# INVERSE SCATTERING TRANSFORM FOR A SUPERSYMMETRIC KORTEWEG-DE VRIES EQUATION

by

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*In this paper, the inverse scattering transform is extended to a super Korteweg-de Vries (KdV) equation with an arbitrary variable coefficient by using Kulish and Zeitlin's approach. As a result, exact solutions of the super KdV equation are obtained. In the case of reflectionless potentials, the obtained exact solutions are reduced to soliton solutions. More importantly, based on the obtained results, an approach to extending the scattering transform is proposed for the supersymmetric KdV equation in the 1-dimensional Grassmann algebra. It is shown the the approch can be applied to some other supersymmetric non-linear evolution equations in fluids.*

Key words: *Supersymmtric KdV equation, Grassmann algebra, inverse scattering transform method, scattering data, soliton solution,*

**Introduction**

As pointed out by Zhang and Liu [1] that soliton equations and their supersymmetric counterparts are relevant both physically and mathematically. From the viewpoint of mathematics, supersymmetry originated from theoretical physics is formulated by extending ordinary space to include anticommuting variables of Grassmann type so that bosons and fermions can be treated in a unified way. As the noncommutative extensions of differential systems, supersymmetric equations can model a variety of physical phenomena and dynamical processes [2].

It is well known that the celebrated KdV equation:

$$u_t = -u_{xxx} + 6uu_x \tag{1}$$

is a completely integrable nonlinear equation for a bosonic field [P. Mathieu]. The supersymmetric extension of the KdV eq. (1) refers to a system coupling a bosonic field and a fermionic field, which reduces to eq. (1) in the limit case when the fermionic field vanishes and keeps invariant under a so-called supersymmetry transformation.

To construct a supersymmetric extension of eq. (1), the classical space and time $(t, x)$

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needs to be extended to a super space and time $(t,x,\theta)$, and the usual field $u(t,x)$ is then replaced with a superfield $\Phi(t,x,\theta)$, here $\theta$ is an anticommuting variable which leads to $\theta^2=0$. Thus, we have a simple Taylor expansion of $\Phi(t,x,\theta)=\xi(t,x)+\theta u(t,x)$, here $u(t,x)$ and $\xi(t,x)$ are called componet fields, the bosonic field $u(t,x)$ is commuting, while the fermionic field $\xi(t,x)$ is anticommuting. Proceeding with a direct extension, we have:

$$u_t \to \Phi_t\,,\quad u_{xxx} \to \Phi_{xxx}\,,\quad 3uu_x \to a\Phi D\Phi_x + (6-a)\Phi_x D\Phi \tag{2}$$

where $a$ is a free constant, $D$ is the superderivative $D=\theta\partial+\partial_\theta$ whose square is the usual derivative $D^2=\partial$. Then the KdV eq. (1) has a extension in terms of superfield:

$$\Phi_t = -\Phi_{xxx} + a(D\Phi_x)\Phi + (6-a)(D\Phi)\Phi_x \tag{3}$$

which is integrable only if $a=3$[3]. In this case, the component version of eq. (3) reads:

$$u_t = -u_{xxx} + 6uu_x - 3\xi\xi_{xx} \tag{4}$$

$$\xi_t = -\xi_{xxx} + 3u\xi_x + 3u_x\xi \tag{5}$$

which reduces to eq. (1) in the limit when the fermionic field $\xi$ vanishes and keeps invariant under the so-called supersymmetry transformation $\delta$ : $x\to x-\eta\theta$ , $\theta\to\theta+\eta$ , here $\eta$ is an anticommuting parameter, or equivalently in the component fields [4]: $\delta u(t,x)=u(t,x)+\eta\xi_x(t,x)$, $\delta\xi(t,x)=\xi(t,x)+\eta u(t,x)$. In fact, it is easy to check:

$$\begin{aligned}\delta\Phi(t,x,\theta) &= \Phi(t,x-\eta\theta,\theta+\eta)\\ &= \Phi(t,x,\theta)-\eta\theta\Phi_x(t,x,\theta)+\eta\Phi_\theta(t,x,\theta)\\ &= \xi(t,x)+\eta u(t,x)+\theta(u(t,x)+\eta\xi_x(t,x))\\ &= \delta\xi(t,x)+\theta u(t,x)=\Phi(t,x,\theta)\end{aligned} \tag{6}$$

Therefore, the system of eqs. (4) and (5) gives a supersymmetric extension and is called supersymmetric KdV equation. However, the following extension of eq. (1) [5]:

$$u_t = -u_{xxx} + 6uu_x - 3\xi\xi_{xx} \tag{7}$$

$$\xi_t = -4\xi_{xxx} + 6u\xi_x + 3u_x\xi \tag{8}$$

is not a supersymmetric KdV equation. It is because that eqs. (7) and (8) are not invariant under the supersymmetry transformation $\delta$. Correspondingly, we call the system of eqs. (9) and (10) a super KdV equation.

With the developments of soliton theory, some analytical methods have been extended to solve supersymmetric non-linear evolution equations, such as Hirota method [6], Bäcklund transformation [1], Darboux transformation [7], and Painlevé analysis [8]. Comparatively speaking, the famous inverse scattering method [9] has not been successfully extended to the supersymmetric non-linear evolution equations in spite of the few pioneer works [10-16]. In 2005, Kulish and Zeitlin [16] extended the inverse scattering transform to a super KdV equation:

$$u_t = -u_{xxx} + 6uu_x - 12\xi\xi_{xx} \tag{9}$$

$$\xi_t = -4\xi_{xxx} + 6u\xi_x + 3u_x\xi \tag{10}$$

in the case of one-dimensional Grassmann algebra.

In the present paper, following Kulish and Zeitlin's approach [16] we shall extend the inverse scattering transform to super non-linear evolution equations with variable coefficients. For such a purpose, we consider the following variable-coefficient version of the super KdV eqs. (13) and (14):

$$u_t = -\alpha(t)u_{xxx} + 6\alpha(t)uu_x - 12\alpha(t)\xi\xi_{xx} \tag{11}$$

$$\xi_t = -4\alpha(t)\xi_{xxx} + 6\alpha(t)u\xi_x + 3\alpha(t)u_x\xi \tag{12}$$

on one hand. More importantly, inspired Kulish and Zeitlin's approach [16], we shall discuss the scattering transform for the supersymmetric KdV eqs. (4) and (5) on the other hand.

**Derivation**

For the super KdV eqs. (11) and (12) we have the following *Theorem 1*.

*Theorem 1*. The super KdV eqs. (11) and (12) have the Lax representation:

$$L_t = [M, L] \tag{13}$$

where

$$M = \alpha(t)\begin{pmatrix} -u_x & -4\xi_x & 4\lambda - 2u \\ 2\xi u - 4\xi\lambda - 4\xi_{xx} & 0 & -4\xi_x \\ -4\xi\xi_x + u_{xx} - 2u^2 + 2u\lambda + 4\lambda^2 & -2\xi u + 4\xi\lambda + 4\xi_{xx} & u_x \end{pmatrix} \tag{14}$$

$$L = \partial_x + \begin{pmatrix} 0 & 0 & 1 \\ -\xi & 0 & 0 \\ u+\lambda & \xi & 0 \end{pmatrix} \tag{15}$$

*Proof*. We substitute eqs. (14) and (15) into eq. (13), then a direct computation gives eqs. (11) and (12). The proof is finished.

**Direct scattering analysis**

It is easy to see that the super KdV eqs. (11) and (12) can be derived from the compatibility condition of the linear spectral problem and its time evolution equation:

$$L\varphi = 0, \quad \partial_t\varphi = M\varphi \tag{16}$$

where $\varphi = (\varphi_1(x), \varphi_2(x), \varphi_3(x))^T$, $\varphi_1(x)$ and $\varphi_3(x)$ belong to the even part of the Grassmann algebra, $\varphi_2(x)$ belongs to its odd part, and the operator $L$ is the scalar Lax operatorthe

$$L = -\partial_x^2 + \lambda + u - \xi\partial^{-1}\xi \tag{17}$$

which is equivalent to the matrix Lax operator (15).

For the completeness, we first recall the analysis of the linear problem $L\varphi = 0$. Take a similarity transformation [16]:

$$ULU^{-1}, \ U = e^{ikX_-}, \ X_- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{18}$$

we get the operator:

$$L' = \partial_x + \begin{pmatrix} -ik & 0 & 1 \\ -\xi & 0 & 0 \\ u & \xi & ik \end{pmatrix} \tag{19}$$

Further taking the matrix:

$$N = \begin{pmatrix} 1 & 0 & -(2ik)^{-1} \\ 0 & 1 & 0 \\ 0 & 0 & (2ik)^{-1} \end{pmatrix} \tag{20}$$

we then reduce $L'$ to a diagonal form

$$\bar{L} = NL'N^{-1} = \begin{pmatrix} -u(2ik)^{-1} & -\xi(2ik)^{-1} & -u(2ik)^{-1} \\ -\xi & 0 & -\xi \\ u(2ik)^{-1} & \xi(2ik)^{-1} & u(2ik)^{-1} \end{pmatrix} + \begin{pmatrix} -ik & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ik \end{pmatrix} + \partial_x \tag{21}$$

In this case, the corresponding linear problem $\bar{L}\varphi = 0$ gives

$$\begin{pmatrix} -u(2ik)^{-1} & -\xi(2ik)^{-1} & -u(2ik)^{-1} \\ -\xi & 0 & -\xi \\ u(2ik)^{-1} & \xi(2ik)^{-1} & u(2ik)^{-1} \end{pmatrix}\varphi + \begin{pmatrix} -ik & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ik \end{pmatrix}\varphi + \partial_x\varphi = 0 \tag{22}$$

and can be rewritten as:

$$\partial_x\varphi = ikh\varphi + Q\varphi \tag{23}$$

where

$$Q = \begin{pmatrix} u(2ik)^{-1} & \xi(2ik)^{-1} & u(2ik)^{-1} \\ \xi & 0 & \xi \\ -u(2ik)^{-1} & -\xi(2ik)^{-1} & -u(2ik)^{-1} \end{pmatrix}\varphi, \ h = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{24}$$

We take a pair of Jost solutions of the linear problem (23):

$$\phi^{+}(x,k)=\begin{pmatrix} e^{ikx} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-ikx} \end{pmatrix}, \quad x\to+\infty \tag{25}$$

$$\phi^{-}(x,k)=\begin{pmatrix} e^{ikx} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-ikx} \end{pmatrix}, \quad x\to-\infty \tag{26}$$

so that $N^{-1}\phi^{\pm}N$ are elements of the group Osp(1|2), *i.e.*, $(N^{-1}\phi^{\pm}N)J(N^{-1}\phi^{\pm}N)^{st}=J$ , here

$$J=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} a & \alpha & b \\ \beta & f & \delta \\ c & \gamma & d \end{pmatrix}^{st}=\begin{pmatrix} a & -\beta & b \\ \alpha & f & \delta \\ b & -\delta & d \end{pmatrix} \tag{27}$$

Secondly, leting the transfer matrix be of the form:

$$T(k)=\begin{pmatrix} a(k) & \gamma(k) & b(k) \\ \zeta(k) & f(k) & \delta(k) \\ c(k) & \eta(k) & d(k) \end{pmatrix} \tag{28}$$

so that

$$\phi^{+}(x,k)T(k)=\varphi^{-}(x,k) \tag{29}$$

then we can determine relation between elements of $T(k)$ [16]:

$$T(k)=\begin{pmatrix} a(k) & \gamma(k) & b(k) \\ \zeta(k) & f(k) & \delta(k) \\ c(k) & \eta(k) & d(k) \end{pmatrix}=\begin{pmatrix} a(k) & \gamma(k) & b(k) \\ \overline{\delta}(k) & f(k) & \delta(k) \\ \overline{b}(k) & \overline{\gamma}(k) & \overline{a}(k) \end{pmatrix} \tag{30}$$

*i.e.*, $\zeta=\overline{\delta}$, $d=\overline{a}$, $\eta=\overline{\gamma}$ and $f=\overline{f}$, here bar denotes taking the reversion of the sign of $k$. Since $N^{-1}TN\in$ Osp(1|2), we have the following constraints:

$$f=1-2ik\overline{\gamma}\gamma, \quad f(a\overline{a}-b\overline{b})=1, \quad \delta=-2ik(\overline{\gamma}b-\gamma\overline{a}) \tag{31}$$

which degenerates into

$$f=1, \quad a\overline{a}-b\overline{b}=1, \quad \delta=-2ik(\overline{\gamma}b-\gamma\overline{a}) \tag{32}$$

in the case of the 1-dimensional Grassmann algebra.

With the help of the factorization $T^{+}(k)=T(k)T^{-}(k)$, here

$$T^{+}(k)=\begin{pmatrix} 1 & \delta(k)(2ik)^{-1} & b(k) \\ 0 & \overline{a}(k) & \delta(k) \\ 0 & 0 & \overline{a}(k) \end{pmatrix}, \quad T^{-}(k)=\begin{pmatrix} \overline{a}(k) & 0 & 0 \\ \overline{\gamma}(k)(2ik)^{-1} & \overline{a}(k) & 0 \\ -\overline{b}(k) & -\overline{\gamma}(k) & 1 \end{pmatrix} \tag{33}$$

we construct the following matrix-valued function:

$$\varphi^{+}(x,k)=\phi^{+}(x,k)T^{+}(k)e^{-ikxh}=\phi^{-}(x,k)T^{-}(k)e^{-ikxh} \tag{34}$$

From eqs. (23), (25) and (33), we have:

$$\varphi^{+}(x,k)=\left[e^{ikxh}-\int_{x}^{+\infty}e^{ik(x-y)h}Q(y,k)\varphi^{+}(y,k)\mathrm{d}y\right]T^{+}(k)e^{-ikxh}$$

$$=e^{ikxh}T^{+}(k)e^{-ikxh}-\int_{x}^{+\infty}e^{ik(x-y)h}Q(y,k)\varphi^{+}(y,k)e^{ik(y-x)h}\mathrm{d}y \tag{35}$$

the first column of which reads:

$$\varphi^{+(1)}(x,k)=\begin{pmatrix}\varphi_{11}\\ \varphi_{21}\\ \varphi_{31}\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}-(2ik)^{-1}\int_{x}^{+\infty}[u(y)(\varphi_{11}(y,k)+\varphi_{31}(y,k))+\xi(y)\varphi_{21}(y,k)]\mathrm{d}y\\ -\int_{x}^{+\infty}e^{-ik(x-y)}[\xi(y)(\varphi_{11}(y,k)+\varphi_{31}(y,k))]\mathrm{d}y\\ (2ik)^{-1}\int_{x}^{+\infty}e^{-2ik(x-y)}[u(y)(\varphi_{11}(y,k)+\varphi_{31}(y,k))+\xi(y)\varphi_{21}(y,k)]\mathrm{d}y\end{pmatrix} \tag{36}$$

In view of eqs. (34) and (36), we get $\phi^{+(1)}(x,k)=\varphi^{+(1)}(x,k)e^{ikx}$ for the first column of the Jost solution (25) and therefore have:

$$\bar{m}_{+}(x,k)=N^{-1}\varphi^{+(1)}(x,k)=\begin{pmatrix}1+\int_{x}^{+\infty}\frac{e^{-2ik(x-y)}}{2ik}(u(y)\bar{m}_{+}^{1}(y,k)+\xi(y)\bar{m}_{+}^{2}(y,k))\mathrm{d}y\\ -\int_{x}^{+\infty}e^{-ik(x-y)}\xi(y)\bar{m}_{+}^{1}(y,k)\mathrm{d}y\\ \int_{x}^{+\infty}e^{-2ik(x-y)}(u(y)\bar{m}_{+}^{1}(y,k)+\xi(y)\bar{m}_{+}^{2}(y,k))\mathrm{d}y\end{pmatrix} \tag{37}$$

which implies:

$$u(x)=2ik\partial_{x}\bar{m}_{+}^{1}(x,k),\quad \xi(x)=ik\partial_{x}\bar{m}_{+}^{2}(x,k),\quad |k|\to+\infty \tag{38}$$

**Inverse scattering analysis**

To restore $u(x)$ and $\xi(k)$, we have:

$$u(t,x)=\partial_{x}(2i\sum_{j=1}^{N}R_{j}(x)+\frac{1}{\pi}\int_{-\infty}^{+\infty}(-r(x)R(-z,x)e^{-2izx})\mathrm{d}z) \tag{39}$$

$$\xi(t,x)=i\sum_{j=1}^{N}\Theta_{j}(x)+\frac{1}{2\pi}\int_{-\infty}^{+\infty}(-r(x)\Theta(-z,x)e^{-2izx}+2iz\rho(z)e^{-izx})\mathrm{d}z \tag{40}$$

by solving the Riemann problem [16], here

$$R_{n}(x)=-\frac{\bar{b}e^{2\kappa_{n}x}}{\bar{a}'(i\kappa_{n})}\left(1+i\sum_{j=1}^{N}\frac{R_{j}(x)}{\kappa_{n}+\kappa_{j}}+\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\mathrm{d}z\frac{-r(z)R(-z,x)e^{-2izx}}{z+i\kappa_{n}}\right) \tag{41}$$

$$\Theta_{n}(x)=-\frac{2\kappa_{n}\bar{\gamma}_{n}e^{\kappa_{n}x}}{\bar{a}'(i\kappa_{n})}-\frac{\bar{b}e^{2\kappa_{n}x}}{\bar{a}'(i\kappa_{n})}\left(i\sum_{j=1}^{N}\frac{\Theta_{j}(x)}{\kappa_{n}+\kappa_{j}}+\frac{1}{2i\pi}\int_{-\infty}^{+\infty}\mathrm{d}z\frac{-r(z)\Theta(-z,x)e^{-2izx}+2iz\rho(z)e^{-izx}}{z+i\kappa_{n}}\right) \tag{42}$$

$$R(x)=1+\sum_{j=1}^{N}\frac{R_{j}(x)}{k-i\kappa_{j}}+\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\mathrm{d}z\frac{-r(z)R(-z,x)e^{-2izx}}{z-(k+i0)} \tag{43}$$

$$\Theta(x)=\sum_{j=1}^{N}\frac{\Theta_j(x)}{k-i\kappa_j}+\frac{1}{2\pi i}\int_{-\infty}^{+\infty}dz\frac{-r(z)\Theta(-z,x)e^{-2izx}+2iz\rho(z)e^{-izx}}{z-(k+i0)} \tag{44}$$

$$r(k)=\frac{\bar{b}(k)}{\bar{a}(k)},\quad \rho(k)=\frac{\bar{\gamma}(k)}{\bar{a}(k)} \tag{45}$$

For the scattering data, we have the following *Theorem 2*.

*Theorem 2*. In the 1-dimensional Grassmann algebra, the transfer matrix (27) has the time-dependence:

$$a(t,k)=a(0,k),\quad b(t,k)=b(0,k)e^{8ik^3\int_0^t\alpha(\tau)\mathrm{d}\tau},\quad \gamma(t,k)=\gamma(0,k)e^{4ik^3\int_0^t\alpha(\tau)\mathrm{d}\tau} \tag{46}$$

$$\bar{a}(t,k)=\bar{a}(0,k),\quad \bar{b}(t,k)=\bar{b}(0,k)e^{-8ik^3\int_0^t\alpha(\tau)\mathrm{d}\tau},\quad \bar{\gamma}(t,k)=\bar{\gamma}(0,k)e^{-4ik^3\int_0^t\alpha(\tau)\mathrm{d}\tau} \tag{47}$$

where $a(0,k)$, $\bar{a}(0,k)$, $b(0,k)$, $\bar{b}(0,k)$, $\gamma(0,k)$ and $\bar{\gamma}(0,k)$ are constants.

*Proof.* From eq. (16), we have:

$$\varphi_{1x}=-\varphi_3,\quad \varphi_{2x}=\xi\varphi_1,\quad \varphi_{3x}=-(u+\lambda)\varphi_1-\xi\varphi_2,\quad \lambda=-k^2 \tag{48}$$

$$\varphi_{1t}=-\alpha(t)u_x\varphi_1+\alpha(t)(4\lambda-2u)\varphi_3-4\alpha(t)\xi_x\varphi_2 \tag{49}$$

$$\varphi_{2t}=(2\alpha(t)u_x-4\lambda\alpha(t)\xi-4\alpha(t)\xi_{xx})\varphi_1-4\alpha(t)\xi_x\varphi_3 \tag{50}$$

$$\begin{aligned}\varphi_{3t}&=(-4\alpha(t)\xi\xi_x+\alpha(t)u_{xx}-2\alpha(t)u^2+2\lambda\alpha(t)u+4\lambda^2\alpha(t))\varphi_1\\&+(-2\alpha(t)\xi u+4\lambda\alpha(t)\xi+4\alpha(t)\xi_{xx})\varphi_2-\alpha(t)u_x\varphi_3\end{aligned} \tag{51}$$

Using the relation $\phi^-(x,k)=\phi^+(x,k)T(k)$ and eqs. (25) and (26) yields:

$$T(t,k)=e^{-4ik^3\alpha(t)h}T(k)e^{4ik^3\alpha(t)h} \tag{52}$$

From eqs. (25), (26) (30) and (52), we then have:

$$\frac{\mathrm{d}a(t,k)}{\mathrm{d}t}=0,\quad \frac{\mathrm{d}b(t,k)}{\mathrm{d}t}=8ik^3b(t,k),\quad \frac{\mathrm{d}\gamma(t,k)}{\mathrm{d}t}=4ik^3\gamma(t,k) \tag{53}$$

Solving eq. (53) we arrive at eq. (46). Consequently, we can easily derive eq. (47) by using the relations $\bar{a}(t,k)=a(t,-k)$, $\bar{b}(t,k)=b(t,-k)$, $\bar{\gamma}(t,k)=\gamma(t,-k)$. Thus, the prove is end.

**Soliton solutions**

Leting $b(t,k)=0$ and $\rho(t,k)=0$, in the case of reflectionless potentials we obtain soliton solutions:

$$u(t,x)=2i\partial_x\sum_{j=1}^{N}R_j(x),\quad \xi(t,x)=i\sum_{j=1}^{N}\Theta_j(x) \tag{54}$$

where

$$R_n(x)=-\frac{\bar{b}e^{2\kappa_n x}}{\bar{a}'(i\kappa_n)}\left(1+i\sum_{j=1}^{N}\frac{R_j(x)}{\kappa_n+\kappa_j}\right),\quad \Theta_n(x)=-\frac{2\kappa_n\bar{\gamma}_n e^{\kappa_n x}}{\bar{a}'(i\kappa_n)}-i\frac{\bar{b}e^{2\kappa_n x}}{\bar{a}'(i\kappa_n)}\sum_{j=1}^{N}\frac{\Theta_j(x)}{\kappa_n+\kappa_j} \tag{55}$$

Particularly, we obtain one-soliton solutions:

$$u(t,x)=-\frac{c}{2}\mathrm{sech}^2\left(\frac{\sqrt{c}}{2}\left(x-c\int_0^t\alpha(\tau)\mathrm{d}\tau\right)\right),\quad \xi(t,x)=-\upsilon\,\mathrm{sech}\left(\frac{\sqrt{c}}{2}\left(x-c\int_0^t\alpha(\tau)\mathrm{d}\tau\right)\right) \tag{56}$$

where $\upsilon$ is a free odd constant element of the Grassmann algebra, and $\bar{a}'(0,\kappa_1)=1$, $\bar{b}(0,\kappa_1)=-i\sqrt{c}$, $\kappa_1=\sqrt{c}/2$ have been used.

For the supersymmetric KdV eqs. (4) and (5), it should be noted that if we suppose $\xi=\upsilon\omega$ then eq. (4) reduces to the KdV eq. (1). Sine the scattering transform for eq. (1) is known, we can solve eq. (5) given the known scattering solution $u$. Extending the scattering transform to supersymmetric non-linear evolution equations with local fractional derivatives [17-22] is worthy of study.

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**Nomenclature**

| | | | |
|---|---|---|---|
| d/d$t$ | —the first derivative, [–] | $n$, $N$ | —positive integers, [–] |
| $e$ | —the base of natural logarithms, [–] | $t$ | —time, [s] |
| $i$ | —imaginary unit, [–] | $T$ | —transposition, [–] |
| $j$ | —natural number, [–] | $x$, $y$, $z$ | —displacements, [m] |
| $k$, $\lambda$ | —spectral parameters, [–] | $\pi$ | —circumference ratio, [–] |

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