THE MOTION OF A NON-NEWTONIAN NANOFLUID OVER A SEMI-INFINITE MOVING VERTICAL PLATE THROUGH POROUS MEDIUM WITH HEAT AND MASS TRANSFER

Nabil T. Eldabe[1], Mahmoud. E. Gabr [2], Abd-Elhafez A. ELshekhipy[3] and Sameh. A. Zaher[4*]

1-Mathematics Department, Faculty of Education, Ain Shams University, Egypt
(2, 4)-Mathematics Department, Faculty of Science, Zagazig University, Egypt*
3- Mathematic Department, Collage of Science,

Imam Abdulrahman Bin Faisal University, Damma, Saudi Arabia

E-mail: eng_mohamed_nabil125@hotmail.com, memgabr@gmail.com
abdelhafeez82@yahoo.com; samehabdelzaher419@yahoo.com

The motion of a non-Newtonian nanofluid over a semi-infinite moving vertical plate through porous medium stressed by an external uniform magnetic field with heat and mass transfer is investigated. The fluid under consideration obeys Eyring-Powell model. The effects of the physical parameters of the problem such as, permeability, chemical reaction as well as the fluid material parameters such as Hartmann number, Eckert number and Reynolds number are discussed. The effects of external cooling (Gr > 0) of the plate by the free convection are considered. Graphical results are presented to highlight effects of various emerging parameters on velocity, temperature and concentration profiles.

Key words: *Boundary layer; Eyring-Powel model; Nanofluids; Moving surface; Magnetic field; Porous media, Heat and mass transfer, chemical reaction, Vertical plate.*

1- Introduction

At the beginning of the nineteenth century, Maxwell introduced the basic concepts of dispersing solid particles in fluids to enhance its thermal conductivity. Maxwell was one of the first to analytically investigate conduction through suspended particles [1]. By the end of the nineteenth century nanotechnology is widely used in engineering industries. In petroleum engineering industries, for example, when oil and gas companies turn their attention to new search areas. Nanotechnology can improve viscosity, density, specific gravity and any other physical properties of the drilling fluid to overcome any difficulties. Throughout any industrial facility, heat is added, removed, or moved from one process stream to another. These processes provide a source for energy recovery and fluid heating/cooling. Because of the rising development in modern technology, there is a need to develop new types of fluids that will be more effective in terms of heat exchange performance. Convectional heat transfer fluids, including oil, water, and ethylene glycol mixtures are poor heat transfer fluids, while the thermal conductivity of these fluids plays an important role in the heat transfer coefficient between the heat transfer medium and the heat transfer surface [2].The thermal conductivity of fluids can be enhanced by adding nanoparticles to the base fluids ([3] –[9]).

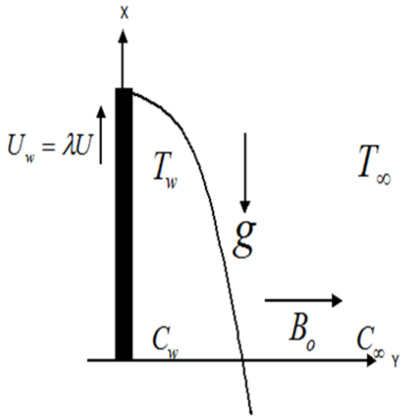
In the last few decides many scientists and engineers studied the boundary layer flow over semi-infinite porous plates [10-17] with concentration on the fluid chemical reactions with heat and mass transfer under the influence of uniform magnetic field. Rashidi et al. [18] investigated heat the transfer of a steady incompressible water based nanofluid flow over a stretching sheet in the presence of transverse magnetic field with thermal radiation and buoyancy effects. Gupta et al. [19] studied the free convection of flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis [20] extended this problem to include mass transfer effects subjected to variable suction or injection. Raptis et al. [21] studied the free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field. Soundalgekar et al. [22] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field. The problem of steady laminar magneto hydrodynamic(MHD) mixed convection heat transfer about a vertical plate is solved numerically by Orhan Aydin and Ahmet Kaya [23] taking into account the effect of Ohmic heating and viscous dissipation. Tripathy *et al.* [24] considered chemical reaction impacts on free convection MHD fluid flow past a moving vertical permeable plate. In [25] Turkyilmazoglu studied Soret and Dufour effect on MHD flow with heat and mass transfer of an electrically conducting viscoelastic fluid past a vertical stretching surface in porous media.

Although different kinds of fluids are discussed by many researchers, other important fluids, such as thixotropic fluids, still needs more study. Thixotropic fluid is a fluid which takes a finite time to attain equilibrium viscosity when introduced to a steep change in shear rate. In 1944 Eyring and Powell [26] introduced a mathematical model, known as Eyring-Powell model, explaining the behavior of these fluids. This mathematical model is used by many researchers. Hayat *et al.* [27] considered heat analysis of Eyring-Powell fluid flow over a continuously moving surface with convective endpoint conditions. Reddy [28] studied the flow and heat transfer of Eyring Powell fluid over a continuously moving surface in the presence of a free stream velocity. Fluids satisfying Eyring-Powell model are studied by many researchers, [29-31]

In this paper we focus on studying the boundary layer flow of a non-Newtonian nanofluid, byes Eyring-Powell model, over a semi-infinite moving vertical plate in the presence of external uniform magnetic. Using appropriate transformation the governing system of partial differential equations of the problem is transformed to a system of ordinary differential equations. The Mathematica Parametric NDSolve package is employed to obtain the results numerically . Moreover the effect of the variation of the permeability, chemical reaction, Hartmann number and Eckert number on velocity distribution, temperature distribution and concentration are obtained and presented in figures.

2. Formulation of the problem

Consider two-dimensional steady boundary-layer flow of an incompressible nanofluid over a moving semi-infinite vertical flat plate. Chose the Cartesian coordinate system such that the x-axis is taken along the continuous surface direction and the y-axis is normal to it (as shown in Figure 1). Let the fluid be influenced by uniform magnetic field $\vec{B}=(0,B_o,0)$ normal to the moving surface.



Figure(1): Physical model and coordinate system

Let U being the velocity of the uniform free stream, $U_w=\lambda U$ being the flat plate velocity, where λ is the plate velocity parameter. Assume that T_w and C_w are the values of the moving surface temperature and concentration respectively while in the ambient fluid these values are T_∞ and C_∞ .

In Eyring-Powell model the stress tensor is written as [26]:

$$,\tau_{ij}=\mu\frac{\partial v_i}{\partial x_j}+\frac{1}{\beta_1}\sinh^{-1}\left(\frac{1}{c}\frac{\partial v_i}{\partial x_j}\right) \tag{1}$$

$$\sinh^{-1}\left(\frac{1}{c}\frac{\partial v_i}{\partial x_j}\right);\ \frac{1}{c}\frac{\partial v_i}{\partial x_j}-\frac{1}{6}\left(\frac{1}{c}\frac{\partial v_i}{\partial x_j}\right)^3,\quad \left|\frac{1}{c}\frac{\partial v_i}{\partial x_j}\right|<<1\ , \tag{2}$$

where, μ is the dynamic viscosity, β_1 and c are the characteristics of the Eyring-Powell model.

Taking in consideration the steady-state flow of the boundary layer, the governing equations of the considered problem are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\left(\frac{\partial u}{\partial y}\right)=(\upsilon+\frac{1}{\beta_1 c\rho_f})\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho_f}(\sigma B_o^{\,2}+\frac{\mu}{k})u-$$
$$\frac{1}{2\beta_1 c^3\rho_f}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty) \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2]+$$

$$\left(\frac{\mu}{(\rho c)_f}+\frac{1}{c\beta_1(\rho c)_f}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma}{(\rho c)_f}B_o^{\,2}u^2-\frac{1}{6c^3\beta_1(\rho c)_f}\left(\frac{\partial u}{\partial y}\right)^4 \qquad (5)$$

$$u\,\frac{\partial C}{\partial x}+v\,\frac{\partial C}{\partial y}=D_B\,\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)+k_\circ(C-C_\infty)\quad , \qquad (6)$$

The boundary conditions of the problem are:

$$v=0,\; u=U_w=\lambda U,\; T=T_w,\; C=C_w \;\; at\, y=0,$$

$$u\to U,\; T\to T_\infty,\; C\to C_\infty \qquad at\, y\to\infty\;. \qquad (7)$$

where u and v are the velocity components along the x and y axis respectively, υ is the kinematic viscosity coefficient, α is the fluid thermal diffusivity $\alpha=k_c/(\rho c)_f$ and $\tau=(\rho c)_p/(\rho c)_f$

.where k_c, k, $k_\circ$ and σ are the fluid thermal conductivity, permeability coefficient , the chemical reaction and fluid electric conductivity , ρ_f, $(\rho c)_f$ and $(\rho c)_p$ are fluid density, fluid heat capacity and nanoparticles effective heat capacity, D_B and D_T are Brownian and thermophoretic diffusion coefficients.

Introducing the stream function ψ defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies the continuity equation (3), we can write equations (4)-(6) as:

$$\frac{\partial\psi}{\partial y}\,\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=(\upsilon+\frac{1}{c\beta_1\rho_f})\,\frac{\partial^3\psi}{\partial y^3}-\frac{1}{\rho_f}(\sigma B_o^{\,2}+\frac{\mu}{k})\frac{\partial\psi}{\partial y}-$$

$$\frac{1}{2c^3\beta_1\rho_f}\left(\frac{\partial^2\psi}{\partial y^2}\right)^2\frac{\partial^3\psi}{\partial y^3}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right) \qquad (8)$$

$$\frac{\partial\psi}{\partial y}\,\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\,\frac{\partial^2 T}{\partial y^2}+\tau[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2]+$$

$$\left(\mu+\frac{1}{c\beta_1(\rho c)_f}\right)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\frac{\sigma}{(\rho c)_f}B_o^{\,2}(\frac{\partial\psi}{\partial y})^2-\frac{1}{6c^3\beta_1(\rho c)_f}\left(\frac{\partial^2\psi}{\partial y^2}\right)^4 \qquad (9)$$

$$.\frac{\partial\psi}{\partial y}\,\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}(\frac{\partial^2 T}{\partial y^2})+k_\circ(C-C_\infty) \qquad (10)$$

By introducing the similarity variable $\eta=y\,(U/2\nu x)^{1/2}$ and the stream function $\psi=(2U\nu x)^{1/2}\,f(\eta)$ together with $\theta(\eta)=(T(\eta)-T_\infty)/(T_w-T_\infty)$ and $\varphi(\eta)=(C(\eta)-C_\infty)/(C_w-C_\infty)$, the system of equations (8)-(10) reduces to the following system of nonlinear ordinary differential equations:

$$(1+\varepsilon)f'''(\eta)+f(\eta)f''(\eta)-\left(M+\frac{1}{k}\right)f'(\eta)-\varepsilon\delta f''^{2}(\eta)f'''(\eta)+2R_{ex}(G_r\theta+G_c\varphi)=0, \tag{11}$$

$$\frac{1}{p_r}\theta''(\eta)+f(\eta)\theta'(\eta)+N_b\theta'(\eta)\varphi'(\eta)+N_t\theta'^{2}(\eta)+E_c(1+\varepsilon)f''^{2}(\eta)+$$

$$M\,E_c f'^{2}(\eta)-\varepsilon\delta E_c f''^{4}(\eta)=0 \tag{12}$$

$$\varphi''(\eta)+\frac{N_t}{N_b}\theta''(\eta)+L_e f\varphi'(\eta)+2\gamma R_{ex}L_e\varphi(\eta)=0, \tag{13}$$

Subject to the boundary conditions:

$f(0)=0,\ f'(0)=\lambda,\ \theta(0)=1,\ \varphi(0)=1,$ as $\eta\to 0$,

and $f'(\eta)\to 1,\ \theta(\eta)\to 0,\ \varphi(\eta)\to 0,$ as $\eta\to\infty$ (14)

Where,

$$M=\frac{2x\sigma B_o^{\,2}}{U\rho_f},\ K=\frac{2x\mu}{U\rho_f k},\ E_c=\frac{U^2}{(C_p)_f(T_w-T_\infty)},\ L_e=\frac{\nu}{D_B},\ R_{ex}=\frac{Ux}{\nu},$$

$$\gamma=\frac{\nu k_o}{U^2},\ P_r=\frac{\nu}{\alpha},\ N_b=\frac{(\rho C)_p D_B(C_w-C_\infty)}{(\rho c)_f\nu},\ N_t=\frac{(\rho C)_p D_T(T_w-T_\infty)}{\nu(\rho c)_f T_\infty}$$

$$.\ \varepsilon=\frac{1}{\mu\beta c},\ \delta=\frac{U^3}{2\upsilon x c^2},\ G_r=\frac{g\beta\upsilon}{U^3}(T-T_\infty),\ G_C=\frac{g\beta^*\upsilon}{U^3}(C-C_\infty) \tag{15}$$

where $M,K,E_c,L_e,R_{ex},\gamma,P_r,N_b,G_r,\ G_c$ and N_t are Hartmann number, permeability parameter, Eckert number, Lewis number, Reynolds number, chemical reaction parameter, Prandtl number, Brownian motion parameter , Grashof number, modified Grashof number and thermophoresis parameter,. ε,δ are the fluid material parameters. Moreover the problem physical quantities c_f, Nu_x *and* Sh_x (skin-friction coefficient, local Nussetl number and Local Sherwood number) are defined as:

$$c_f=\frac{\tau_w}{\rho U^2},\ Nu_x=\frac{xq_w}{k(T_w-T_\infty)},\ Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)},\tau_w=\left(\mu+\frac{1}{\beta c}\right)\frac{\partial u}{\partial y}-\frac{1}{6\beta}\left(\frac{1}{c}\frac{\partial u}{\partial y}\right)^3$$

$$q_w=-\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0}\quad ,q_m=-D_B\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{16}$$

Also, with the help of the similarity transformations we can write:

$$\sqrt{2R_{ex}}C_f=(1+\varepsilon)f''(0)-\varepsilon\delta f''^{3}(0),\quad \theta'(0)=-\sqrt{\frac{2}{R_{ex}}}Nu_x,\quad \varphi'(0)=-\sqrt{\frac{2}{R_{ex}}}Sh_x. \tag{17}$$

Where, C_f , τ_w , q_w and q_m are Skin-friction coefficient, Shear stress, plate heat flux and plate mass flux.

3. Results and Discussion

In this problem we studied the motion of non-Newtonian nanofluid with heat and mass transfer over a semi-infinite vertical flat plate stressed by a uniform external magnetic field. The problem is formulated mathematically by a system of nonlinear partial differential equations (3)-(7). Using suitable similarity variable the governing equations (3)-(7) are transformed into a system of non-liner ordinary differential equations (11)-(14). The Mathematica ParametricNDSolve package is employed to solve the obtained system (11)-(14) numerically.

Using the default values of the physical parameters $P_r = 1, R_{ex} = 2,\ N_t = 0.1, N_b = 0.5$, $L_e = 2,\ \varepsilon = 1, \delta = 0.5, G_r = 0.5,\ G_c = 0.5, \gamma = 0.5,\ \lambda = 0.2,\quad M = 0.5, K = 0.5, E_c = 2$, the velocity, temperature and concentration distributions as well as skin fraction and rate of the heat and mass transfer are discussed and illustrated graphically through set of figures (2-7):

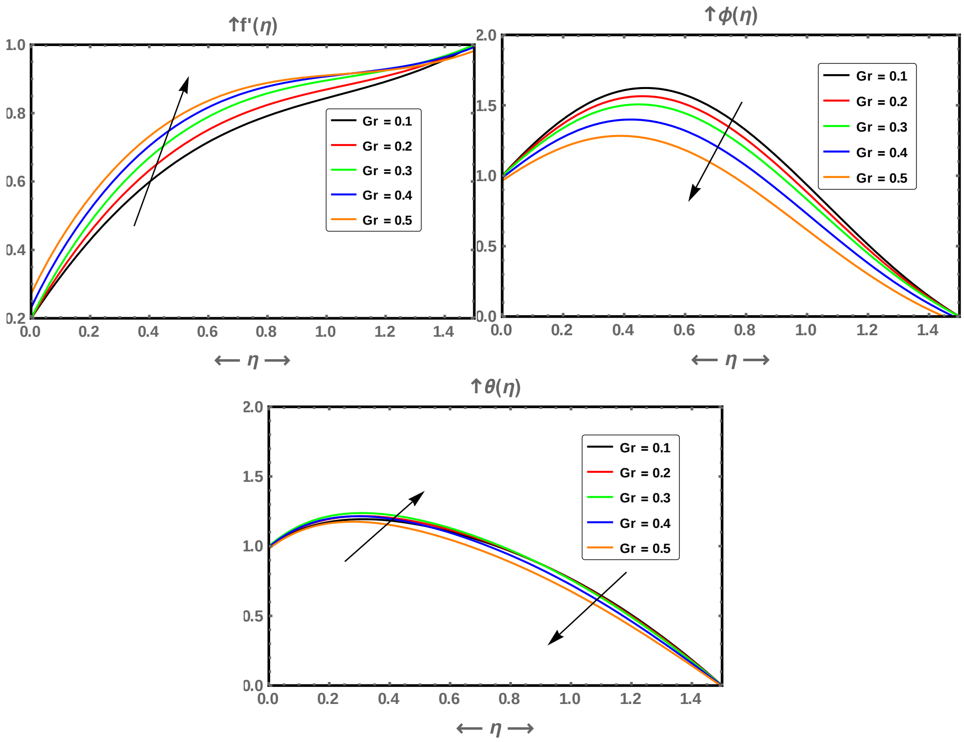


Figure (2): Study the variation of the velocity distribution, temperatures and concentration against the Grashof number Gr.

Figure(2): Shows the relation between the velocity $f'(\eta)$,the temperature $\theta(\eta)$ and the concentration $\varphi(\eta)$ with respect to the Grashof number G_r. It is observed that the Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force in the boundary layer. The positive values of G_r correspond to cooling of the plate and the negative values of G_r correspond to heating of the plate by free convection. As expected, it is found that the increase in the Grashof number leads to increase the velocity, decrease the concentration and troubled increase in temperature due to enhancement in the buoyancy force.

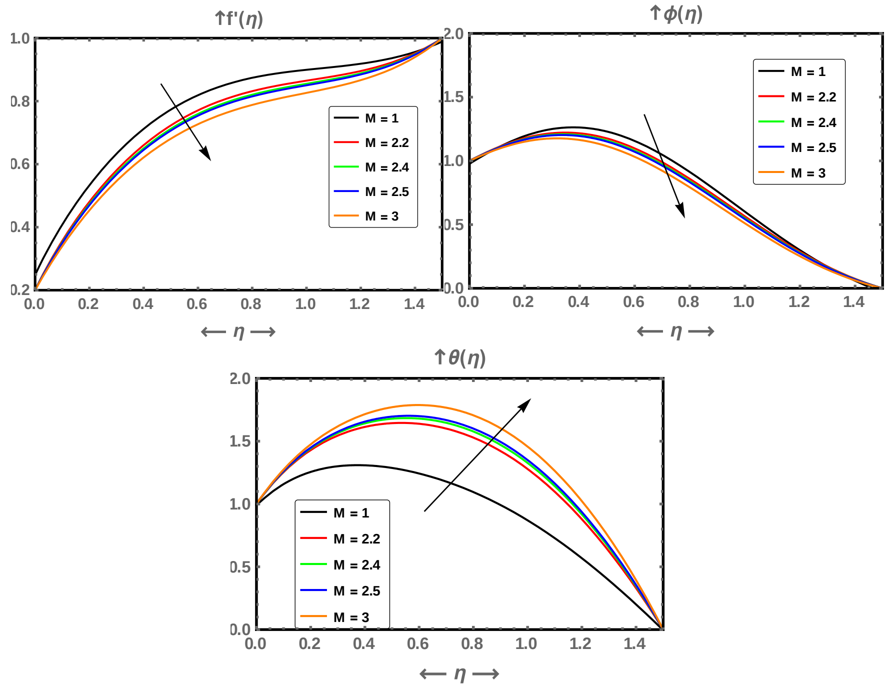


Figure (3): Study the variation of the velocity distribution, temperatures and concentration against the Magnetic field M.

Figure (3) Illustrate the magnetic field effects on velocity, temperature and concentration. The Lorentz force introduced by the magnetic field normal to the fluid motion decreases fluid velocity. This type of resisting force reduces fluid velocity, increase temperature and decrease concentration.

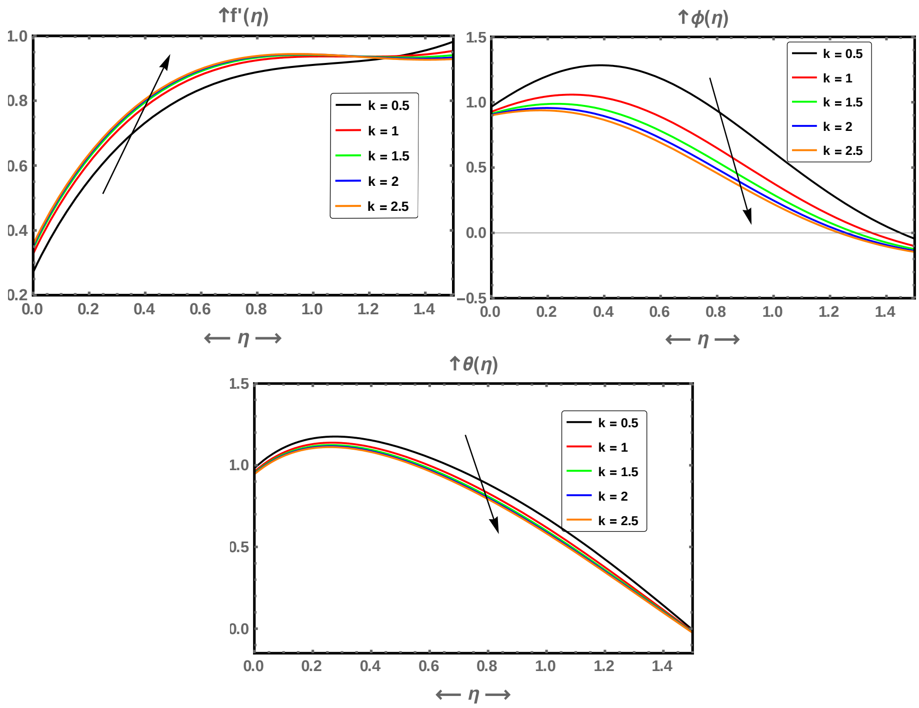


Figure (4): Study the variation of the velocity distribution, temperatures and concentration against the permeability parameter k.

Figure (4) represents the influence of permeability parameter K on velocity, temperature and concentration. Increasing the permeability parameter K increases velocity and temperature while decreases concentration. The porous media creates huge resistance to fluid flow and consequently minor changes occur in momentum boundary layer.

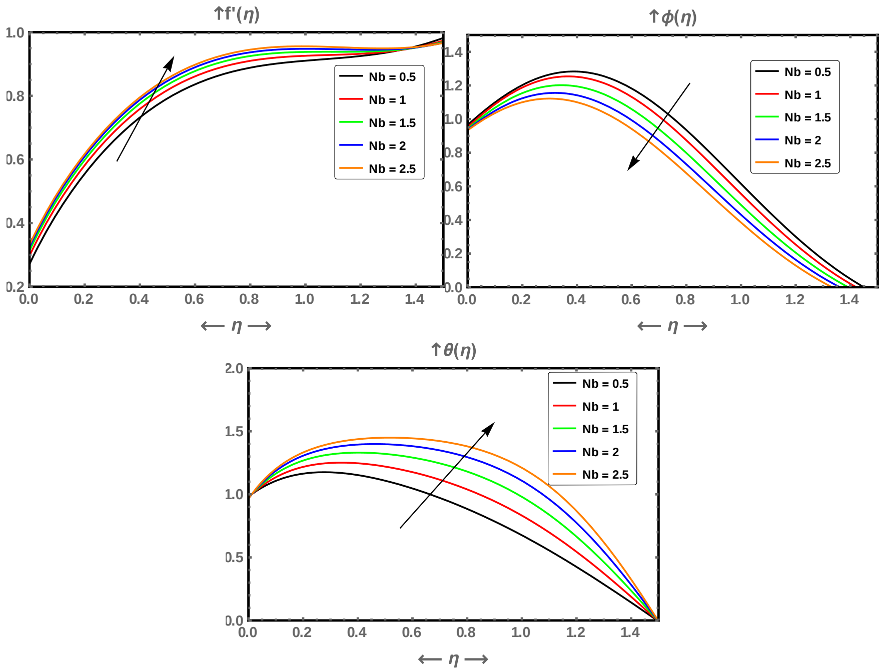


Figure (5): Study the variation of the velocity distribution, temperatures and concentration against the Brownian motion parameter N_b.

Figure (5) illustrate the result of effects of Brownian motion parameter N_b on velocity, temperature and concentration. The study shows that the increase in the Brownian motion parameter N_b increases velocity, temperature and decrease fluid concentration. This is due to the Brownian motion parameter is a random movement of microscopic particles suspended in liquids or gases resulting from the impact of molecules of the fluid surrounding the particles.

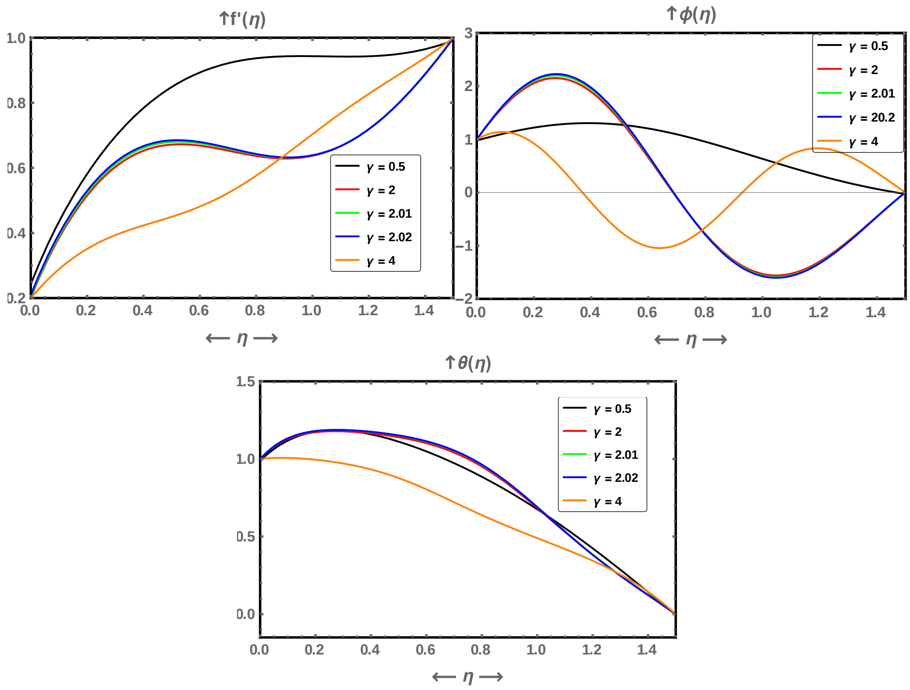


Figure (6): Study the variation of the velocity distribution, temperatures and concentration against the Chemical reaction parameter ɣ.

Figure (6) Study the relation between velocity, temperature and concentration for different values of the chemical reaction parameter γ. Based in this study the increase in chemical reaction γ increases the velocity, temperature and concentration decrees as a result of the converted chemical force into energy

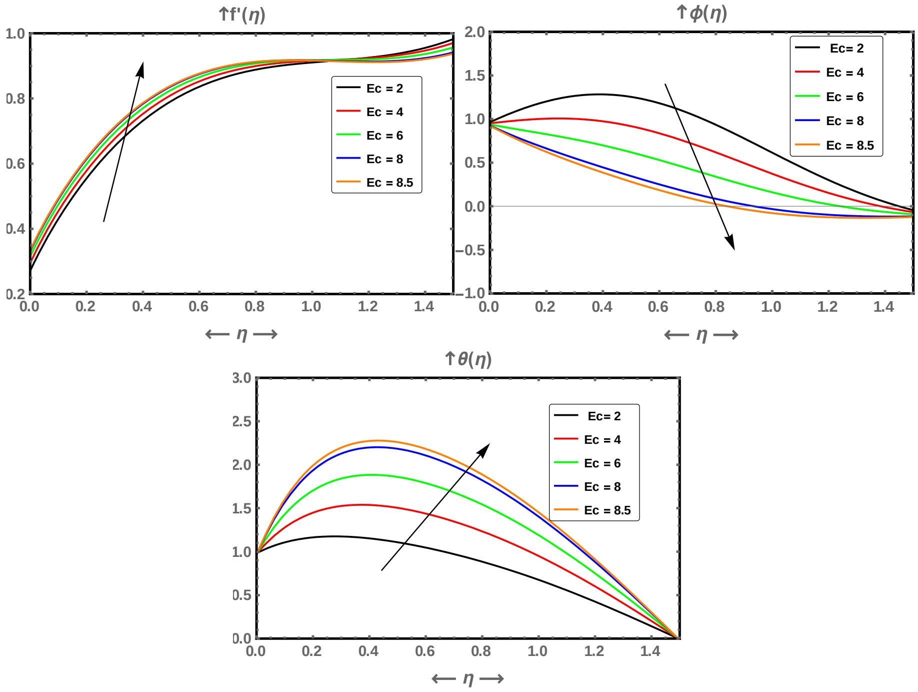


Figure (7): Study the variation of the velocity distribution, temperatures and concentration against the Ecert number (Ec).

Figure (7) shows the effect of Eckert number E_c on velocity, temperature and concentration.

The positive values of Eckert number indicates plate cooling, that is, loss of heat from the surface to the fluid so that the greater values of viscous dissipative heat cause enhancement of temperature and velocity profile as well. In addition, the magnitude of the thermal boundary layer became large because of viscous dissipation effect. This leads to the increase in velocity and temperature but decreasing concentration.

Conclusion:

Based on the above study we conclude the following results:

•The effects of some physical quantities(Eckert number E_c, Prandtl number P_r ,Lewis number L_e , Reynolds number R_{ex}, , Chemical reaction γ, Brownian motion Parameter N_b, Thermophoretic parameter N_t and the fluid material parameter ε, δ) it has significant effects on both the velocity , temperature and concentration.

•The Grashof number signifies the relative effect of the buoyancy force to the hydrodynamic viscous force in the boundary layer.

•Any increase in the Grashof number increases the velocity but decrease concentration and make trouble increase in the temperature due to the enhancement in the buoyancy force.

•Increasing the permeability parameter K increases the velocity profile throughout the boundary layer.

•The presence of porous media creates huge resistance to fluid flow due to which trifling changes occurs in momentum boundary layer and hence inciting of fluid temperature and decreasing in concentration appeared.

• Increase Eckert number increase fluid temperature and velocity.

Nomenclature

$\vec{B}$ external uniform magnetic field[$kg s^{-2} A^{-1}$]

C nanoparticle volume fraction[-]

C_f skin-friction coefficient[-]

C_w nanoparticle volume fraction at the plate[-]

C_N ambient nanoparticle volume fraction[-]

D_B Brownian diffusion coefficient [-]

D_T thermophoretic diffusion coefficient[-]

f dimensionless stream function [-]

k thermal conductivity [$Wm^{-1} K^{-1}$]

G_r Grashof number[-]

G_c modified Grashof number[-]

u, v velocity components along x- and y-axes[$m s^{-1}$]

Uuniform velocity of the free stream Low[-]

Uw uniform velocity of the moving plate[-]

x, y Cartesian coordinates measured along the wall and normal to it, respectively[-]

α thermal diffusivity[-]

φ dimensionless nanoparticle volume Fraction[-]

η similarity variable [-]

L_e Lewis number [-]

N_b Brownian motion parameter, [-]

N_t thermophoresis parameter [-]

N_{ux} local Nusselt number [-]

p_r Prandtl number [-]

p pressure[Nm^{-2}]

q_w plate heat flux [-]

q_m plate mass flux [-]

R_{ex} local Reynolds number [-]

sh_x Shx local Sherwood number [-]

T temperature[K]

T_w temperature at the plate [K]

T_N ambient temperature [K]

$\vec{V}$ velocity vector [$m\, s^{-1}$]

σ electrical conductivity [sm^{-1}]

λ plate velocity ratio[-]

μ dynamic viscosity [$kgm^{-1}s^{-1}$]

θ dimensionless temperature [ ]

ρ_f fluid density [kgm^{-3}]

ρ_p nanoparticle mass density [kgm^{-3}]

$(\rho c)_f$ heat capacity of the fluid [kgm^{-3}]

$(\rho c)_p$ effective heat capacity of the nanoparticle material [kgm^{-3}]

τ heat capacity ratio [-]

ν kinematic viscosity[$kgm^{-1}s^{-1}$]

ψ stream function [-]

β^* coefficient of expansion with concentration [K^{-1}]

β coefficient of thermal expansion[K^{-1} c_p specific heat at constant pressure [$Jkg^{-1}K^{-1}$]

Applications:

The most interesting applications of the considered problem are in petroleum industries, human blood flow, heat and mass transfer and rheology, as following:

• The demand of a better drilling fluid increases when oil and gas companies turn their attention to new search areas such as the unstable shale areas and high pressure. Nanotechnology can improve viscosity, density, specific gravity and any other physical properties of the drilling fluid to overcome any difficulties.

• Human blood can be treated as a kind of a considered fluid due to the presence of several substances like, protein, fibrinogen and globulin in aqueous base plasma.

•The importance of rheology science comes from its industrial applications where synthetic polymers and their solutions in different solvents is necessary for polymers and industrial application.

•A wide variety of industrial processes involve the transfer of heat energy. Throughout any industrial facility, heat must be added, removed, or moved from one process stream to another and it has become a major task for industrial necessity. These processes provide a source for energy recovery and process fluid heating/cooling.

Acknowledgements

The authors are thankful to the referees for their critical comments which help in improving the presentation of the paper.

References

[1] Maxwell, J.C., Electricity and Magnetism, Clarendon Press, (1873)Oxford, UK

[2] Kakaç S., Pramuanjaroenkij A., *Int. J. Heat Mass Transf* ,52, (13–14), (2009) pp.3187–3196

[3] Muthtamilselvan M., Commun. Nonlinear Sci. Numer. Simul, 15 (2010) pp. 1501–1510

[4] Choi S.U.S., Enhancing thermal conductivity of fluids with nanoparticles, *Proceedings of the ASME International Mechanical Engineering Congress and Exposition*, Vol. 66, (1995), pp. 99–105

[5] Eastman J.A., et al., Thompson, Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, *Appl. Phys. Lett.* 78 (2001) , pp.718–720

[6] Eldabe N.T.M., et al., Peristaltic transport of magneto-hydrodynamic carreau nanofluid with heat and mass transfer inside asymmetric channel. *Am J Comput Math*, (2017) , 7, pp.1–20

[7] EldabeN.T.M., et al., Magnetohydrodynamic peristaltic flow of Jeffry nanofluid with heat transfer through a porous medium in a vertical tube,*Appl Math Inf Sci*(2017) ,11,(4) ,pp.1097–1103

[8] EldabeN.T.M., et al., Magnetohydrodynamic peristaltic flow with heat and mass transfer of micropolar biviscosity fluid through a porous medium between two co-axial tubes, *Arab J. Sci. Eng*, 39, (2014) , pp.5045–5062

[9] Eldabe N.T.M., et al., Two dimensional boundary layer flow with heat and mass transfer of magneto hydrodynamic non-Newtonian nanofluid through porous medium overa semi-infinite moving plate, *Microsystem Technologies* ,(2018)*https://doi.org/10.1007/s00542-018-3717-5123456789*

[10] Venkateswarlu, M., et al., Effects of chemical reaction andheat generation onMHD boundary layer flow of a moving vertical plate withsuction and dissipation. *Eng Int* (2013), 1

[11] Reddy G.V.R., et al., Similarity transformations heat and mass transfer effects on steady MHD free convection dissipative fluid flow past an inclined porous surface with chemical reaction,*J Naval Archit Mar Eng* (2014), *http://dx.doi.org/10.3329/jname.v11i2.18313.*

[12] Malik M.Y., Rehman K.U., Effects of second order chemical reaction on MHD free convection dissipative fluid flow past an inclined porous surface by way of heat generation: a Lie group analysis. *Inf Sci Lett* (2016); *5 pp: 35–45*

[13] Nayak M.K., et al., Heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction, *Propul Power Res* (2016),5 pp:70–80

[14] Eid M.R., Chemical reaction effect on MHD boundary-layer flow of two-phasenanofluid model over an exponentially stretching sheet with a heat generation.*J Mol Liq*(2016), 220:7, pp.18–25

[15] Seth G.S., et al., Natural convection heat and mass transfer flow with Hall current, rotation, radiation and heat absorption past an accelerated moving vertical plate with ramped temperature, *Journal of Applied Fluid Mechanics*, 8 (1), (2015), pp. 7-20

[16] Seth G.S., et al., MHD natural convection heat and mass transfer flow past a time dependent moving vertical plate with ramped temperature in a rotating medium with

Hall effects, radiation and chemical reaction, *Journal of Mechanics*, 31, (2015), pp. 91- 104
[17] Seth G.S., et al., Effects of Hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer past an impulsively moving vertical plate in the presence of radiation and chemical reaction, *Bulgarian Chemical Communications*, 46 (4), (2014), pp. 704-718
[18] Rashidi M. M., et al., Buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation, *J. Mol. Liq.*198 (2014)234–238.
[19] Gupta A.S., et al., Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. *Rev. Roum. Sci. Techn.- Mec. Apl* ,24, (1979) ,pp: 561-568
[20] Kafousias N.G., Rapits A.A., Mass Transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection. *Rev. Roum. Sci. Techn.- Mec. Apl* ,(1981) ,26 , pp.11-22
[21] Rapits A.A., Hydromagnetic free convection flow past an accelerated vertical infinite plate withvariable suction and heat flux ,*Letters in Heat and Mass transfer*(1981) ,8, pp. 137-143
[22] Soundalgekar,V.M., et al., Free convection effects on the MHD Stokes problem for a vertical plate, *Nucl. Eng. Des.*,, 1979 ,51, pp. 403–407
[23] O. Aydin, A. Kaya, MHD mixed convection of a viscous dissipating fluid about a permeable vertical flat plate, *Appl. Math. Model.*, 33 **,**11, (2009), pp. 4086–4096
[24] Tripathy R.S., et al., Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium, *Alexandria Engineering Journal*. 54,(2015),pp. 673-679
[25] Turkyilmazoglu ,M., The analytical solution of Mixed convection heat transfer and fluid flow of a MHD viscoelastic fluid over a permeable stretching surface, *International Journal of Mechanical Sciences,*(2013),77, pp. 263 -268
[26] Powell R.E, et al, Mechanism for the relaxation theory of viscosity, Nature, 154, pp.427-428(1944).
[27] Hayat T., et al., Steady Flow of an Eyring Powell Fluid over a Moving Surface with Convective Boundary Conditions, *International Journal of Heat and Mass Transfer*, 55,(2012), pp. 1817-1822
[28] Reddy M.G, Scaling Transformation for Heat and Mass Transfer Effects on Steady MHD Free Convection Dissipative Flow Past an Inclined Porous Surface, *International Journal Applied Mathematics Mechanics.* 9, (2013), pp. 1-18
[29] Singh, G., et al , Effects of Ohmic Heating and Viscous Dissipation on Steady MHD Flow Near a Stagnation Point on an Isothermal Stretching Sheet, Thermal Science, 13 (2009), 1, pp. 5-12
[30] Ghani, F., et al.: Unsteady Magnetohydrodynamics Thin Film Flow of a third grade fluid over an oscillating in-clined belt embedded in a porous medium ,Thermal Science, 2016, No. 5, pp. 875-887
[31] Hayat, T., et al.: Magnetohydrodynamic Flow of Powell-Eyring Fluid by by a stretching cylinder with Newtonian heating, Thermal Science, 2018, Vol. 22, No. 1B, pp. 371-382