IMPACT OF TEMPERATURE DEPENDENT HEAT SOURCE AND NONLINEAR RADIATIVE FLOW OF THIRD GRADE FLUID WITH CHEMICAL ASPECTS

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Abstract: This communication explores the magnetohydrodynamic (MHD) flow of third grade fluid bounded by a stretching surface with homogeneous-heterogeneous reactions. Incompressible fluid is electrically conducting in the presence of constant magnetic field. Heat transfer is performed through exponential based space internal heat source, non-linear thermal radiation and convective boundary condition. Nonlinear differential systems are computed by homotopic technique. Intervals of convergence through numerical data and plots are explicitly determined. The dimensionless velocity, temperature and concentration distributions manifesting the characteristics of various influential parameters are addressed. The skin friction coefficient and local Nusselt number are also addressed. Clearly temperature is enhanced by radiation, temperature ratio and magnetic parameters.

Keywords: Non-linear thermal radiation; MHD; Exponential based heat source; Homogeneous-heterogeneous reactions.

1. Introduction

Investigation of non-Newtonian fluids such as paints, cosmetic products, colloidal fluids, suspension fluids, shampoos, blood at low shear rate, ice cream, mud, polymers etc is current area of research for the recent researchers. It is due to their wide uses in engineering and industrial processes. The non-Newtonian fluids in comparison to Newtonian fluids are not easy to analyze. Certainly the diverse behaviors of non-Newtonian fluids cannot be described by Newton's law of viscosity. There is always non-linear link between the shear stresses and shear rate in the case of non-Newtonian fluids. Several models of non-Newtonian fluids for their diverse characteristics are suggested. Mainly such liquids have been classified into three main branches namely the integral, differential and rate types. A simple subclass of differential type fluids is known as second grade describing normal stress only. Note that second grade fluid do not predict shear thinning and shear thickening. Third grade fluids capture shear thinning/shear thickening effects even in one-dimensional steady flow. No doubt there is a sizeable informations for flows of third grade fluids at present. Few representative studies in this direction can be seen by the attempts [1–11].

The heat transfer in boundary layer flow over a stretching surface has relevance in processes like food processing, cooling of large metallic plat in a bath, glass fiber production, manufacturing of rubber and plastic sheets, wire drawing and many others. Crane [12] was the first who explored the boundary layer flow caused by stretching sheet. Bhattacharyya [13] numerically discussed the heat transfer in boundary layer flow induced by an exponentially stretching surface. Here shooting method is utilized for the solution procedure. Mukhopadhyay [14] analyzed the slip effects in unsteady mixed convective flow and heat transfer over a stretching surface. Hydromagnetic flow and heat transfer in flow of viscoelastic fluid is examined by Turkyilmazoglu [15]. Heat transfer and partial slip in MHD flow past a porous shrinking surface explored Zheng et al. [16]. Hayat et al. [17] developed the series solutions for heat transfer in

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unsteady flow of Jeffrey fluid over a stretching sheet. Mixed convection flow of viscoelastic fluid over a surface with heat transfer presented by Hayat et al. [18]. Some other related attempts are communicated in [19-28].

In nature there is a wide range of chemical reactions having useful practical applications. Some of the reactions have the capability to proceed moderately or do not react at all without catalyst. In general the contact between the heterogeneous and homogeneous chemical reactions is very complicated. Such reactions link the consumption and production of the reactant materials at distinct rates both within the fluid and on the surface of catalyst. Merkin [29] studied a problem for isothermal homogeneous-heterogeneous reactions in boundary layer flow over a flat plate. The effects of forced convection and homogeneous-heterogeneous reactions in stagnation point flow was explored by Chaudhary and Merkin [30]. Khan and Pop [31] investigated the flow of viscoelastic fluid over a stretching sheet with heterogeneous reactions. Flow of Maxwell fluid over a stretching surface with homogeneous-heterogeneous reactions was investigated by Hayat et al. [32]. The characteristics of melting heat and heterogeneous-homogeneous reactions in the flow of viscoelastic fluid was presented by Hayat et al. [33].

Here our prime focus for four aspects. Firstly to consider third grade fluid model with characteristics of MHD. Secondly to utilize the convective heat transport analysis in flow by stretched surface. Thirdly to perform analysis in the presence of homogeneous-heterogeneous reactions. Fourth to develop the series solutions through homotopic technique [34-40]. Sketch of different parameters are presented and discussed in detail. The results of skin friction coefficient and local Nusselt number are also analyzed.

2. Mathematical modeling and constitutive expression

Here we intend the 2D flow of an incompressible third grade fluid over a linear stretching sheet. A constant magnetic field $B_0$ is implemented parallel to the $y$ axis. Induced magnetic field is not accounted due to the consideration of small Reynolds number. Heat transport is inspected through exponential based space internal heat source and nonlinear thermal radiation. A Cartesian coordinate is assumed in such a manner that the $x -$ axis is selected along the stretched surface with velocity $U_w = cx$ and $y -$ axis is transverse to it. The convectively heated surface is expressed by heat transport coefficient $h_f$ and hot fluid temperature $T_f$. Aspects of heterogeneous-homogeneous reactions are intended. The isothermal cubic autocatalytic reaction (homogeneous) and the first order reaction (heterogeneous) on the surface of catalyst is demonstrated by [29, 30]:

$$A + 2B \rightarrow 3B, \quad rate = K_{ab}a^2b,$$

$$A \rightarrow B, \quad rate = K_a.$$

The concentrations of the chemical species $A$ and $B$ are $a$ and $b$ while $K_a$ and $K_f$ are the constants. Both the reaction processes are assumed to be isothermal. The governing boundary layer expressions can be put into the form [43-45]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left( u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial x \partial y^2} \right) + \frac{2\alpha_2 \nu}{\rho} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{6\alpha_3}{\rho} \left( \frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u,$$
\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + Q_o \frac{(T_w - T_\infty)}{\rho c_p} \exp \left\{-n \left( \frac{c}{v} \right)^{1/2} y \right\},
\]

(5)

\[
u \frac{\partial a}{\partial x} + v \frac{\partial a}{\partial y} = D_A \frac{\partial^2 a}{\partial y^2} - K_a a b^2,
\]

(6)

\[
u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} = D_B \frac{\partial^2 b}{\partial y^2} + K_b a b^2.
\]

(7)

The related boundary conditions are
\[
u = U_w = cx, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T), \quad D_A \frac{\partial a}{\partial y} = K_a a, \quad D_B \frac{\partial b}{\partial y} = -K_b a \quad \text{at} \quad y = 0,
\]

(8)

\[
u \rightarrow 0, \quad T \rightarrow T_\infty, \quad a \rightarrow a_0, \quad b \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty.
\]

(9)

In above expressions \((u, v)\) corresponds to the velocity components parallel to \(x\) – and \(y\) – directions, \(\mu\) designates the dynamic viscosity, \(\nu\) shows the kinematic viscosity, \(\rho\) the fluid density, \(n\) the exponential index, \(\alpha_1, \alpha_2\) and \(\alpha_3\) the material constants, \(\sigma\) the electrical conductivity, \(\alpha_m\) the thermal diffusivity of fluid, \(c\) the stretching rate, \(D_B\) and \(D_A\) the diffusion coefficients of \(B\) and \(A\) and \(a_0\) the positive dimensionless constant, \(a\) and \(b\) the concentration of chemical species, \(T\) and \(T_\infty\) the surface and ambient temperatures respectively. Through Rosseland’s approximation the radiative heat flux \(q_r\) is [46]:

\[q_r = -\frac{4\sigma^*}{3m^*} \frac{\partial(T^4)}{\partial y} = -\frac{16\alpha^*}{3m^{**}} T^3 \frac{\partial T}{\partial y},\]

(10)

in which \(\sigma^*\) shows the Stefan-Boltzman and \(m^{**}\) designates the coefficient of mean absorption. Invoking Eq. (10) the energy equation can be reduced to the form

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left( 16\sigma^* T^3 \frac{\partial T}{\partial y} \right) + Q_o \frac{(T_w - T_\infty)}{\rho c_p} \exp \left\{-n \left( \frac{c}{v} \right)^{1/2} y \right\}.
\]

(11)

The transformations are taken in the form

\[
u = c x f(\eta), \quad v = -\left( c v \right)^{1/2} f(\eta), \quad a = a_0 g(\eta), \quad b = b_0 h(\eta),
\]

(12)

\[
u \rightarrow \infty, \quad T \rightarrow T_\infty, \quad a \rightarrow a_0, \quad b \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty.
\]

Invoking above definitions, the continuity eq. (1) is now identically satisfied and eqs. (4) – (9) and (11) become

\[
\begin{align*}
&f'''' + f'' + f'' + \beta_1 \left( 2 f''' - f''' \right) + \left( 3 \beta_3 + 2 \beta_2 \right) f'^{1/2} + 6 \varepsilon_1 \varepsilon_2 f''' - M^2 f' = 0, \\
&(1 + \frac{\gamma}{Rd}) + \frac{\gamma}{Rd} \left[ (\theta - 1) (3 \theta^2 + \theta^2 + \theta^2 \theta^2) + 3 (\theta - 1) (2 \theta^2 + \theta^2 \theta^2) \right] + 3 (\theta - 1) (\theta^2 + \theta^2 \theta^2) + \Pr f \theta' + \Pr \delta \exp(-\eta) = 0, \\
&\frac{1}{Sc} g'' + fg' - K_1 gh^2 = 0, \\
&\frac{\delta}{Sc} h'' + fh' + K_1 gh^2 = 0,
\end{align*}
\]

(13) – (16)
\[ f = 0, \quad f' = 1, \quad \theta' = -\gamma (1 - \theta), \quad g' = K_2 g, \quad \delta h' = -K_2 h \text{ at } \eta = 0, \quad (17) \]
\[ f' \rightarrow 0, \quad \theta \rightarrow 0, \quad g \rightarrow 1, \quad h \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (18) \]

Where \( \beta_1, \beta_2 \) and \( \varepsilon_1 \) are the material parameters for third grade fluid, \( \varepsilon_2 \) is the local Reynolds number, \( M \) represents the magnetic parameter, \( \delta \) the internal heat source parameter, \( \gamma \) the Biot number, \( \theta_w \) the temperature ratio parameter, \( K_1 \) denotes the strength of homogeneous reaction parameter, \( \delta \) the diffusion coefficient ratio, \( K_2 \) the strength of heterogeneous reaction, \( \text{Sc} \) the Schmidt number and prime designates differentiation with respect to \( \eta \). These quantities are expressed as follows:

\[
\beta_1 = \frac{c_m}{\mu}, \quad \beta_2 = \frac{c_m}{\mu}, \quad \varepsilon_1 = \frac{c_m}{\mu}, \quad M^2 = \frac{\sigma \beta_1}{\rho c}, \quad \text{Pr} = \frac{v}{\alpha}, \quad K_1 = \frac{K_1 \alpha_1}{c}, \quad \text{Rd} = \frac{4 \sigma \beta_1 \alpha_1}{m\kappa} \}
\]
\[
\delta = \frac{\theta_w}{\alpha_1}, \quad \theta_w = \frac{\tau}{R}, \quad \varepsilon_2 = \frac{c}{v}, \quad K_2 = \frac{K_2}{D_1}, \quad \text{Sc} = \frac{\nu}{D_1}, \quad \delta = \frac{D_1}{D}, \quad \text{and } \gamma = \frac{h_j}{c}. \quad (19) \]

Here comparable size is presumed for the coefficients of diffusion of chemical species \( B \) and \( A \). This fact provides us to establish further supposition that the coefficients of diffusion \( D_A \) and \( D_B \) are same i.e. \( \delta = 1 \) and thus:

\[ g(\eta) + h(\eta) = 1. \quad (20) \]

Now eqs. (13) and (14) give

\[ \frac{1}{\text{Sc}} g'' + f g' - K_1 g (1 - g)^2 = 0, \quad (21) \]

and the corresponding boundary conditions are

\[ g'(0) = K_2 g(0), \quad g(\infty) \rightarrow 1. \quad (22) \]

Expressions for velocity gradient \( (C_f) \) and temperature gradient \( (\text{Nu}_x) \) are

\[ C_f = \frac{2\tau_w}{\rho U_w^2}, \quad \text{Nu}_x = \frac{x q_w}{k(T_f - T_0)} + (q_r)_w, \quad (23) \]

where

\[
\tau_w = \left[ \mu \frac{\partial u}{\partial y} + \alpha_1 \left( u \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + K \frac{\partial^2 u}{\partial y^2} \right) + 2 \alpha_2 \frac{\partial u}{\partial y} \right]_{y=0}, \quad q_w = -k \frac{\partial T}{\partial y} \bigg|_{y=0}. \quad (24) \]

The dimensionless form of Eq. (23) gives

\[
\left( \text{Re} \right)^{1/2} C_f = \left( f'' + \beta_1 (3 f f'' - f f''') + 2 \varepsilon_2 f^{(1)} \right)_{\eta=0}, \quad (25) \]

where \( \text{Re} \) is the Reynolds number defined by \( \text{Re} = U_w x / \nu \).

### 3. Series solutions and convergence

The initial guesses and operators are given below:

\[ f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = \frac{\gamma}{1 + \gamma} e^{-\eta}, \quad g_0(\eta) = 1 - \frac{1}{2} e^{-K_1 \eta}, \quad (26) \]
\[ L_f = f'' - f', \quad L_\theta = \theta'' - \theta, \quad L_g = g'' - g, \quad (27) \]
\[ L_f \left[ C_1 + C_3 e^\eta + C_3 e^{-\eta} \right] = 0, \quad L_\theta \left[ C_4 e^\eta + C_4 e^{-\eta} \right] = 0, \quad L_g \left[ C_6 e^\eta + C_7 e^{-\eta} \right] = 0, \quad (28) \]
in which the constants $C_i$ ($i = 1-7$) are defined by

$$
\begin{align*}
C_2 &= C_4 = C_6 = 0, \\
C_3 &= \frac{\partial f_m^*(\eta)}{\partial \eta} |_{\eta=0}, \\
C_1 &= -C_3 - f_m^*(0), \\
C_5 &= \frac{1}{1+\frac{1}{\nu_f}} \frac{\partial f_m^*(\eta)}{\partial \eta} |_{\eta=0} - \frac{1}{1+K_2} \frac{\partial f_m^*(\eta)}{\partial \eta} |_{\eta=0} - K_2 g_m^*(0). \\
\end{align*}
$$

(29)

In homotopic solutions, the rate of deformation and convergence region highly depend upon $h_f$, $h_\theta$ and $h_g$. For such interest, the $h$-curves have been plotted in Fig. 1 at 16-th order of deformations. Such curves provide the admissible ranges of these auxiliary parameters. It is clear from this fig. 1 that the suitable ranges of these parameters are $[-1.0, -0.1]$, $[-1.4, -0.4]$ and $[-1.1, -0.2]$. Besides that the series solutions are convergent in all region of $\eta$ when $h_f = -0.4 = h_\theta = h_g$. Table 1 shows that 30 th order of approximations up to 4 decimal places are adequate for good agreement regarding convergence.

![Fig. 1: The $h$-curve for $f(\eta)$](image1)

![Fig. 2: The $h$-curves for $\theta(\eta)$ and $g(\eta)$](image2)

Table 1: Convergence of homotopic solutions when $Rd = 0.4 = \delta$, $\theta_w = 0.3$, $\varepsilon_1 = \varepsilon_2 = 0.2 = M = \gamma$, $\beta_1 = 0.1 = \beta_2$, $Pr = 1.0$, $K_1 = 0.5 = K_2$, $Sc = 0.9$

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4. Results and Discussion

This portion focuses for the impact of distinct influential parameters on the dimensionless velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $g(\eta)$ profiles. These outcomes are interpreted via graphs
in the Figs. (3) – (14). The consequences of magnetic parameter $M$ on the velocity distribution $f'(\eta)$ can be seen in Fig. 3. For larger magnetic parameter $M$ the velocity field reduces close to the surface and it vanishes away from the surface. It is quite obvious because larger magnetic parameter corresponds to enhancement of Lorentz forces thereby reducing the velocity profile $f'(\eta)$. Fig. 4 investigates the impact of material parameter $\beta_1$ on $f'(\eta)$. It is seen that velocity distribution increases when we enlarge $\beta_1$. The characteristics of material parameter $\beta_2$ on the velocity field is sketched in Fig. 5. It is concluded that higher values of $\beta_2$ corresponds to an enhancement in the fluid velocity. In fact higher values of material parameter tend to enhance the normal stresses and it reduces the viscous forces which lead to increase the fluid velocity. Fig. 6 declared the aspect of $Rd$ on $\theta(\eta)$. Here bigger values of $Rd$ augments the thermal field. As expected heat is produced due to radiation process in the working fluid so thermal distribution enhances. Fig. 7 presents the variations in magnetic parameter $M$ on the temperature distribution $\theta(\eta)$. It is well known established fact that the magnetic field intensity tends to create drag force which restricts the fluid motion and heats up the fluid. There is rise in the temperature and thickness of thermal boundary layer. Note that the case $M = 0$ corresponds to hydrodynamic flow situation. Feature of $\delta$ on $\theta(\eta)$ is pointed out in Fig. 8. It is figure out that thermal field is enhances via $\delta$. Fig. 9 illustrates the consequences of Biot number $\gamma$ on the temperature filed $\theta(\eta)$. It is noticed that both temperature and thermal layer thickness are increasing functions of Biot number $\gamma$. Fig. 10 depicts the change of temperature in response to change in the $\theta_u$. Both thermal field and associated layer thickness are enhance when $\theta_u$ is enlarged. Fig. 11 portrays the influence of Prandtl number $Pr$ on the temperature profile $\theta(\eta)$. Fluids of higher Prandtl have minimum thermal conductivities so that heat can spread away from the plate slower than for smaller $Pr$ fluids. Hence rise in Prandtl number substantially decreases the temperature and thickness of thermal boundary layer. Larger strength of homogeneous reaction $K_1$ shows a reduction in concentration distribution $g(\eta)$ (see Fig. 12). Fig. 13 explores the impacts of strength of the heterogeneous reaction $K_2$ on the concentration profile $g(\eta)$. There is an increase in concentration field $g(\eta)$ for larger values of $K_2$. Fig. 14 shows the impact of Biot number $\gamma$ and Prandtl number $Pr$ on Nusselt number. It is examined that by increasing $\gamma$ and $Pr$ the Nusselt number enhances. Numerical data of skin friction coefficient $-\left(Re\right)^{1/2}C_f$ for pertinent flow parameters including $\varepsilon_1$, $\varepsilon_2$, $M$, $\beta_1$, $\beta_2$, $Pr$ and $\gamma$ are presented in tab. 2. It is found that the coefficient of skin friction increases for higher values of $Pr$ and $M$ whereas opposite effect is observed for $\gamma$. 
Fig. 3. Impact of $M$ on $f'(\eta)$.

Fig. 4. Impact of $\beta_1$ on $f'(\eta)$.

Fig. 5. Impact of $\beta_2$ on $f'(\eta)$.

Fig. 6. Impact of $R_d$ on $\theta(\eta)$.

Fig. 7. Impact of $M$ on $\theta(\eta)$.

Fig. 8. Impact of $\delta$ on $\theta(\eta)$. 
Fig. 9. Impact of $\gamma$ on $\theta(\eta)$.

Fig. 10. Impact of $\theta_w$ on $\theta(\eta)$.

Fig. 11. Impact of $\text{Pr}$ on $\theta(\eta)$.

Fig. 12. Impact of $K_1$ on $g(\eta)$.

Fig. 13. Impact of $K_2$ on $g(\eta)$.

Fig. 14. Impacts of $\text{Pr}$ and $\gamma$ on Nusselt number.
Table 2: Numerical estimations of drag force $-\left(\text{Re}\right)^{1/2} C_{f, x}$ for various values of $\varepsilon_1$, $\varepsilon_2$, $M$, $\beta_1$ and $\beta_2$ when $Rd = 0.4 = \delta$, $\theta_m = 0.3$, $Pr = 0.9$, $\gamma = 0.1$, $K_1 = 0.5 = K_2$ and $Sc = 0.9$.

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5. Conclusions
Homogeneous-heterogeneous reactions in magnetohydrodynamic (MHD) flow of third grade fluid over a starching sheet are explored. Main results are reported below.
- The characteristics of material parameters $\beta_1$ and $\beta_2$ has similar effects on the velocity field.
- Increasing values of $M$ show opposite behavior for the velocity and temperature fields.
- An enhancement in convective parameter $\gamma$ shows an increment in temperature field.
- Temperature field is enhancing function of $\delta$, $Rd$ and $\theta_m$.
- The heterogeneous and homogeneous reactions strengths show an opposite behavior on concentration field.
- Magnitude of skin friction coefficient is increasing functions of $\beta_1$ and $M$.
- Local Nusselt number represents the opposite behavior for higher values of $M$ and $Pr$.

References


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