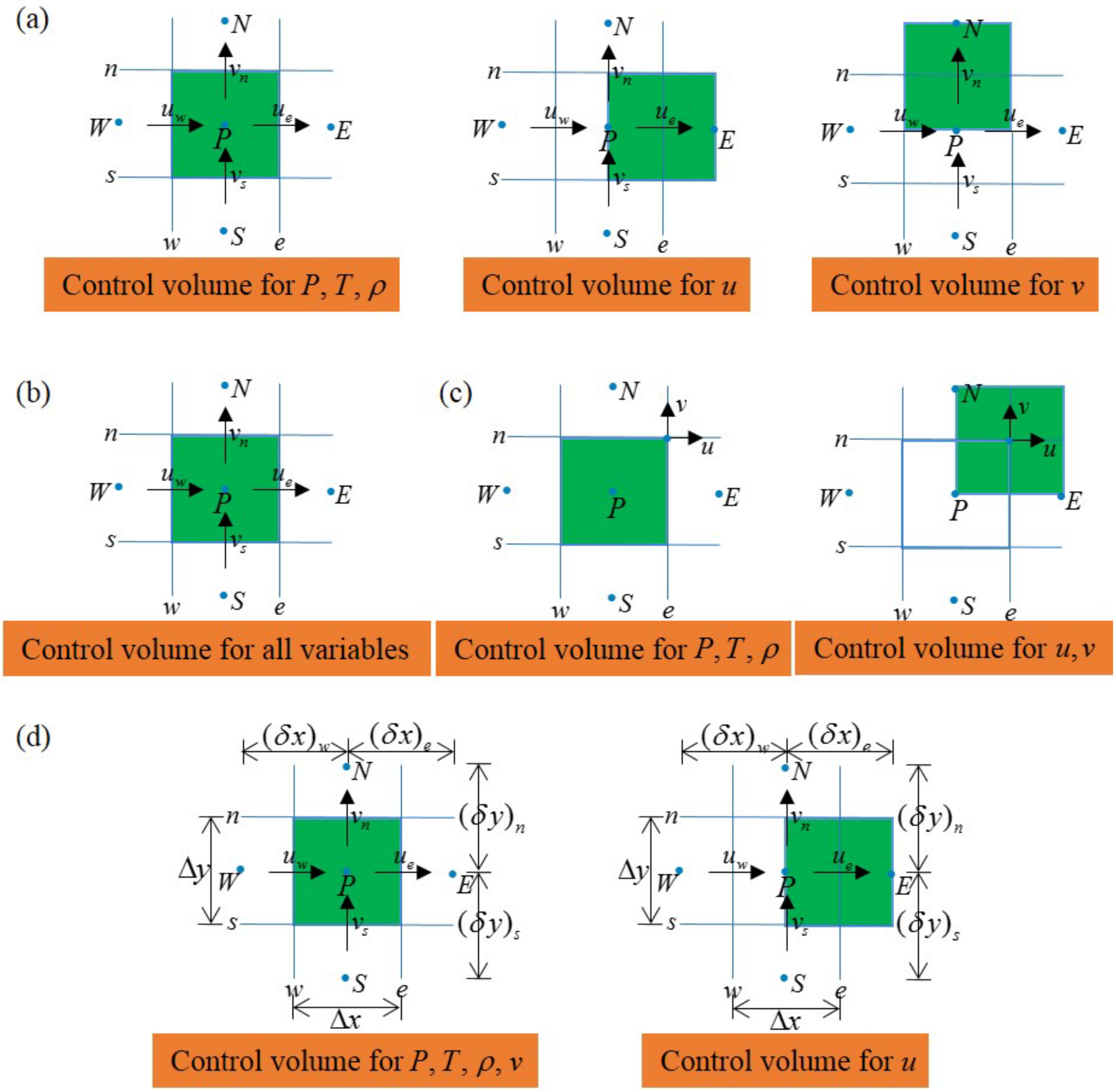

Figure. 1 Sketch maps of different grid systems for the FVM: (a) staggered grid system (b) collocated grid system (c) semi-staggered grid system (d) single staggered grid system

The discretization of fluid dynamic equations on the single staggered grids

The general expression of the fluid dynamic equations is [6]

$$\frac{\partial(\rho\phi)}{\partial t} + div(\rho\mathbf{u}\phi) = div(\Gamma \bullet grad\phi) + S \tag{1}$$

where ρ is the fluid density, ϕ is the variables to be solved, Γ is the diffusion coefficient, S is the source term and div represents the divergence.

When ϕ, Γ and S take different values, Eq. (1) represents different types of fluid dynamic equations. The details are listed in Tab. 1.

Table 1. Basic fluid dynamic equations [6]

The types of equations	ϕ	Γ_ϕ	S_ϕ
Continuity equation	1	0	0
Momentum equation	u_i	μ	$-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_i}(\mu\frac{\partial u_i}{\partial x_i})+\frac{\partial}{\partial x_j}(\mu\frac{\partial u_j}{\partial x_i})+S_{u_i}$
Energy equation	T	k	S_T

The discrete process of basic fluid dynamic equations on the single staggered grid system are displayed as follows.

Considering a two-dimensional steady state problem of fluid dynamic, Eq. (1) becomes [6]

$$\frac{\partial(\rho u\phi)}{\partial x}+\frac{\partial(\rho v\phi)}{\partial y}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+S \tag{2}$$

where u and v are the velocity components.

If ϕ is the scalar variables, the integral of Eq. (2) on the scalar control volume of the single staggered grid system (Fig. 1(d)) is expressed as [6]

$$\int_{\Delta V}\frac{\partial}{\partial x}(\rho u\phi)dV+\int_{\Delta V}\frac{\partial}{\partial y}(\rho v\phi)dV=\int_{\Delta V}\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)dV+\int_{\Delta V}\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)dV+\int_{\Delta V}\frac{\partial}{\partial x}SdV \tag{3}$$

With the aid of central difference method and the linear interpolation method, Eq. (3) is dispersed as [6]

$$\begin{aligned}&[(\rho u\phi)_e-(\rho u\phi)_w]\Delta y+[(\rho v\phi)_n-(\rho v\phi)_s]\Delta x\\&=\left[\frac{\Gamma_e}{(\delta x)_e}(\phi_E-\phi_P)-\frac{\Gamma_w}{(\delta x)_w}(\phi_P-\phi_W)\right]\Delta y\\&+\left[\frac{\Gamma_n}{(\delta x)_n}(\phi_N-\phi_P)-\frac{\Gamma_s}{(\delta x)_s}(\phi_P-\phi_S)\right]\Delta x+\bar{S}\Delta x\Delta y\end{aligned} \tag{4}$$

Simplifying Eq. (4) yields

$$a_P\phi_P=a_W\phi_W+a_E\phi_E+a_S\phi_S+a_N\phi_N+S_o \tag{5}$$

where a_P, a_W, a_E, a_S, a_S, a_N and S_o are variables related to ρ, Δx, Δy, δx, δy, Γ, u and v.

If ϕ is the velocity component u, and the pressure gradient term $\partial p/\partial x$ is separated from the source term S, Eq. (1) becomes [6]

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x}+S_u \tag{6}$$

Similarly, Eq. (6) is dispersed on the single staggered grid system as

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N + b(P_E - P_P) + S_o{}' \quad (7)$$

where P_E and P_P are the pressures of point E and F on single staggered grids (Fig. 1(d)).

If ϕ is the velocity component v, Eq. (1) becomes [19]

$$\frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho vv)}{\partial y} = \frac{\partial}{\partial x}\left(\Gamma\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial v}{\partial y}\right) + S \quad (8)$$

The discrete form of Eq. (8) is same to Eq. (5). Here, to avoid the pressure oscillations, the momentum interpolation method is applied to solve the term a_P. The expression of the momentum interpolation is [19]

$$v_n = \frac{\alpha_v(\sum_i a_i u_i + b_p)_n}{(a_p)_n} - \frac{\alpha_v \Delta x(P_N - P_P)}{(a_p)_n} \quad (9)$$

The SIMPLE algorithm

The SIMPLE algorithm is known as the effective method to solve incompressible flow field [8, 20]. In this paper, after performing the discretization of the fluid dynamic equations on the single staggered grids, the SIMPLE algorithm shown in Fig.2 is used to solve the discrete equations. The general steps of the SIMPLE algorithm are as follows [8, 20]:

(1) It is assumed that the initial velocity distribution and pressure distribution of the flow field are u^*, v^* and p^* respectively. The differences between u^*, v^*, p^* and the actual velocity distribution u, v and pressure distribution p are u', v' and p' respectively. Here, u', v' and p' are called the velocity correction terms and pressure correction term.

(2) Substituting u, v, p, v^*, u^* and p^* into the discrete momentum equations yields

$$\begin{cases} u' = \lambda_1 p' \\ v' = \lambda_2 p' \end{cases} \quad (10)$$

where λ_1 and λ_2 are the variables related to ρ, Δx, Δy, δx, δy, Γ.

(3) Substituting the velocity correction terms u' and v' into the discrete continuity equations and obtaining the pressure correction term p'.

(4) The u, v and p are corrected as

$$\begin{cases} p = p^* + p' \\ u = u^* + \lambda p' \\ v = v^* + \lambda p' \end{cases} \quad (11)$$

(5) Solve the other discrete fluid dynamic equations.

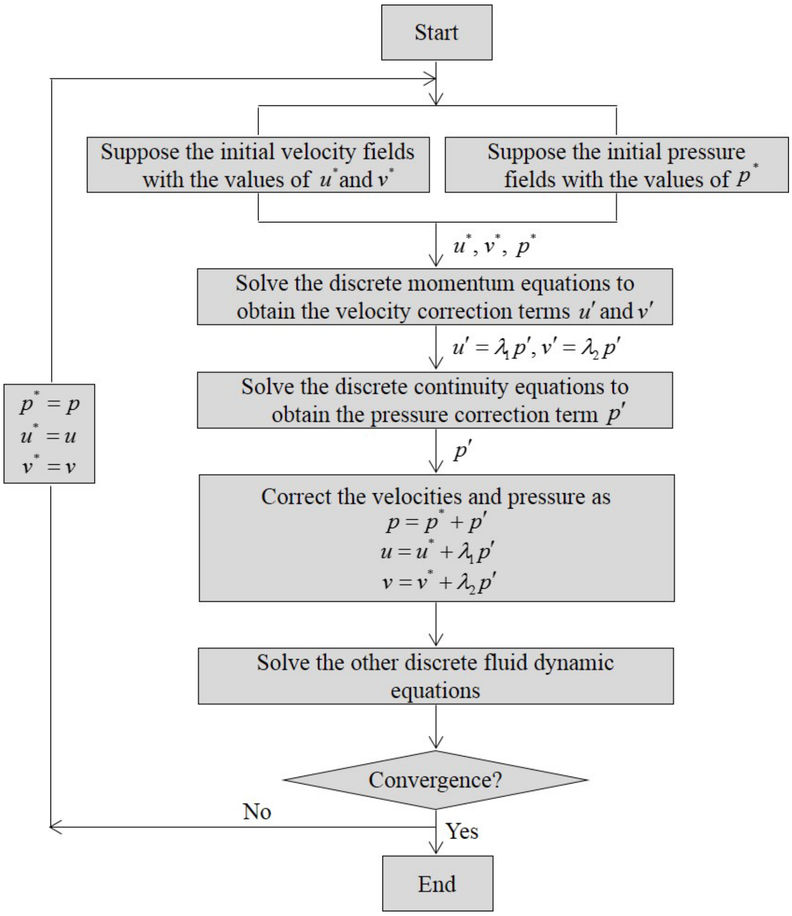

Figure. 2 Flow diagram of the SIMPLE algorithm

Solving the heat-transfer problems

To verify the validity of the new single staggered grid method, a heat-transfer problem of fluid flow at a right angle was solved in this paper. Fig. 3 shows the geometric model. The boundary-value condition is $T = x + y + xy$.

Based on the FORTRAN language, the calculation programs for the above heat-transfer problem were designed by the single staggered grid method and the SIMPLE algorithm. For the comparison, the same heat-transfer problem was also solved by the commercial software COMSOL based on the finite element method. The results are shown in Fig. 4. Obviously, the temperature contours acquired from the single staggered grid method is same to the results from the COMSOL. It is indicated that the new single staggered grid method is effective to solve the heat-transfer problems.

Further, to explore the sensitivity of the new single staggered grid method to the of grid scales, quadrilateral meshes with different scales (5×5, 10×10 and 20×20) were applied to solve the above heat-transfer problem. Fig. 5 shows the temperature distribution along the diagonal line of the model under different grid scales. In addition, the correlation coefficients r between the results based on the single staggered grids with different grid scales and the COMSOL were calculated. As shown in Tab. 2, all the correlation coefficients are more than 0.99. Clearly, the results from the 20×20 single staggered grid scales are the closest to results from the COMSOL. The new single staggered grid method is slightly sensitive to grid scales.

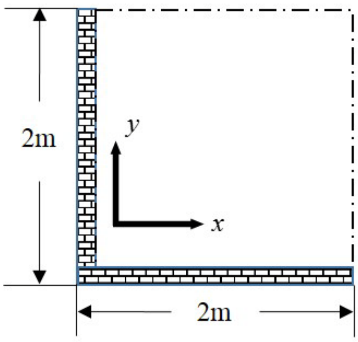

Figure. 3 Geometric model of the heat-transfer problem of fluid flow at a right angle

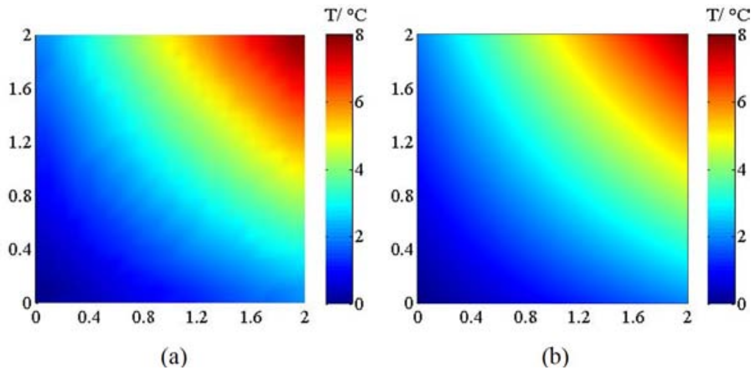

Figure. 4 Temperature contours: (a) numerical results based on the single staggered grid method (b) numerical results based on the COMSOL

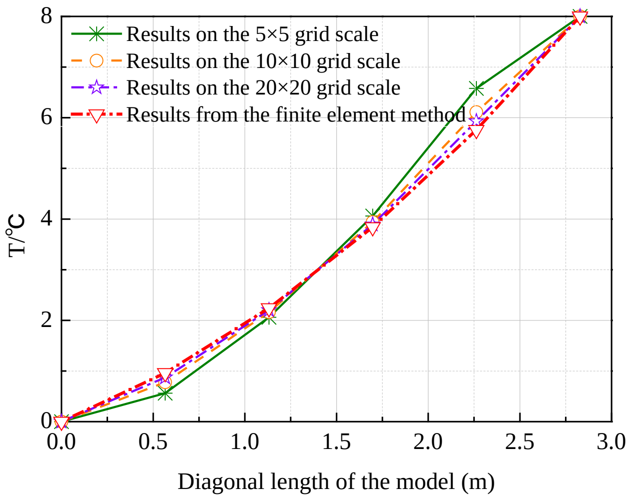

Figure. 5 The temperature distribution along the diagonal line of the model

Table 2. The correlation coefficient r between the results from the single staggered grid method and the COMSOL

Grid scales	5×5	10×10	20×20
Correlation coefficients r	0.9940	0.9987	0.9997

Conclusion

In this paper, a new single staggered grid method was proposed to solve the heat-transfer problems numerically. The single staggered grid method is of less grid control volumes than classical staggered grid system and higher accuracy than the classical collocated grid system. Based on the single staggered grid system and the SIMPLE algorithm, the programs by the FORTRAN language was designed to solve the heat-transfer problem of fluid flow at a right angle. The numerical results are almost the same as those calculated by the COMSOL. It is indicated that the new single staggered grid method is valid and effective to solve the fluid dynamic problems.

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Nomenclature

p - fluid pressure, [Pa]	T - temperature, [℃]
ρ - fluid density, [kg/m^3]	μ - fluid dynamic viscosity, [Pa·s]
Δx - grid interval in x direction, [m]	Δy - grid interval in y direction, [m]
u - fluid velocity component in x direction, [m/s]	k - thermal diffusion coefficient, [m^2/s]
v - fluid velocity component in y direction, [m/s]	t - time, [s]

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