An iterative approach to viscoelastic boundary layer flows with heat source/sink and thermal radiation

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Abstract: In this study effect of radiation on the viscoelastic Walter-B fluid is investigated with heat sink/source. Sakiadis, Blasius and stagnation point flows are considered at constant surface temperature. Some suitable similarity variables have been utilized to transform governing equations into ordinary differential equations. An iterative approach based on the Legendre wavelet spectral collocation method (LWSCM) is applied for the solution of the resulting equations. The obtained results are validated by plotting the residual error curves in each case. Temperature and heat transfer rate at wall are analyzed to investigate the influence of involved parameters. It is found that the Legendre wavelet spectral collocation method (LWSCM) is very efficient and can be employed for the solutions of various non- Newtonian flow problems.

Keywords: Boundary layer flow; heat transfer; radiation effects; wavelets; collocation method; shooting method.

1. Introduction

The boundary layer phenomenon in non-Newtonian fluids is of great interest for the recent researchers due to its applications in the industry and applied sciences. The boundary layer flow on the flat plate was studied by [1-6]. Sakiadis [2, 3] was the first who studied the flow on a moving plate by applying boundary layer assumptions to the two-dimensional flow. The same problem by assuming a stretching velocity at the surface was first analyzed by Crane [4]. Heat transfer analysis for the boundary layer flow due to continuously moving plate was investigated by Tsou et al. [7]. Takar et al. [8] discussed the impact of fluid properties for the boundary layer flow of a viscous fluid due to a moving surface. Hiemenz [9] considered the boundary layer approximation to study the flow towards a stagnation point. Motivated by these pioneering works the two-dimensional flows utilizing the boundary layer approximations have been investigated extensively in the literature [10–22] and references therein.

The influence of heat transfer along with thermal radiation effects on the flows inside boundary layer in different situations have been investigated by several researchers [23- 28]. In studies [23, 24] Cortell investigated the effects of radiations on the Sakiadis and Blasius flows for different emerging parameters. Impact of thermal radiation on the flow due to boundary layer over an exponentially stretching surface is examined by Sajid and Hayat [29]. These studies are important in solar power technology, electrical power generation, cooling of electronic tools and nuclear reactors, satellites, space and other industrial areas.

Viscoelastic fluids have importance in industry, biological fluids, geophysics etc. Due to this fact in recent years, the study of viscoelastic fluids gains considerable attention of researchers working in this area. Some important investigations reflecting the viscoelastic effects are presented by [30-36]. Heat transfer in viscoelastic fluids due to boundary layer in the presence of thermal radiations and constant suction is discussed by [37-39].

The present investigation is devoted to investigate radiation and heat source/sink effects for Blasius, Sakiadis and stagnation point flows of Walter-B fluid [40]. The velocity profile overshoot in these flows have already been discussed by Sajid et al. [41]. The same method presented in [40] is adopted here for solving the highly nonlinear boundary value problems.

2. Mathematical formulation

The boundary layer flow of viscoelastic Walter-B fluid is governed by [40]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dP}{dx}+\frac{\mu_0}{\rho}\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{k_0}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right], \tag{2}$$

where u and v are respectively the horizontal and vertical velocity components, ρ is the density, P is the pressure, μ_0 and k_0 are coefficient of viscosity and viscoelastic parameter, respectively. The energy equation with heat source/sink and radiations is given by

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_1\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+Q(T-T_\infty), \tag{3}$$

in which T and T_∞ are the fluid and free stream temperatures respectively. Furthermore, c_p, k_1, Q and q_r denote the specific heat, thermal conductivity, volumetric rate of heat absorption/ generation and radiative heat flux, respectively. Employing Rosseland approximations [42], we can write

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4}$$

where k^* and σ^* are respectively, the mean absorption coefficient and Stefan-Boltzmann constant. Following Bataller [42], T^4 can be expressed using Taylor series as

$$T^4\cong 4T_\infty^3 T-3T_\infty^4. \tag{5}$$

Using Eqs. (4) and (5), Eq. (3) implies

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k_1+\frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+Q(T-T_\infty). \tag{6}$$

We are aiming to discuss the Blasius, Sakiadis and stagnation point flows. Boundary conditions for the considered flow situations for Blasius and stagnation point flows are

$$u(0)=v(0)=0,\ T(0)=T_w\ ,\quad u(\infty)=U_\infty,\ T(\infty)=T_\infty, \tag{7}$$

and for Sakiadis flow boundary conditions reads as

$$u\,(0)=U_w,\ v(0)=0,\ T(0)=T_w\ ,\ u(\infty)=0,\ \ T(\infty)=T_\infty, \tag{8}$$

in which, U_∞ and U_w are respectively the free stream and wall velocities. Furthermore, free stream velocity for the stagnation point flow is $U_\infty=ax$. Also T_w is the surface temperature.

Eq. (2) can be transformed to ordinary differential equation by a choice of suitable transformations [42]

$$\psi=\sqrt{\nu x U_\infty}f(\eta),\quad \eta=\sqrt{U_\infty/\nu x}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{9}$$

such that $\frac{\partial\psi}{\partial y}=u$ and $-\frac{\partial\psi}{\partial x}=v$.

Therefore, in the transformed variables we have

$$f'''+\frac{1}{2}ff''+K\left(ff^{iv}+2f'f'''-f''^2\right)=0,\qquad \text{(for Blasius and Sakiadis flow)} \tag{10}$$

$$f''' + ff'' - f'^2 + 1 + K\left(ff^{iv} - 2f'f''' + f''^2\right) = 0. \qquad \text{(for stagnation point flow)} \qquad (11)$$

Here $K = k_0 U_\infty / 2\mu x$ denotes the local Weissenberg number in the case of Blasius and Sakiadis flows and is a constant Weissenberg number for stagnation point flow. Energy Equation in transformed coordinates takes the form

$$\theta'' + Pr_{eff}\left(\frac{1}{2}f\theta' + \lambda\theta\right) = 0, \qquad \text{(for Blasius and Sakiadis flow)} \qquad (12)$$

$$\theta'' + Pr_{eff}(f\theta' + \lambda\theta) = 0, \qquad \text{(for stagnation point flow)} \qquad (13)$$

in which $Pr_{eff} = Pr/(1 + Nr)$ denotes the effective Prandtl number as discussed by [39], $Nr = 16\sigma^* T_\infty^3 / 3k^* k_1$ represents radiation parameter and $\lambda = Qx/\rho c_p U_\infty$ is the heat sink ($\lambda < 0$) or source ($\lambda > 0$) parameter. Boundary conditions for the considered flow problems are

$$f(0) = f'(0) = 0, \theta(0) = 1, f'(\infty) = 1, \theta(\infty) = 0, \ \text{(Blasius and stagnation point flows)} \qquad (14)$$

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1,\ f'(\infty) = 0,\ \theta(\infty) = 0,\ \text{(Sakiadis flow)} \qquad (15)$$

3. Numerical solutions

In this section we briefly explain the numerical technique known as LWSCM [41] used to solve the considered problems. In the first step Eqs. (10) and (12) are converted into initial value problems via shooting method. Letting $f''(0) = s$ and $\theta'(0) = s_1$ and differentiating Eq. (10) w.r.t. s and Eq. (12) w.r.t s_1 along with their boundary conditions, we get

$$g''' + \frac{1}{2}gf'' + \frac{1}{2}fg'' + K\left(gf^{iv} + fg^{iv} + 2f'g''' + 2g'f''' - 2f''g''\right) = 0, \qquad (16)$$

$$t'' + Pr_{eff}\left(\frac{1}{2}ft' + \lambda t\right) = 0, \qquad (17)$$

$$g(0) = 0, \quad g'(0) = 0, \quad g''(0) = 1, \quad t(0) = 0, \quad t'(0) = 1. \qquad (18)$$

In the next step the domain $0 \le \eta < \eta_\infty$ is divided into subintervals $[(n-1)/2^{k-1}, n/2^{k-1})$ in which $n = 1, \dots, 2^{k-1}\eta_\infty$. Therefore,

$$\chi_{lj}(\eta) = 0 \ \text{ for any } \ l \ne n \ \text{ and } \ \eta \in [(n-1)/2^{k-1}, n/2^{k-1}), \qquad (19)$$

where $\chi_{lj}(\eta) = \left\{\begin{matrix} \left(m + \frac{1}{2}\right)^{\frac{1}{2}} 2^{\frac{k}{2}} L_m(2^k\eta - 2n + 1), & \frac{n-1}{2^{k-1}} \le \eta < \frac{n}{2^{k-1}} \\ 0, & \text{otherwise} \end{matrix}\right\}$, are discrete wavelets in which $k = 1, 2, \dots, \eta_\infty$, $\quad n = 1, 2, \dots, 2^{k-1}\eta_\infty$. Legendre wavelet interpolation approximation to the functions $f(\eta)$ and $\theta(\eta)$ on the nth subinterval is given by

$$f(\eta) \simeq F_n(\eta) = \sum_{l=1}^{2^{k-1}T} \sum_{j=0}^{M-1} I_{;j}(\eta) f_{;j} = \sum_{j=0}^{M-1} I_{nj}(\eta) y_{nj}\,,\ for\ \ \eta \in \left[\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}}\right), \qquad (20)$$

$$\theta(\eta) \simeq \theta_n(\eta) = \sum_{l=1}^{2^{k-1}T} \sum_{j=0}^{M-1} I_{;j}(\eta) \theta_{;j} = \sum_{j=0}^{M-1} I_{nj}(\eta) \theta_{nj}\,,\ for\ \ \eta \in \left[\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}}\right), \qquad (21)$$

Similarly, one can define

$$g(\eta) \simeq G_n(\eta) = \sum_{j=0}^{M-1} I_{nj}(\eta) g_{nj}\,, \quad t(\eta) \simeq T_n(\eta) = \sum_{j=0}^{M-1} I_{nj}(\eta) t_{nj}, \qquad (22)$$

where $I_{nj}(\eta) = \bar{w}_j \sum_{n=0}^{2^{k-1}T} \chi_{n,m}(x_{nj}) \psi_{n,m}(\eta), \quad j = 0, \dots, M-1, \quad n = 1, \dots, 2^{k-1}\eta_\infty,$ (23)

and $\bar{w}_j = \frac{w_j}{2^k}, \quad x_{nj} = \frac{x_j}{2^k} + \frac{2n-1}{2^k}, \quad j = 0, \dots, M-1, \quad n = 1, \dots, 2^{k-1}\eta_\infty.$ (24)

$$w_j = \frac{2}{\left(1 - x_j^2\right)\left(L_M'(x_j)\right)^2}, \ \ j = 0, 1, \dots, M-1, \qquad (25)$$

where x_j are the Legendre-Gauss collocation points and w_j the corresponding weights. In fact, x_j are the roots of $L_m(x)$ in the interval $(-1,1)$ arranged in ascending order. Applying the points $\{x_{nj} | n = 1, \dots, 2^{k-1}\eta_\infty,\ \ j = 3, \dots, M-1\}$ into governing initial value problems, we get

$$F_n''' + \frac{1}{2}F_n F_n'' + K\left(F_n F_n^{iv} + 2F_n' F_n''' - F_n''^2\right) = 0, \qquad (26)$$

$$G_n{}''' + \frac{1}{2} G_n F_n{}'' + \frac{1}{2} F_n G_n{}'' + K\left(G_n F_n{}^{iv} + F_n G_n{}^{iv} + 2F_n{}' G_n{}''' + 2G_n' F_n''' - 2F_n'' G_n''\right) = 0, \quad (27)$$

$$\theta_n{}'' + Pr_{eff}\left(\frac{1}{2} F_n \theta_n{}' + \lambda \theta_n\right) = 0, \quad (28)$$

$$T_n{}'' + Pr_{eff}\left(\frac{1}{2} F_n T_n{}' + \lambda T_n\right) = 0, \quad (29)$$

$$F_1(0) = 0, \quad F_1{}'(0) = 0, \quad F_1''(0) = s, \quad \theta_1(0) = 1, \quad \theta_1'(0) = s_1, \quad (30)$$

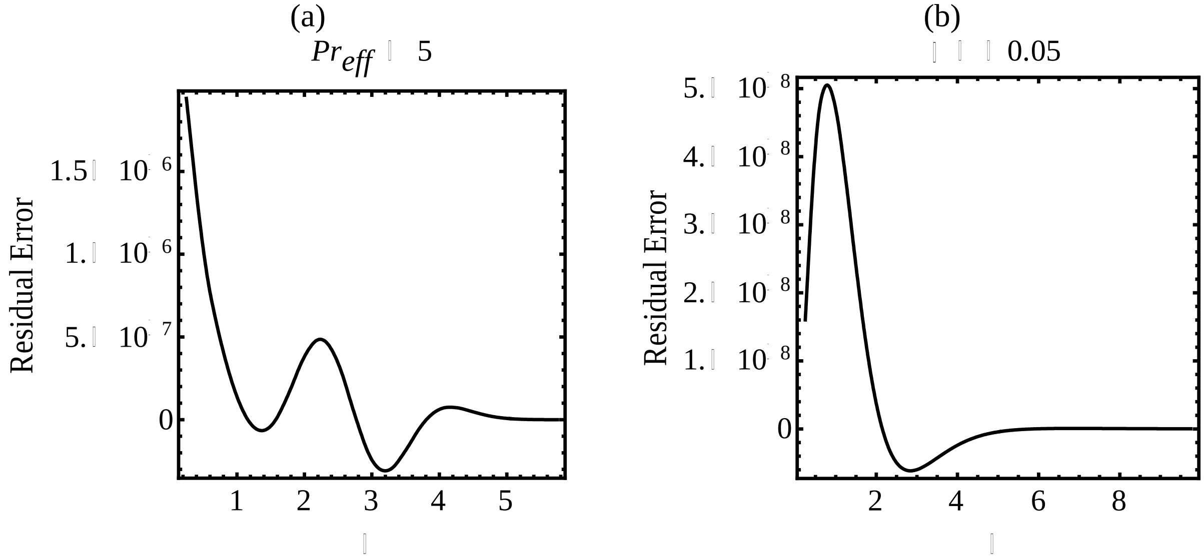
$$G_1(0) = 0, \quad G_1{}'(0) = 0, \quad G_1{}''(0) = 1, \quad T_1(0) = 0, \quad T_1'(0) = 1. \quad (31)$$

The initial value problems (26)-(31) are solved using parallel shooting technique in which solutions in the preceding subinterval provides the initial conditions for the next subinterval. The values of s and s_1 are modified using Newton's method so that $F_{2^{k-1}\eta_\infty}{}'(\eta_\infty) = 1.,\theta_{2^{k-1}\eta_\infty}{}'(\eta_\infty) = 0$. The appropriate values of M and k are chosen to obtain an accuracy of 10^{-6}. Sakiadis and stagnation point flow problems can be solved in the same way.

4. Numerical results and discussion

The mathematical models developed for analysis of heat transfer in Blasius, Sakiadis and stagnation point flows are solved numerically by implementing LWSCM technique. To ensure that the obtained solutions are convergent and accurate residual errors in all the cases have been plotted in Fig.1. The figure elaborates that the obtained solutions are within an accuracy of 10^{-6}.

(a) (b)



(c)

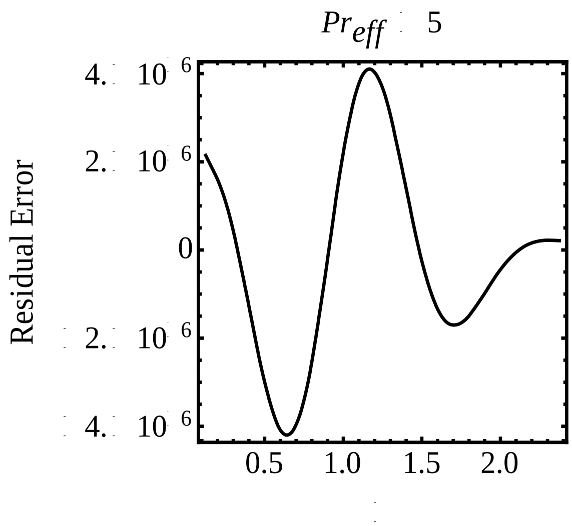
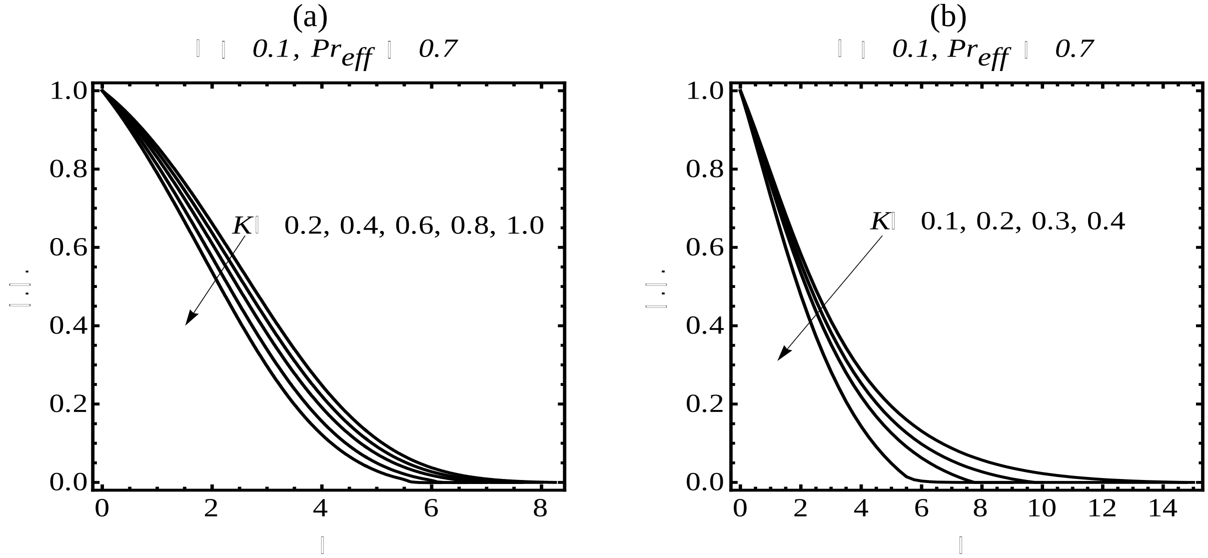


Fig. 1. Residual errors in case of (a): Blasius flow, (b): Sakiadis flow and (c): stagnation point flow.

Variation in the fluid temperature θ against the physical parameters is illustrated in Figs. 2–4. Our main attention is focused to see the influence of Pr_{eff}, λ and K on the fluid temperature θ inside the boundary layer. Effects of Weissenberg number K on the θ for the Blasius flow are displayed in Fig. 2(a). This figure depicts that temperature and thermal boundary layer thickness decrease by increasing K. It is concluded from Fig. 2(a) that the viscoelastic nature of the fluid reduces the temperature. The similar observation is noted in the case of Sakiadis and stagnation point flows (see Figs. 2(b) and 2(c)).

(a) (b)



(c)

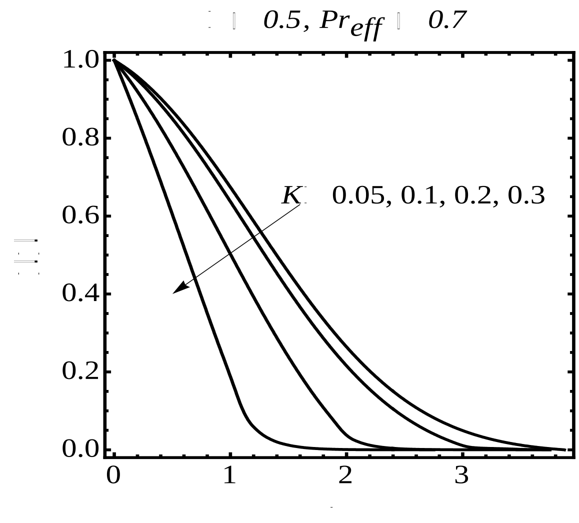
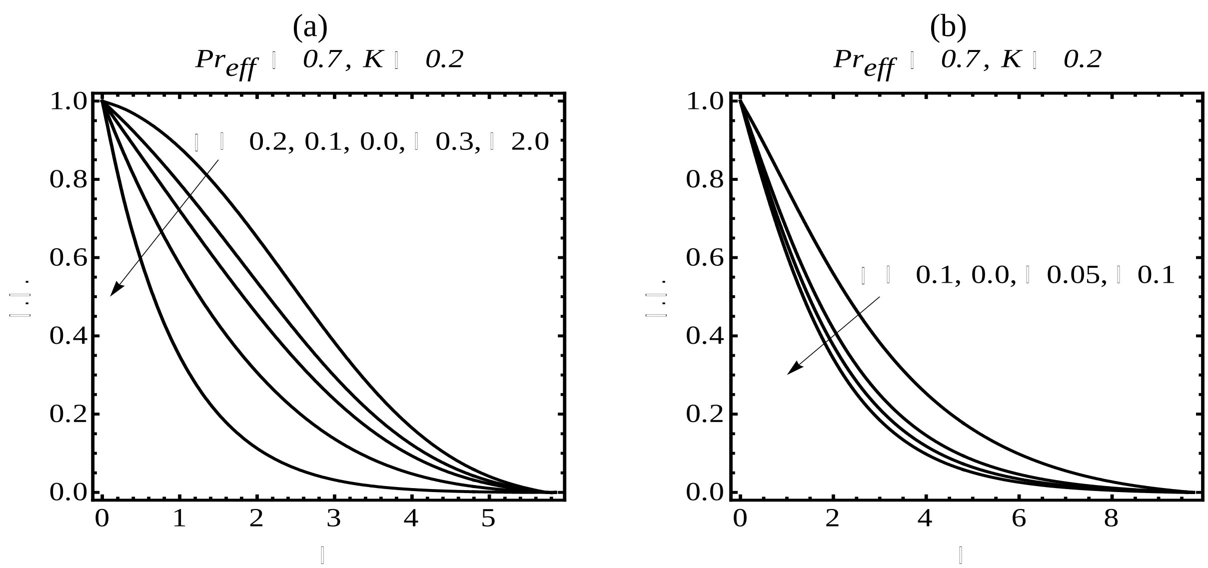


Fig. 2. Influence of Weissenberg number K on the temperature for (a): Blasius flow, (b): Sakiadis flow and (c): stagnation point flow.

Figure 3 depicts an increase / decrease in the temperature and thermal boundary layer thickness with heat source/sink in all the three considered flow situations.



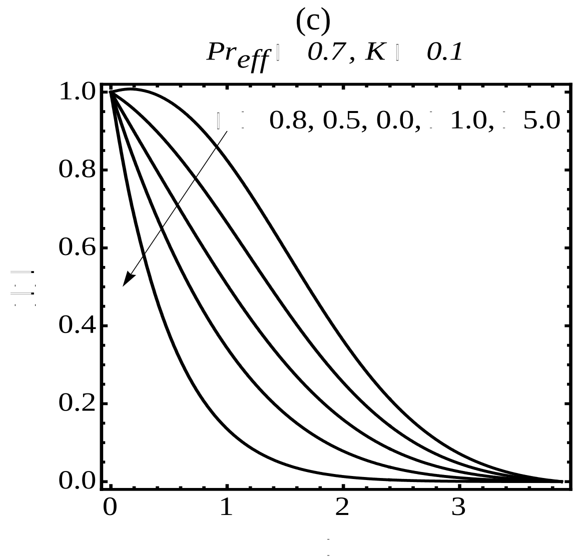
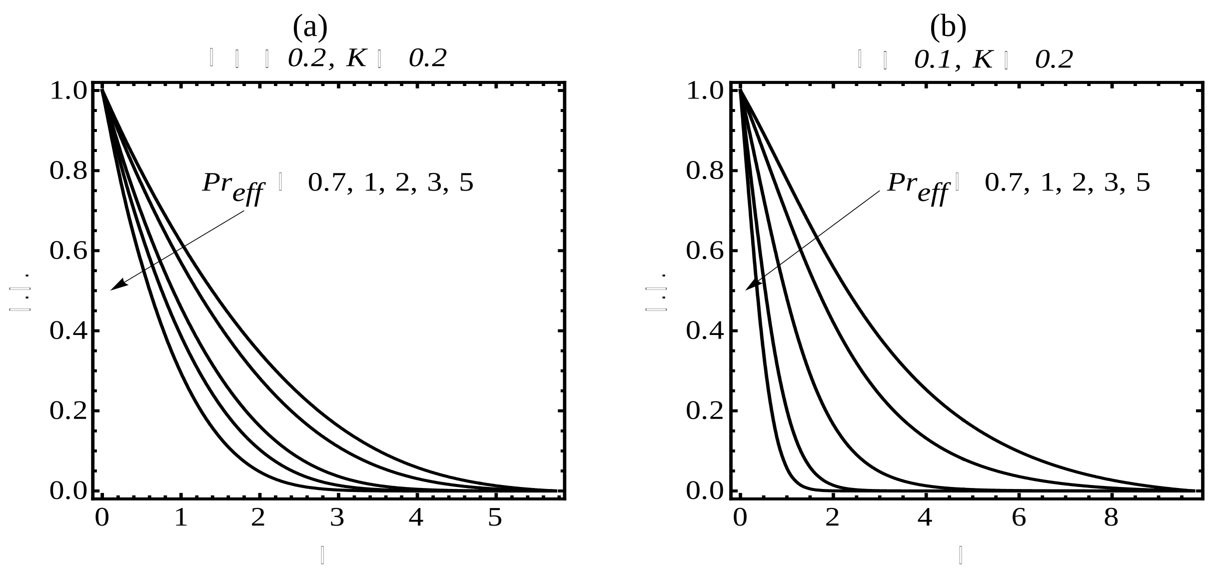


Fig. 3. Influence of parameter λ on the temperature for (a): Blasius flow, (b): Sakiadis flow and (c): stagnation point flow.

Variation in the temperature against the effective Prandtl number Pr_{eff} is elaborated in Fig. 4. According to this figure, fluid temperature and thermal boundary layer thickness decrease with an increase in Pr_{eff}. This decrease in the value of θ is due to the fact that Pr_{eff} is directly proportional to the Prandtl number and inversely proportional to the radiation parameter.



(c)

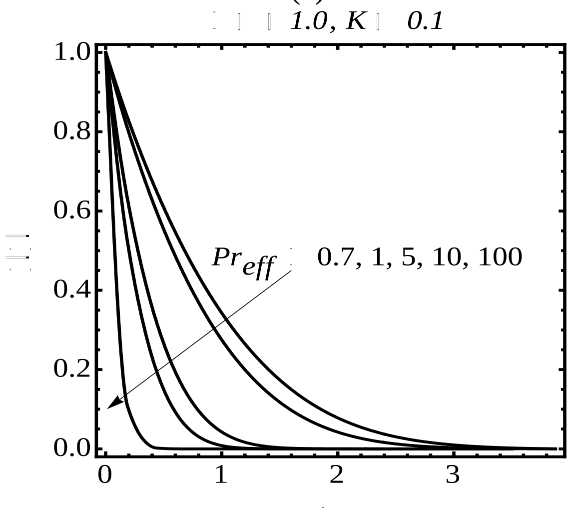


Fig. 4. Influence of Pr_{eff} on the temperature for (a): Blasius flow, (b): Sakiadis flow and (c): stagnation point flow.

To analyze the surface heat transfer rate against Pr_{eff} and λ, Tables 1-3 have been plotted for Blasius, stagnation point and Sakiadis flows. This table shows that heat transfer rate increases by increasing heat source and decreases by increasing heat sink. The table also depicts that heat transfer rate increases by increasing Pr_{eff}. Similar behavior is noted in Tables 2 and 3 for the Sakiadis and stagnation point flows.

Table 1. Variation in $\theta'(0)$ against Pr_{eff} and λ for Blasius flow when $K = 0.2$.

Pr_{eff}	$\lambda = -0.3$	$\lambda = -0.2$	$\lambda = 0$	$\lambda = 0.05$	$\lambda = 0.2$
0.7	−0.508088	−0.441539	−0.280936	−0.232344	−0.051440
1	−0.598573	−0.516683	−0.315068	−0.252399	−0.007691

Table 2. Variation in $\theta'(0)$ against Pr_{eff} and λ for stagnation point flow when $K = 0.1$.

Pr_{eff}	$\lambda = -5$	$\lambda = -1$	$\lambda = 0$	$\lambda = 0.1$
0.7	−1.898130	−0.948779	−0.560127	−0.513127
1	−2.264172	−1.112076	−0.614666	−0.551946
5	−5.031365	−2.372691	−1.076984	−0.890872
10	−7.103287	−3.308321	−1.383130	−1.092058

Table 3. Variation in $\theta'(0)$ against Pr_{eff} and λ for Sakiadis flow when $K = 0.2$.

Pr_{eff}	λ =-0.05	λ =-0.01	$\lambda = 0$	$\lambda = 0.05$	$\lambda = 0.1$
0.7	-0.397338	-0.356621	-0.345555	-0.283274	-0.202051
1	-0.495604	-0.450697	-0.438551	-0.371873	-0.288115
3	-0.941025	-0.879018	-0.862491	-0.775317	-0.679052
5	-1.249001	-1.171096	-1.150972	-1.045640	-0.931322
10	-1.809997	-1.704664	-1.677702	-1.536881	-1.385855

5. Conclusion

In this study, we have applied an iterative approach based on the LWSCM to present a boundary layer analysis for the heat transfer in a Walter-B viscoelastic fluid for three cases namely Blasius, Sakiadis and stagnation point flows. Numerical solutions are obtained to discuss heat transfer characteristics during the flow. The results are given for temperature distribution for the influence of various pertinent parameters. It is found that the temperature of the fluid is decreased for the Weissenberg Prandtl numbers. Also it is noticed that the proposed algorithm is very efficient and one can apply it on various flow problems regarding in non-Newtonian fluids.

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