+DARCY FORCHHEIMER FLOW OF JEFFREY NANOFLUID WITH HEAT GENERATION/ABSORPTION AND MELTING HEAT TRANSFER

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Abstract: *This study reports Darcy-Forchheimer flow of magnetohyrodynamic (MHD) Jeffrey nanofluid bounded by non-linear stretching sheet with variable thickness. Thermophoresis and Brownian motion are studied. Heat transfer is accounted with melting heat and heat absorption/generation. Optimal homotopy analysis method (OHAM) is utilized for the solutions development of nonlinear ordinary differential system. Outcomes of parameters involved in equation are studied through graphs. Outcomes indicate that ratio parameter declines the velocity. Melting parameter enhances temperature and concentration. Nusselt number increases in the occurrence of thermophoresis Brownian motion.*

Keywords: *Jeffrey fluid; MHD; Melting heat transfer; Porous medium; Heat generation/absorption*

Introduction

Non-Newtonian fluid dynamics is popular area of research for the investigation. Recent researchers have exposed deep attention in this field of research due to its utilization in industry and in many other fields. The viscous fluids can be elaborated by single constitutive equation whereas non-Newtonian fluid due to its different structures cannot be debated by single constitutive expression. Therefore numerous models of non-Newtonian materials exist. In past much attention is devoted to subclasses of differential and rate type liquids. Jeffrey material is one of the non-Newtonian liquid which can be predict the retardation and relaxation effects. Jeffrey fluid model due to its application in bio-engineering, geophysics, oil reservoir process and chemical and nuclear technologies has remarkable importance [1-8]. Hayat et al. [9] exmained stratifications in radiative flow of Jeffrey fluid. MHD Jeffrey fluid flow with variable fluid properties is investigated by Mabood et al. [10]. Gaffar et al. [11] reported influence of mixed convection in Jeffrey fluid flow by a non-isothermal segment.

Flow saturating permeable medium is significant in fields like thermal engineering, geothermal processes, chemical and petroleum equipment etc. Much attention in permeable space is given by darcy's law. However Darcy law is not meaningful over those area where permeable medium takes higher flow rates due to non-uniformness near the wall area. Therefore non-Darcian effect due to porous medium becomes necessary to investigate the heat transfer and flow analysis. Tamayol et al. [12] addressed thermal exploration of fluid flow in a permeable medium. Hong et al. [13] reported convective flow under the influence of non-Darcian effects. Khani et al. [14] discussed fluid flow saturating a non-Darcy permeable media with heat transfer. Thermal radiation impact in non-Darcian fluid flow is explored by Pal et al. [15]. Hayat et al. [16] considered convective CNTs nanofluid flow through non-Darcy porous medium.

Technologies and industries have widespread utilizations of melting phenomenon. Researchers have paid full consideration to improve effective, sustainable and energy depot technologies. Such technologies are mutually connected with excess heat repossession, planetary, power and plants heat. Three procedures have been implemented for energy storage for example latent, sensible heat and chemical energy. The economically sound storage of heat energy is latent heat through the adjustment of material phase. In hydraulic processes, the thermal energy is deported by latent heat i.e. melting and regain again by

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freezing it. Melting phenomenon has its application in many fields namely heat exchanger coils, based pump, the freeze treatment, solidification, welding processes and many others. Rahman et al. [17] addressed radiative effects in MHD flow over a extended surface. Melting temperature of ice piece in the stream of hot air is addressed by Robert [18]. Das [19] reported MHD flow with melting and radiation influences. Hayat et al. [20] investigated MHD flow of Cu-nanofluid with viscous dissipation and joule heating.

The current study investigates Darcy Forchheimer flow of MHD Jeffrey nanofluid. Melting heat transfer and heat generation/absorption are also incorporated for heat transfer. The nonlinear PDEs are distorted to nonlinear ODEs with the help of similarity transformations. Optimal homotopy analysis method is utilized [21-31] for solutions development. The outcomes of Nusselt and Sherwood number are argued through graphs.

Statement

Two dimensional boundary layer flow of Jeffrey nanofluid is under consideration. Flow generated is by nonlinear stretching sheet with variable thickness at $y=\delta(x+b_1)^{\frac{1-n}{2}}$. Stretching velocity has velocity $U_w = a_1(x+b_1)^n$. Our interest here is to discuss melting heat and heat generation/absorption. Porous medium is categorized by Darcy-Forchheimer relation. The problem statement are;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\left\{\begin{array}{c} u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=\dfrac{\nu}{1+\lambda_2}\left[\dfrac{\partial^2 u}{\partial y^2}+\lambda_1\left\{v\dfrac{\partial^3 u}{\partial y^3}+u\dfrac{\partial^3 u}{\partial x\partial y^2}+\dfrac{\partial u}{\partial y}\dfrac{\partial^2 u}{\partial x\partial y}-\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial y^2}\right\}\right] \\ -\dfrac{\sigma}{\rho}B_o^2 u-\dfrac{\nu\varepsilon}{k}u-\dfrac{c_b\varepsilon}{\sqrt{k}}u^2, \end{array}\right\} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha^*\frac{\partial^2 T}{\partial y^2}+\tau\left(\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2+D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{Q_o(T_o-T_m)}{(\rho c)_p}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)+D_B\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

$$u=U_w=a_1(x+b_1)^n,\quad v=0,\quad C=C_w,\ T=T_m \text{ at } \quad y=\delta(x+b_1)^{\frac{1-n}{2}}, \tag{5}$$

$$u\to 0,\ \ C\to C_\infty,\ T\to T_\infty \text{ when } y\to\infty. \tag{6}$$

$$k^*\left(\frac{\partial T}{\partial y}\right)=\rho\left[\lambda^*+C_S\left(T_m-T_0\right)\right]v(x,0) \text{ at } y=\delta(x+b_1)^{\frac{1-n}{2}}. \tag{7}$$

Considering

$$\psi=\sqrt{(\frac{2}{n+1})\upsilon a_1(x+b_1)^{n+1}}F(\eta),\quad \eta=\sqrt{(\frac{n+1}{2})\frac{a_1}{\upsilon}(x+b_1)^{n-1}}\,y,$$

$$u=a_1(x+b_1)^n F'(\eta),\ v=-\sqrt{(\frac{n+1}{2})\upsilon a_1(x+b_1)^{n-1}}\left[F(\eta)+\eta(\frac{n-1}{n+1})F'(\eta)\right], \tag{8}$$

$$\Theta(\eta)=\frac{T-T_m}{T_\infty-T_m},\quad G(\eta)=\frac{C-C_\infty}{C_\infty}. \tag{9}$$

Incompressibility condition (1) is automatically satisfied. The additional equations and conditions give

$$\left.\begin{aligned} F''' - \left(\tfrac{2n}{n+1}\right)(1+\lambda_2)F'^2 + (1+\lambda_2)FF'' - K\left[\left(\tfrac{n+1}{2}\right)F'F^{iv} - (n-1)F'F''' - \left(\tfrac{3n-1}{2}\right)F''^2\right] \\ -\left(\tfrac{2}{n+1}\right)(1+\lambda_2)\left[(Ha)^2F' - (Da)F' - \beta F'^2\right] = 0 \end{aligned}\right\} \tag{10}$$

$$\Theta'' + \Pr\left[F\Theta' + Nb\Theta'\Phi' + Nt\Theta'^2 + \left(\frac{2}{n+1}\right)\lambda\Theta\right] = 0, \tag{11}$$

$$\Phi'' + \Pr LeF\Phi' + \left(\frac{Nt}{Nb}\right)\Theta'' = 0, \tag{12}$$

$$\left.\begin{aligned} F'(\alpha) = 1,\ \ \Theta(\alpha) = 0,\ \ M\Theta'(\alpha) + \Pr F(\alpha) + \Pr\eta\left(\tfrac{n-1}{n+1}\right) = 0,\ \Phi(\alpha) = 0, \\ F'(\infty) = 0,\ \Theta(\infty) = 1,\ \Phi(\infty) = 0 \end{aligned}\right\}. \tag{13}$$

Defining $F(\eta) = f(\eta-\alpha) = f(\xi), \Theta(\eta) = \theta(\eta-\alpha) = \theta(\xi),$ $\Phi(\eta) = \phi(\eta-\alpha) = \phi(\xi)$ equation $(10-13)$ become

$$\left.\begin{aligned} f''' - \left(\tfrac{2n}{n+1}\right)(1+\lambda_2)f'^2 + (1+\lambda_2)ff'' - K\left[\left(\tfrac{n+1}{2}\right)f'f^{iv} - (n-1)f'f''' - \left(\tfrac{3n-1}{2}\right)f''^2\right] \\ -\left(\tfrac{2}{n+1}\right)(1+\lambda_2)\left[(Ha)^2f' - (Da)f' - \beta f'^2\right] = 0, \end{aligned}\right\} \tag{14}$$

$$\theta'' + \Pr\left[f\theta' + Nb\theta'\phi' + Nt\theta'^2 + \left(\frac{2}{n+1}\right)\lambda\theta\right] = 0, \tag{15}$$

$$\phi'' + Le\Pr f\phi' + \frac{Nt}{Nb}\theta'' = 0, \tag{16}$$

$$\left.\begin{aligned} f'(0) = 1,\ \ \theta(0) = 0,\ \ M\theta'(0) + \Pr f(0) + \Pr\alpha\left(\tfrac{n-1}{n+1}\right) = 0,\ \phi(0) = 0, \\ f'(\infty) = 0,\ \theta(\infty) = 1,\ \phi(\infty) = 0 \end{aligned}\right\}, \tag{17}$$

with

$$\Pr = \frac{\nu}{\alpha},\ K = \lambda_1 a(x+b^1)^{n-1},\ Ha = \sqrt{\frac{\sigma}{\rho a}}B_0,\ Da = \frac{\varepsilon\upsilon}{ka^1(x+b^1)^{n-1}},$$

$$\lambda = \frac{Q_O}{a\rho c_p},\ \beta = \frac{C_b\varepsilon(x+b^1)}{\sqrt{k}},\ M = \frac{C_p(T_\infty - T_m)}{\lambda^* + C_s(T_m - T_0)}$$

$$Nt = \frac{\tau D_T(T_\infty - T_m)}{\nu T_\infty},\ Nb = \frac{\tau D_B(C_\infty - C_m)}{\nu},\ Le = \frac{\alpha}{D_B}$$

The Skin friction, Nusselt and Sherwood number are

$$C_f = \frac{2\tau_w}{u_w^2\rho},\ \ N_u = \frac{q_w(x+b)}{k(T_\infty - T_m)},\ Sh = \frac{q_m(x+b)}{D_B(C_\infty)}. \tag{18}$$

In dimensionless coordinates one has

$$C_f\left(\mathrm{Re}_x\right)^{1/2} = \frac{1}{1+\lambda_2}\left[f''(0) + K\left(f'(0)f''(0) - \left(\frac{n+1}{2}\right)f(0)f'''(0)\right)\right], \tag{19}$$

$$\frac{Nu}{\sqrt{\mathrm{Re}_x}} = -\sqrt{\frac{n+1}{2}}\theta'(0), \tag{20}$$

$$\frac{Sh}{\sqrt{\mathrm{Re}_x}} = -\sqrt{\frac{n+1}{2}}\phi'(0), \tag{21}$$

where $\mathrm{Re}_x = \frac{a_1(x+b_1)^{n+1}}{\nu}$ the Reynolds number.

Solutions by OHAM

The initial guesses and operators satisfy

$$\begin{gathered} f_0(\eta) = (1-\exp(-\xi)) - \tfrac{M}{\mathrm{Pr}} - \alpha\,\tfrac{n-1}{n+1}, \\ \theta_0(\eta) = 1-\mathrm{e}^{(-\xi)}, \\ \phi_0(\eta) = \mathrm{e}^{(-\xi)}. \end{gathered} \tag{22}$$

$$\mathbf{L}_f(f) = -\left(\frac{df}{d\xi} - \frac{d^3 f}{d\xi^3}\right), \ \mathbf{L}_\theta(\theta) = \left(\frac{d^2\theta}{d\xi^2} - \theta\right), \ \mathbf{L}_\phi(\phi) = \left(\frac{d^2\phi}{d\xi^2} - \phi\right), \tag{23}$$

with

$$\mathbf{L}_f\left[D_1 + D_2\,\mathrm{e}^{\xi} + D_3\,\mathrm{e}^{\xi}\right] = 0, \tag{24}$$

$$\mathbf{L}_\theta\left[D_4\,\mathrm{e}^{\xi} + D_5\,\mathrm{e}^{-\xi}\right] = 0, \tag{25}$$

$$\mathbf{L}_\phi\left[D_6\,\mathrm{e}^{\xi} + D_7\,\mathrm{e}^{-\xi}\right] = 0, \tag{26}$$

where $D_i\ (i=1-7)$ are the arbitrary constants. The total square residual error (ε_k^t) is arranged by the following expressions:

$$\varepsilon_k^f(h_f) = \frac{1}{N+1}\sum_{j=0}^{N}\left[\sum_{i=0}^{k}(f_i)_{\xi=j\Pi\xi}\right]^2, \tag{27}$$

$$\varepsilon_k^\theta(h_f,h_\theta,h_\phi) = \frac{1}{N+1}\sum_{j=0}^{N}\left[\sum_{i=0}^{k}(f_i)_{\xi=j\Pi\xi},\ \sum_{i=0}^{k}(\theta_i)_{\xi=j\Pi\xi},\ \sum_{i=0}^{k}(\phi_i)_{\xi=j\Pi\xi}\right]^2, \tag{28}$$

$$\varepsilon_k^\phi(h_f,h_\theta,h_\phi) = \frac{1}{N+1}\sum_{j=0}^{N}\left[\sum_{i=0}^{k}(f_i)_{\xi=j\Pi\xi},\ \sum_{i=0}^{k}(\theta_i)_{\xi=j\Pi\xi},\ \sum_{i=0}^{k}(\phi_i)_{\xi=j\Pi\xi}\right]^2, \tag{29}$$

$$\varepsilon_k^t = \varepsilon_k^f + \varepsilon_k^\theta + \varepsilon_k^\phi \tag{30}$$

The complete squared residual error is reduced by using Mathematica (BVPh2.0) case has been considered. The optimal values of convergence-control variables are $h_f = -0.967169$, $h_\theta = -0.518451$, $h_\phi = -1.36582$ and averaged squared residual error is $(\varepsilon_k^t = 7.15033\times 10^8)$.

Discussion

We secure the values of nondimensional variables for numerical solutions as $n=0.5$, $\lambda=0.1$, $\beta=0.1$, $Da=0.1$, $\alpha=0.2$, $\lambda_2=0.1$, $K=0.4, Ha=0.3$, $Nb=M=0.2$, $\mathrm{Pr}=Le=1$. and $Nt=0.4$. These values are taken as constant besides the variable in the figures. Fig. 1 shows the plots for velocity via (n). Clearly velocity is an increasing for larger values of power index (n). In fact that stretching velocity improves for (n). This develops more distortion in fluid. Fig. 2 displays velocity for melting variable. Velocity profile $f'(\xi)$ enhances via melting parameter (M). Fig. 3 exhibits the plots for velocity via inverse Darcy number. Here resistive force enhances via inverse Darcy number and the velocity of fluid decreases. Similar behavior is shown via inertia parameter β (see Fig. 4). Impact of Deborah number (K) on $f'(\xi)$ is shown in Fig. 5. Higher retardation time improves fluid flow and thus velocity increases. Fig 6 illustrate the plots via ratio parameter (relaxation to retardation) for velocity field. Here velocity declines via ratio parameter. Fig. 7 addresses the significances of heat generation parameter (λ) on $\theta(\xi)$. An enhancement in λ corresponds to improve the thermal layer and temperature. Fig. 8 depicts the plots for temperature via melting parameter (M). Temperature improves via (M). The plots for temperature via $\Pr$ is shown in Fig. 9. Temperature profile reduces via Prandtl number. In fact $(\Pr)$ and thermal diffusivity are inverse relation with each other. Fig. 10 depicts the temperature via Brownian motion parameter. Temperature boosts when the values of Nb are increased. Behavior of Nt for temperature distribution is noted similar to that of Nb (see Fig. 11). Fig. 12 represents the concentration via melting parameter (M). Concentration is higher in presence of melting. Fig. 13 addresses that higher values of Lewis number reduces $\phi(\xi)$. Lewis number directly relates to Brownian diffusion coefficient. Larger values of Lewis number yield lower Brownian diffusion coefficient and thus concentration decreases. Fig. 14 reveals that concentration declines via thermophoresis parameter.

Fig. 15 illustrates the plots for skin friction via shape parameter (n) and (Ha). The skin friction improves via (n) and (Ha). Fig. 16 shows the skin friction via Deborah number (K) and ratio parameter (λ_2). The skin friction increases via (K) and it decreases for (λ_2). Fig. 17 addresses the plots for Nusselt number via (λ) and $(\Pr)$. Magnitude of Nusselt number enhances via (λ) and $(\Pr)$. The plots for Nusselt number through (Nt) and (Nb) are shown in Fig. 18. Same trend is noted for (Nt) and (Nb) here. The plots for Sherwood number against (Nt) and (Nb) are addressed in Fig. 19. Here we can see that Sherwood number (Sh) reduces for (Nt) and it increases through (Nb) . Fig. 20 shows the magnitude of mass transfer against $(\Pr)$ and (Le). Magnitude of mass transfer is improved via $(\Pr)$ and (Le).

Table (1) specifies the individual average squared residual error. The error decreases with higher order of approximations increases.

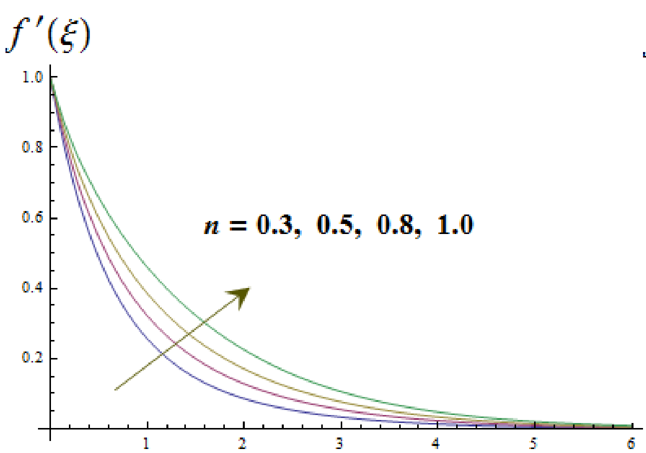

Fig. 1: Influence of n on f'.

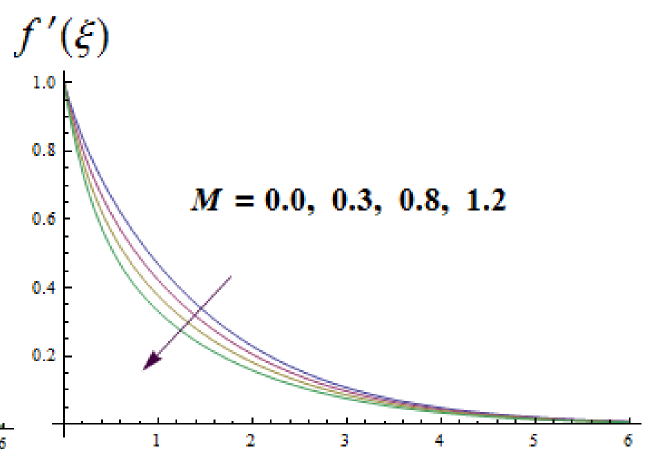

Fig. 2: Influence of M n f'.

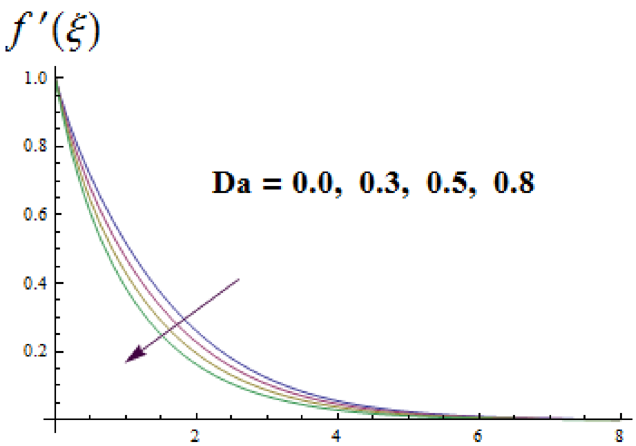

Fig. 3: Influence of Da on f'.

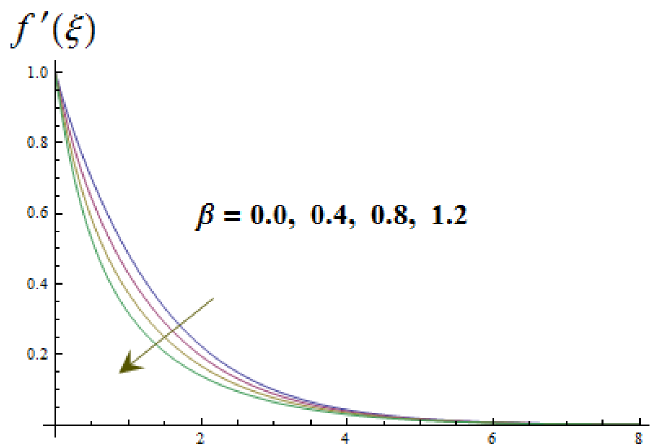

Fig. 4: Influence β **on** f'.

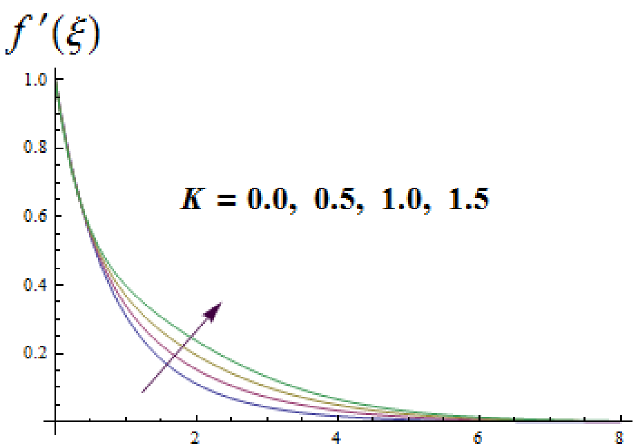

Fig. 5: Influence of K on f'.

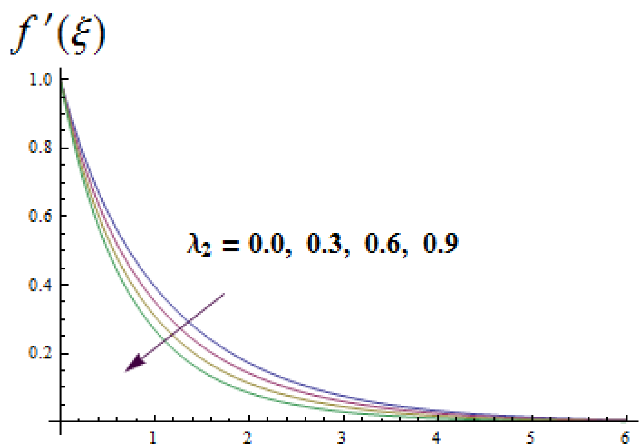

Fig. 6: Influence of λ_2 **on** f'.

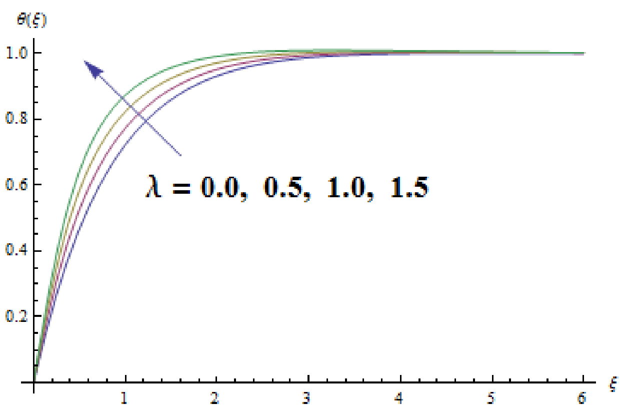

Fig. 7: Influence of λ **on** θ.

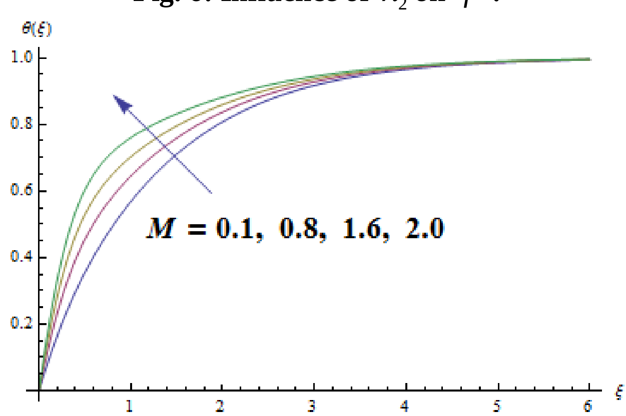

Fig. 8: Influence of M on θ.

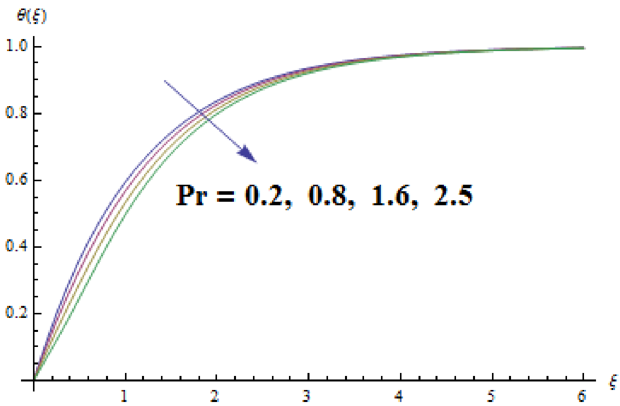

Fig. 9: Influence of Pr on θ.

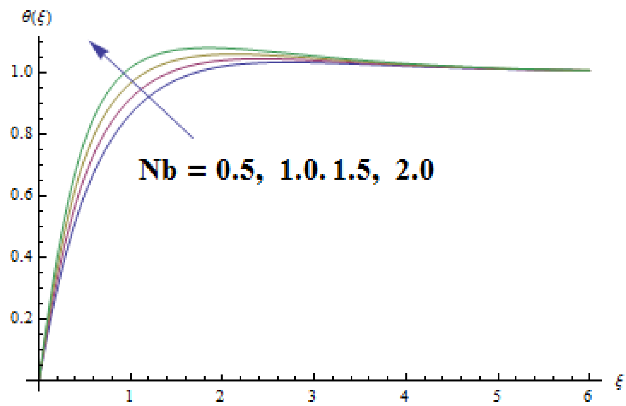

Fig. 10: Influence of Nb on θ.

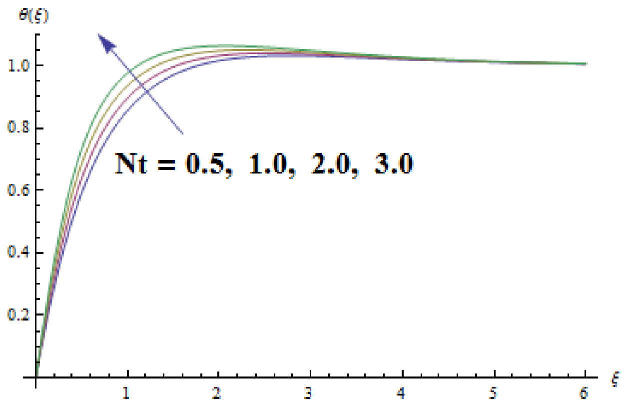

Fig. 11: Influence of Nt on θ.

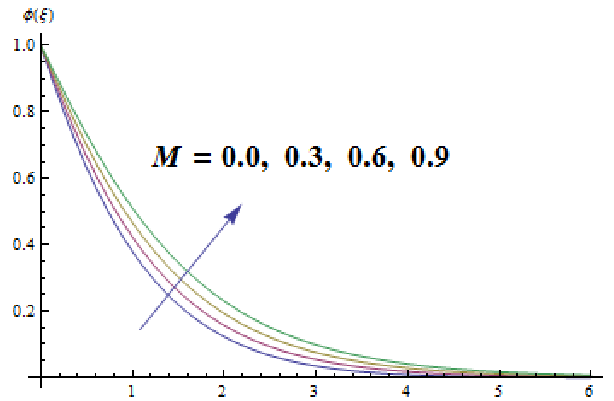

Fig. 12: Influence of M ϕ.

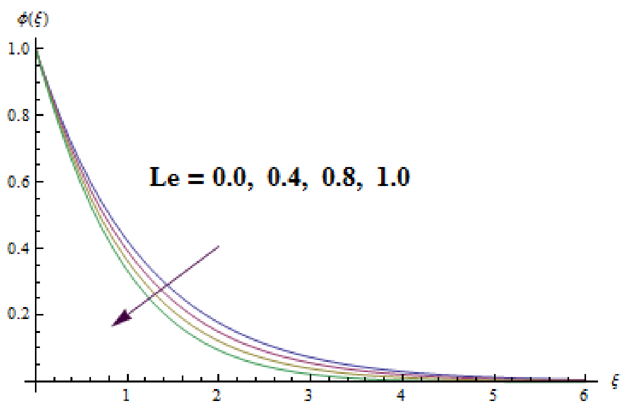

Fig. 13: Influence of Le on ϕ.

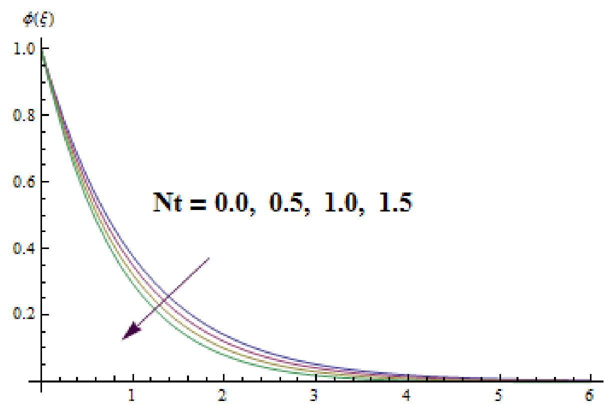

Fig. 14: Influence of Nt on ϕ.

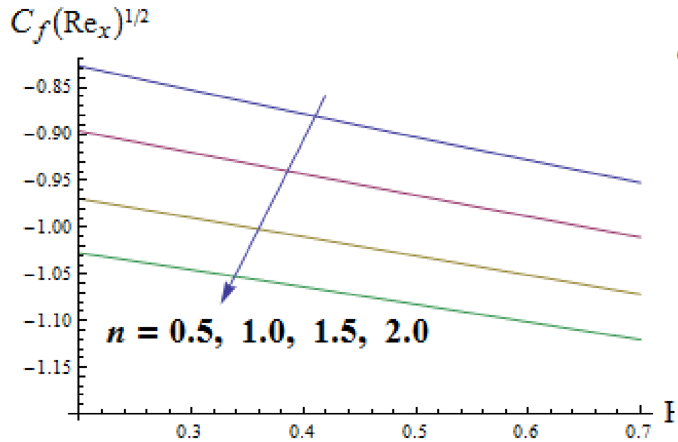

Fig. 15: Plots for C_f via n and Ha.

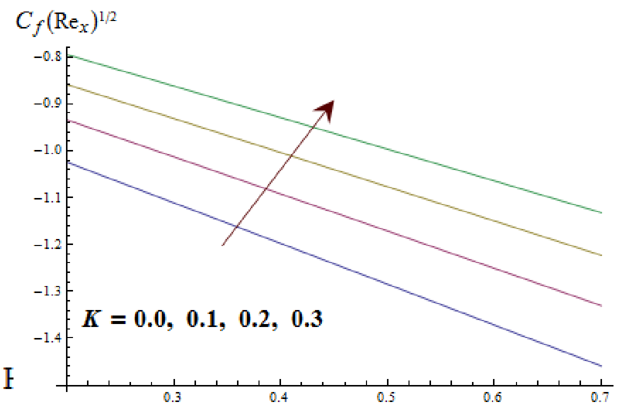

Fig. 16: Plots for C_f via K and λ_2.

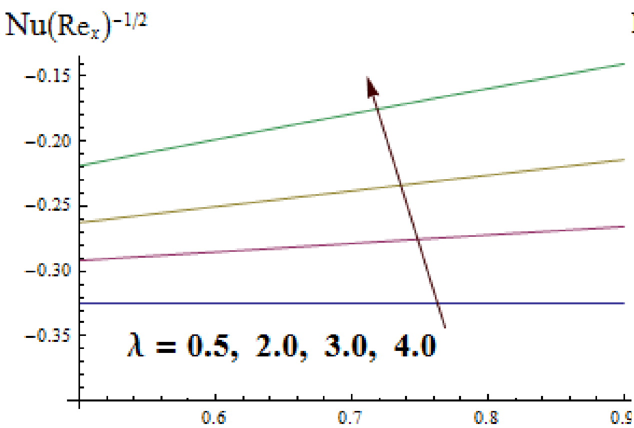

Fig. 17: Plots for Nu via Pr and λ.

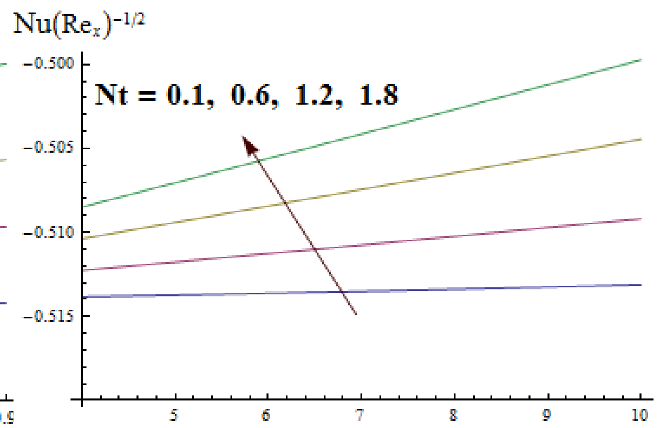

Fig. 18: Plots for Nu via Nt and Nb.

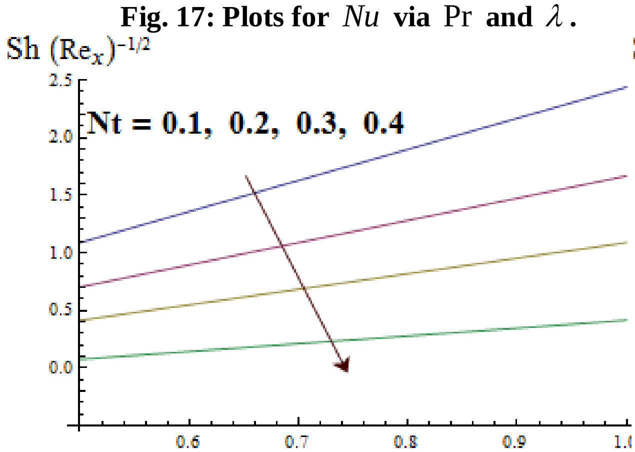

Fig. 19: Plots for Sh via Nb and Nt.

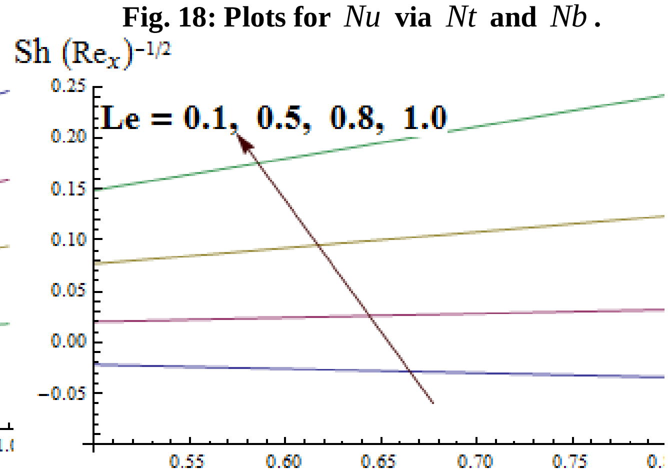

Fig. 20: Plots for Sh via Pr and Le.

Table; 1 Specific averaged squared residual errors in view of optimal values of auxiliary parameters.

k	ε_k^f	ε_k^θ	ε_k^ϕ
2	8.4516×10^{-5}	6.81238×10^{-4}	4.4501×10^{-7}
6	3.2315×10^{-9}	4.1032×10^{-6}	3.39785×10^{-7}
10	1.9392×10^{-11}	1.1216×10^{-9}	2.03086×10^{-9}
16	3.71564×10^{-15}	3.51569×10^{-10}	3.80485×10^{-13}
22	5.12567×10^{-19}	7.42344×10^{-12}	4.58971×10^{-16}
26	6.23623×10^{-26}	9.94954×10^{-16}	5.97298×10^{-19}

Closing remarks

Melting heat and heat generation/absorption in flow of Jeffrey nanofluid are discussed. Main points of present study include the following.

- Velocity enhances via K and n. Trend of velocity for β, M, Da and λ_2 is opposite.
- Temperature improves via M, λ, Nb, Nt while reverse trend is observed for $\Pr$.
- Concentration declines through M and it improves for Nb and Nt.
- Skin-friction coefficient reduces through λ_2 and it increases for K and Ha.
- Heat transfer enhances through $\Pr$, Nb and Nt.
- Mass transfer reduces through Nt and it increases for Nb.

Numenclature

u, v – velocity components, [$\mathrm{LT^{-1}}$]	α^* – thermal diffusivity, [$\mathrm{L^2T^{-1}}$]
x, y – Cartesian coordinates, [L]	α – thickness parameter, [-]
T_m – melting temperature, [K]	ε –.porosity, [-]
D_B – coefficient of Brownian diffusion, [$\mathrm{ML^{-1}T^{-1}}$]	η – dimensionless variable, [-]
C_∞ – ambient concentration, [-]	λ_1 – retardation temperature, [-]
k – permeability of porous medium, [-]	Q – heat generation/absorption, [-]
Nt, Nb – Thermopherasis and Brownian motion parameters, [-]	μ – fluid dynamical viscosity, [$\mathrm{ML^{-1}T^{-1}}$]
λ_2 – ratio parameter, [-]	υ – kinematic viscosity, [$\mathrm{L^2T^{-1}}$]
U_w – stretching velocity, [$\mathrm{LT^{-1}}$]	ϕ – dimensionless concentration, [-]
T – fluid temperature, [-]	ρ_f – fluid density, [$\mathrm{ML^{-3}}$]
C – concentration of the nanomaterial, [-]	n – shape parameter, [-]
D_T – coefficient of themophoretic diffusion, [$\mathrm{ML^{-1}T^{-1}K^{-1}}$]	θ – dimensionless temperature, [-]

$\Pr$ – Prandtl number, [-]	Ha – Hartman number, [-]
K – Deborah number, [-]	M – melting parameter, [-]
Re_x – local Reynolds number, [-]	Le – lewis number, [-]
T – temperature of the fluid, [K]	Nu_x – local nusselt number, [-]
c_s, c_p – heat capacity and specific heat, [-]	C_f –.local skin friction, [-]
τ – ratio of heat capacities, []	q_m – mass transfer, [-]
f' – dimensionless velocity	q_w – surface heat temperature

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