

## FLOW AND HEAT TRANSFER CHARACTERISTICS OF NANOFLUIDS IN SUDDEN EXPANSION STRUCTURE BASED ON SLA METHOD

by

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*The current work aims at a fundamental understanding of the concept of head loss coefficient,  $K$ , of nanofluids flowing in sudden expansion pipe. While so far several articles have applied this concept to the laminar flow regime of water, it is extended here to the mechanics of nanofluids. To describe the flow dissipation, a thermodynamic model is built based on the Second law analysis approach to calculate the overall entropy generation with the assistance of appropriate single-phase models used to get viscosity values of nanofluids. Then, specific values of  $K$  can be determined by the integration of entropy generation field. In addition, considering the thermodynamic irreversibility caused by temperature gradients due to heat transfer processes, a new concept of thermodynamic loss coefficient,  $K_E$ , has been applied to calculate total dissipation. The correlations between  $K$  and Reynolds number of sudden expansion flows are also derived. It is interesting to note that the results reveal some striking similarities among nanofluids of various volume concentrations. This unexpected phenomenon shows that the  $K$  value is independent of the volume concentration (within the scope of the study). Furthermore, the results show that with an increase in both nanofluid concentration and temperature rise in the heated section, the  $K_E$  and Nusselt number increases accordingly.*

*Key words: nanofluids, sudden expansion, Second law analysis, Nusselt number, entropy generation,*

### Introduction

The nanofluids are dilute liquid suspensions containing nanopowders with dimensions smaller than 100 nm. Due to their improved thermal characteristics, nanofluids have been attracting increasing research interest since their first application by Choi *et al.* [1], at Argon national laboratory two decades ago. Pak and Cho [2] used experiment method to perform viscosity, pressure loss, and heat transfer measurements on titanium dioxide nanofluids for up to 3% volume fraction. Li and Xuan [3] and Xuan and Li [4] investigated the heat transfer coefficient and friction factor of Cu-water nanofluid up to 2% volume concentration.

In addition experimental method, the numerical simulation is also applied in many studies with the assistance of CFD approach on convective heat transfer of nanofluids. In general, numerical studies can be categorized into two types. One is a two-phase modelling and the second is a single-phase model. Many illustration works have been done by comparing the results of heat transfer of nanofluids obtained through the single-phase model or two-phase

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model with the experimental data [5, 6]. Some studies even make further efforts to compare the accuracy of the heat transfer of nanofluids between the two models [7, 8].

However, all the mentioned studies analyze the heat transfer process from the same view, *i. e.* the convective heat transfer view, which means the key elements in the study are heat transfer coefficient and Nusselt number. Meanwhile, it has been proven that the entropy production approach, *i. e.* the Second law approach is a powerful tool in many applications, including the loss coefficient of laminar or turbulent flows. This forms the motivation of this work.

In this article, the flows in sudden expansion are investigated by a combination of single-phase model and thermodynamic model based on the SLA, which will be introduced in detail in next section.

### A combination of theoretical model of nanofluids

#### *The viscosity of nanofluids*

Because of the simplicity of single-phase approach and the fact that nanofluids can be easily fluidized [5, 7], single-phase model is applied to this simulation.

#### *Classical theoretical models*

With great efforts to extend the Einstein model to concentrated suspensions, numerous models were developed during the past few decades. Among variety models, because of the simplicity and certain accuracy, Brinkman model has been widely used in many works [9]. Therefore, this model is used for suspension with low concentration in our study, which is written [10]:

$$\eta_{\text{eff}} = \eta_0(1 - \varphi)^{-2.5} \quad (1)$$

where  $\eta_{\text{eff}}$  is the effective viscosity of the suspension, and  $\eta_0$  – the dynamic viscosity of the base fluid.

#### *New theoretical model*

Because of neglecting the consideration of some very salient characteristics such as nanolayer, pH and agglomeration that influence the thermal properties of suspensions, most classical models failed to predict the viscosity accurately. With the consideration of agglomeration, an important factor that affected the viscosity, Chen *et al.* [11] gave a greatly improved model:

$$\eta_{\text{eff}} = \eta_0 \left( 1 - \frac{\varphi}{0.605} 3^{1.2} \right)^{-1.5125} \quad (2)$$

The scope of volume fraction according to most application and simulation in articles ranges from 0.5-6%, including six circumstances ( $\varphi = 0.5\%$ ,  $0.8\%$ ,  $1.0\%$ ,  $2.0\%$ ,  $4.0\%$ , and  $6.0\%$ ). The volume concentration of  $0.5\%$  and  $0.8\%$  is used Brinkman model, while the Chen model is applied to the rest for relatively high concentrations of suspension particles.

### Thermodynamic model

#### *The $K$ and $K_E$ concept for laminar flow and Re-dependence*

A general definition of the head loss coefficient,  $K$ :

$$K = \frac{h_w g}{u_m^2 / 2} = \frac{2g h_w}{u_m^2} \quad (3)$$

where  $h_w$  associated with the flow between two cross-sections. Since  $u^2/2$  is the specific kinetic energy (*i. e.* the kinetic energy per mass,  $m$ ),  $K$  – the ratio of the dissipated and the kinetic energy in a conduit component. For conduit components with fluids flowing in it keeping laminar state, the relationship between  $K$  and Reynolds number can be divided into two circumstances. When it is creeping flow, *i. e.*,  $u_m \rightarrow 0$ ,  $Re \rightarrow 0$ , the inertia forces can be neglected by definition and the friction forces are proportional to velocity. With Reynolds number value getting larger (keeping laminar state), then the inertia forces become dominant in the flow, and the frictions forces change to be proportional to the square of velocity. Hence, for higher accuracy, we may tentatively assume that  $K$  has a general form proposed by Churchill and Usagi [12]:

$$K = \left[ C_1^n + \left( \frac{C_2}{Re} \right)^n \right]^{1/n} \quad (4)$$

From the analyzation, we have realized that losses in a flow field occur due to the dissipation of mechanical energy by the viscous flow. Thus, (keeping at environmental temperature) it is also a corresponding conversion of exergy into energy with the accompanying of entropy production. But, if the dissipation occurs on a different temperature level, *i. e.*,  $T_m \neq T_0$ , the situation is different. The  $K$  based on eq. (3) quantifies the head loss, but not corresponding to exergy loss, which is the real loss in the thermodynamic point of view. Therefore, we define an exergy loss coefficient using thermodynamics, which is given [12]:

$$K_E = \frac{2T_0 \dot{S}_D}{\dot{m} u_m^2} \quad (5)$$

where  $\dot{S}_D$  is the overall entropy generation due to the conduit component.

#### Determination of the $K$ -value and $K_E$ -value

Next, we need to build a calculation method about the overall entropy production. In a laminar flow field, the local entropy generation rate in Cartesian co-ordinates [13]:

$$\dot{S}_D^m = \frac{\mu}{T} \left\{ 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right\} \quad (6)$$

with the integral form, the specific dissipation can be expressed:

$$\varphi_{12} = \frac{1}{\dot{m}} \int_{V_c} \Phi^m dV \quad (7)$$

where  $\varphi_{12} = gh_w$  is as the specific dissipation. It is necessary to note that the influence caused by conduit component will be extended to upstream and downstream of flow field. Therefore, the total dissipation occurs in the overall control volume can be expressed in another form [12]:

$$\frac{1}{\dot{m}} \int_{V_c} T_m \dot{S}_{tot}^m dV = \left\{ \frac{1}{\dot{m}_u} \int_{V_u} T_m (\dot{S}^m - \dot{S}_0^m) dV + \frac{1}{\dot{m}_c} \int_{V_c} T_m \dot{S}^m dV + \frac{1}{\dot{m}_d} \int_{V_d} T_m (\dot{S}^m - \dot{S}_0^m) dV \right\} \quad (8)$$

Apparently, the eq. (8) can be simplified to eq. (9):

$$\int_{V_c} \dot{S}_{tot}^m dV = \int_V \dot{S}^m dV - \int_{V_u} \dot{S}_0^m dV - \int_{V_d} \dot{S}_0^m dV \quad (9)$$

where  $V = V_c + V_d + V_u$  is the total volume of the flow field. Therefore, the head loss coefficient  $K$  can be re-expressed [12]:

$$K = \frac{2T_m}{\dot{m}u_m^2} (\dot{S}_V - \dot{S}_{u,0} - \dot{S}_{d,0}) = \frac{2T_m \dot{S}_{tot}}{\dot{m}u_m^2} \quad (10)$$

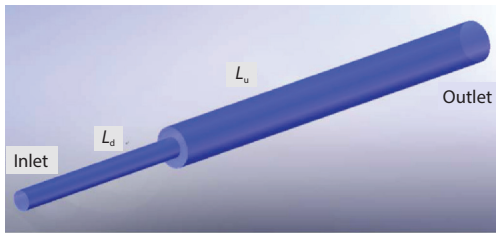
where  $\dot{S}_V$ ,  $\dot{S}_{u,0}$  and  $\dot{S}_{d,0}$  are the entropy production rate of the whole flow system, upstream and downstream of the component in the undisturbed flow, respectively, [12]. In order to determine the value of  $K$ , it is necessary to compute the entropy production rate according to eqs. (8)-(10). As for the entropy production rate of undisturbed flow in the upstream and downstream pipes, the exact values can be obtained from the corresponding analytic solutions. With the consideration of temperature effect, the values of  $K_E$  need to be calculated. The same method of calculating  $K$ -value can be used to this process.

#### Calculation of the heat transfer property

If nanofluids flow through a heating pipe, the temperature effect is an important factor to influence flow dissipation and it is necessary to calculate heat transfer property such as the Nusselt numbers varying with Reynolds numbers. Thermal conductivity of nanofluids containing spherical nanoparticles can be calculated using Maxwell's model [14]:

$$\frac{k_{nf}}{k_{bf}} = 1 + \frac{3(k_p - k_{bf})\phi}{(k_p + 2k_{bf}) - (k_p - k_{bf})\phi} \quad (11)$$

where  $k_{bf}$ ,  $k_p$ , and  $k_{nf}$  are thermal conductivity of base fluid, particle and nanofluids, respectively.



**Figure 1.** Geometrical details of the sudden-expansion  $L_u = 10 D$ ,  $L_d = 20 D$  ( $D$  is the inner diameter of inlet section)

After building the complete theory model, next step is to determine the geometry size. Figure 1 shows the geometry of a flow system (laminar in sudden expansion) with a smaller circular cross-section whose  $D_{h,u} = 4$  mm. The larger one corresponding to the outlet section is  $D_{h,d} = 8$  mm. The flow in the pipe is assumed to be isothermal with the fluid field temperature keeping at ambient level. As for the process of calculating Nusselt number, the up section wall is under 373 K heating condition, while the down section is under adiabatic condition.

The actual calculation of  $\dot{S}_{tot}$  based on eqs. (9) and (10) is performed using the commercial CFD package ANSYS. Meanwhile, grid independence has been validated.

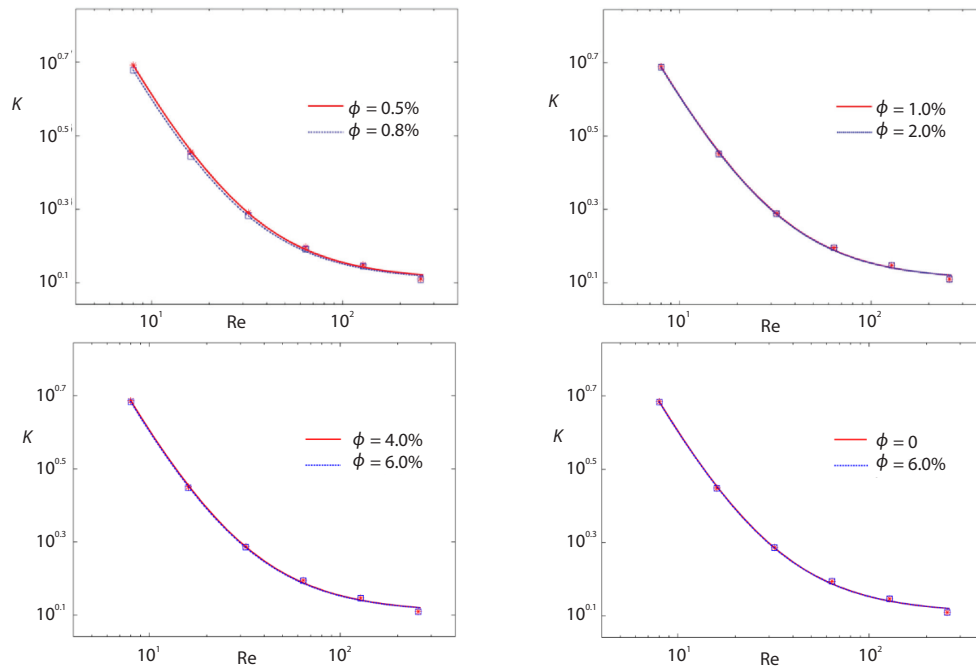
**Table 1.** The  $K$  and entropy production rate  $\dot{S}_{tot}$  for different Reynolds number of the concentration  $\phi = 1.0\%$  and  $2.0\%$  in sudden expansion

$\phi = 1.0\%$			$\phi = 2.0\%$		
Re	$\dot{S}_{tot}$ [WK <sup>-1</sup> ]	$K$	Re	$\dot{S}_{tot}$ [WK <sup>-1</sup> ]	$K$
8	0.09753e-11	4.887	8	0.11507e-11	4.877
16	0.45315e-11	2.838	16	0.53354e-11	2.827
32	2.46834e-11	1.947	32	2.93450e-11	1.943
64	1.60534e-10	1.571	64	1.89562e-10	1.569
128	1.14950e-09	1.406	128	1.35772e-09	1.405
256	8.43208e-09	1.289	256	9.96005e-09	1.288

#### Calculation results of $K$ -value

With the models having been built from analyzation of previous section, the data of head loss coefficient  $K$  for different Reynolds number under different volume concentrations can be obtained from calculation and presented in tab. 1.

Then the  $K$  (Re) equation can be determined by fitting the data presented in the tab. 1 with the eq. (10). Same with the concentration with 1.0% and 2.0%, other fitting curves with different concentration conditions can be obtained in the same way. Therefore, the constants  $C_1$ ,  $C_2$ , and  $n$  can be derived, the fitting curve is shown in fig. 2.



**Figure 2. Comparison of different head loss fitting curves with different volume concentrations;**  
 (a) fitting of the head loss coefficient  $K = [C_1^n + (C_2/Re)^n]^{1/n}$  ( $\phi = 0.5\%$ ,  $0.8\%$ ),  
 (b) fitting of the head loss coefficient  $K = [C_1^n + (C_2/Re)^n]^{1/n}$  ( $\phi = 1.0\%$ ,  $2.0\%$ ),  
 (c) fitting of the head loss coefficient  $K = [C_1^n + (C_2/Re)^n]^{1/n}$  ( $\phi = 4.0\%$ ,  $6.0\%$ ), and  
 (d) fitting of the head loss coefficient  $K = [C_1^n + (C_2/Re)^n]^{1/n}$  ( $\phi = 6.0\%$ , water)

The values of three constants are listed in tab. 2.

**Table 2. Values of three constants**

Concentration	Constants		
$\phi$	$C_1$	$C_2$	$n$
0	1.2830	34.2705	1.4067
0.5%	1.2865	34.9482	1.4057
0.8%	1.2803	33.6634	1.4053
1.0%	1.2838	34.6435	1.4024
2.0%	1.2852	34.6358	1.4125
4.0%	1.2838	34.4327	1.4055
6.0%	1.2836	34.1788	1.4089

In order to get the mechanism influence of nanoparticles on the fluid-flow dissipation, the data of pure water is also calculated. From the four pictures and the  $K$ -value listed in the previous charts, we note that nanofluids with different concentrations obey the same regularity, which indicates that the  $K$ -value is independent of the volume concentration. With Reynolds

number getting larger, the absolute values of energy dissipation increase as well, while the values of head loss coefficient decrease in sudden-expansion. It is apparent from the information supplied that the nanofluids with higher volume concentrations generate more flow dissipation. This phenomenon is due to the increasing in viscosity caused by the suspension of nanoparticles.

### Calculation results of heat transfer process

The results of exergy loss coefficient for different Reynolds numbers under 4% volume concentration can be obtained from calculation and presented in tab. 3.

**Table 3. Exergy loss coefficient of nanofluids**

Re	$\dot{S}_i$ [WK <sup>-1</sup> ]	$\dot{S}_{u,0}$ [WK <sup>-1</sup> ]	$\dot{S}_{d,0}$ [WK <sup>-1</sup> ]	$\dot{S}_{tot}$ [WK <sup>-1</sup> ]	$K_E$
8	3.66603e-12	3.16138e-12	6.50117e-12	-5.99652e-12	-575
16	1.59928e-11	1.37132e-11	2.60529e-11	-2.37733e-11	-142
32	7.282e-11	6.09195e-11	1.04645e-10	-9.27445e-11	-68.1
64	3.54669e-10	2.74373e-10	4.22238e-10	-3.41942e-10	-30.4
128	1.7524e-09	1.25274e-09	1.71841e-09	-1.21875e-10	-1.31

As for the heat transfer process, the calculation results of Nusselt number are shown in tab. 4.

**Table 4. Nusselt number varying with Reynolds number**

Re	Surface [m <sup>2</sup> ]	$\Delta T$ [K]	$q$ [W]	Nu
8	0.002511	0.63	3.7791	13.5
16		1.8	6.5061	8.12
32		7.3	11.3145	3.48
64		18.7	17.983	2.16
128		30.4	26.353	1.95

From the tab. 4 we know that, the Nusselt number is decreasing with the increasing of Reynolds number. Therefore, the heat transfer is worse when the velocity of nanofluids increase. It is interesting to note that, from the tab. 3, the  $K_E$  is negative, which means under heating condition, the dissipation of sudden expansion conduct is smaller than the total exergy loss of up section and down section. Therefore, in the high temperature heat transfer process, nanofluids can apparently decrease the total entropy production. However, it is important to note that, the down section calculation is based on the adiabatic condition. So, the down section's temperature is much lower when calculate separately. Therefore, the real absolute value of  $K_E$  should be smaller than the calculated.

### Conclusion

The work based on the Second law analyzation approach, has studied the head loss coefficient,  $K$ , for different Reynolds number under different volume concentrations of nanofluids flowing in the sudden-expansion pipe. The comparison results in each group show that the  $K$ -value is independent of the volume concentration (within the scope of the study). We also find that with Reynolds number getting larger, the  $K$ -value and energy dissipation show opposite variation trends. This may caused by the rapid square growth of the specific kinetic energy. In addition, the relationship between Reynolds number and  $K$  has been given, which can be used to predict the head loss of sudden-expansion with nanofluids flowing in it. Meanwhile, a new criterion of evaluation of sudden-expansion utilization from the entropy production view

has been built. The analyzation of  $K_E$  behavior proves the advantage of nanofluids in the high temperature heat transfer process.

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### Nomenclature

$C_i$ – constants, [-]	<i>Greek symbol</i>
$D_h$ – hydraulic diameter, [m]	$\Phi$ – dissipation rate, [W]
$g$ – gravitational acceleration, [ $\text{ms}^{-2}$ ]	<i>Subscripts and superscripts</i>
$h_w$ – energy dissipation, [m]	$'''$ – value per volume, [-]
$K$ – head loss coefficient, [-]	$d$ – dissipation effect, [-]
$K_E$ – exergy loss coefficient, [-]	$d,0$ – undisturbed flow, [ $\text{ms}^{-2}$ ]
$T_m$ – mean temperature, [K]	$c$ – within the component, [-]
$\dot{m}$ – mass-flow rate, [ $\text{kgs}^{-1}$ ]	
$u_m$ – mean velocity, [ms]	
$V$ – volume of the conduit, [ $\text{m}^3$ ]	

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