

EXPERIMENTAL STUDY ON GAS-LIQUID-COAL FINES THREE-PHASE FLOW IN UNDULATING PIPELINE

by

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The aggregation of coal fines particles at the bottom of the coal-bed methane well is a common occurrence during production, which could inhibit the flow in the bottom of wells and have adverse effect on the downhole equipment. In this work, gas-liquid-solid three-phase flow experiments were carried out to investigate the migration and discharge of coal fines particles in undulating pipeline. The experiments were conducted in downbent V-shaped pipes with different inclination angles. Based on the conductivity method, the real-time liquid holdup at three positions of the elbow was measured by the developed software. The slugs were identified on the time series curves of liquid holdup, and the characteristics of each slug were calculated, such as length and translational velocity. Meanwhile, the moving of particles with different size and concentration can be observed through visualized flow channel. The dyed coal fines particles are injected into the multiphase flow loop. By observing whether they can be discharged from the V-shaped pipe, the lower limits of superficial gas and liquid velocities to avoid particle retention at the elbow were determined. A correlation to predict the critical gas and liquid velocity was presented, and the accuracy of the calculation model was verified in comparison with the experimental results.

Key words: slug flow, coal fines, liquid holdup, critical velocity, undulating pipeline

Introduction

In the process of production, there would be coal fines particles in coal-bed methane (CBM) wells with the influence of rock stress and fluid erosion. The performance of downhole equipment would be affected, while the concentration of coal fines reaches a certain threshold. The low velocity of fluid would cause the aggregation of coal fines in the wellbore, which could reduce the cross-sectional area of flow and increase pressure drop and fluctuation.

Horizontal wells have been used to exploit CBM reservoirs in the last twenty years [1, 2]. The actual tracks of the wellbores are not completely horizontal, and the elbow section of undulating pipeline provides favorable conditions for particles aggregation [3, 4].

A great deal of researches has been done about slug flow in undulating pipeline. The experimental results of Al-Safran *et al.* [5] show that there exist five possible flow behavior categories along a hilly-terrain section. Wave growth and wave coalescence were found as two

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main mechanisms by the statistical analyses of slug parameters. Subsequently, Al-Safran *et al.* [6] adopted critical liquid level as the slug initiation criterion at the lower elbow, based on which a predictive model for initiated slug length distribution at the lower elbow of a hilly-terrain pipeline is developed by probabilistic modeling. Al-Safran *et al.* [7] developed a simple transient model to predict the development of slug velocity in time and space beyond the initiation point based on experimental investigation of the unsteady velocity behavior of newly initiated slugs at a pipeline dip.

All the studies of slug are based on the research findings of two-phase flow and three-phase flow. There are plenty of studies on two-phase flow and three-phase flow. Mandal *et al.* [8] adopted the conductivity probe technique to perform experiments for phase velocities ranging from 0.28 m/s to 1.12 m/s for air and from 0.01 m/s to 1.12 m/s for water to investigate the flow characteristics of gas-liquid two-phase flow through an undulated pipeline consisting of interconnected horizontal, upward and downward inclined sections.

Ersoy *et al.* [9] performed experiment for gas-water-oil three-phase slug flow in undulating pipelines and analyzed the slug flow with respect to average liquid holdup, phase distributions and slug characteristics which include translational velocity, slug length and frequency along the system.

Bassani *et al.* [10] proposed a three-phase solid-liquid-gas slug flow mechanistic model coupling hydrate dispersion formation with heat and mass transfer. The model couples mass, momentum and energy balances for the slug flow unit cell and provides analytic expressions for temperature and pressure distributions along the pipeline.

Stevenson *et al.* [11] and Najmi *et al.* [12] studied the transition conditions of the low concentration sand particles from the stationary state to the moving state in stratified and intermittent flow, respectively. In their experiments, the velocity of sands had been measured for various gas and liquid flow rates to produce an equation that predicts the point of incipient-sand deposition. Moreover, the effect of liquid- and gas-flow rate and physical parameters such as sand concentration, sand size and liquid viscosity were also experimentally investigated.

Yan *et al.* [13], Osho *et al.* [14], and Osho [15] conducted gas-water-sand three-phase flow experiments in undulating pipeline of large inclination angle (24°). They studied sand transport characteristics and minimum transport conditions in water and air-water at different sand concentration using both visual observation and statistical parameters. For air-water experiments, the sand behavior observed in the uphill and downhill pipe was different due to the different air-water flow regime. The slug flow and aerated slug flow were found to be most efficient flow regime for sand transport in uphill pipe.

The studies on the solid particle movement in multiphase flow mostly use sand, gravel, or other inorganic particles. Coal fines particles, a combination of organic and inorganic matter, has different characteristics with sand. Han [16] carried out three-phase flow experiments (air, water, and coal fines) in an undulating pipeline. The different flow patterns for coal fines migration in water flow were fully described, and the relations between coal fines critical moving velocity and discharge velocity and the size of coal fines are investigated.

In this work, we performed a series of gas-liquid-solid three-phase flow experiments in visualized V-shaped pipes. The flow regularity of coal fines particles in undulating pipeline, such as aggregation and discharge, has been analyzed based on the slug flow. The calculation model which can be used to predict the critical velocity of gas and liquid that can make the coal fines particles carried away under different conditions, such as inclination angle, particle size and concentration, has been proposed.

Experimental design

Particle characteristics of coal fines

According to the experience of sampling the particles during the stage of CBM production, it is not difficult to find that the volume concentration of coal fines in the liquid is generally less than 5%. The samples of coal fines used in present experiments were collected from the Qinshui Basin, the main CBM reservoir in China. In this experiment, the coal fines particles were sieved 16-20 mesh, 20-40 mesh, and 40-60 mesh, respectively, corresponding to the average diameters of 0.99 mm, 0.58 mm, and 0.30 mm. The average density of those particles is 1340 kg/m^3 , fig. 1.

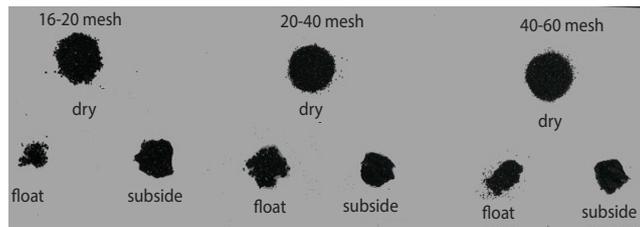


Figure 1. Coal fines samples in different states

Experimental set-up

Gas-liquid-solid three-phase flow loop

Air and water were used as flow media in experiments, fig. 2. The function of Loop 1 is to help the mixer quickly form uniform coal-water slurry in the tank, and the slurry is driven by a screw pump which has an adjustable flow rate range from 0.1 to $1.0 \text{ m}^3/\text{h}$. Via the sampling point, we can know whether the concentration of the slurry in Loop 1 is stable. If the concentration of coal particles in the fluid do not change after repeated sampling, the slurry in Loop 1 and pure water will be mixed and injected into Loop 2.

In Loop 2, gas, liquid and solid particles are injected into the experimental test section together and circulate in it. The visualized test segment of the pipeline is a PMMA pipe with internal diameter $D = 50 \text{ mm}$, and the whole line consists of three sec-

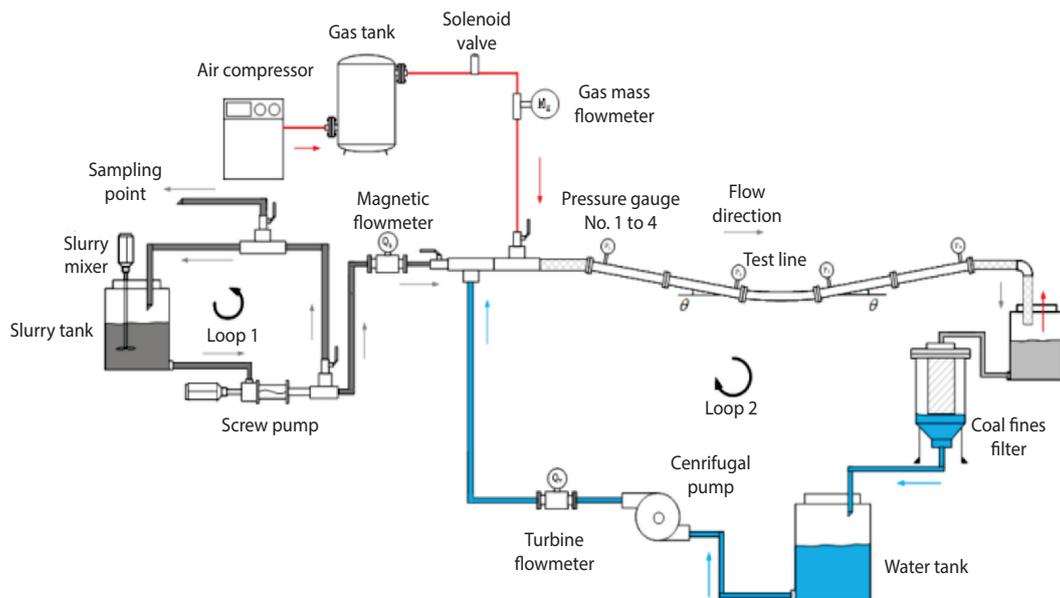


Figure 2. Schematic diagram of gas-liquid-solid three-phase flow experimental apparatus

tions: a downward straight section, a V-shaped section, and an upward straight section. The V-shaped pipe is the main channel of fluid, through which the movement, aggregation and discharge of coal fines particles can be observed directly. Also the flow parameters can be measured through the measurement equipment installed on it, such as conductivity sensors and pressure gauges. The inclination angle of the two straight sections can be changed by replacing the V-shaped elbow. Both the downhill section and uphill section length is 2 m ($40 D$), and the length of the elbow is 0.75 m ($15 D$). We used a solenoid valve to control gas-flow rate within the range of 15-1000 L per minute. Pure water is driven by an electric centrifugal pump, and the pump can supply water 0.25-1.5 m³/h by adjusting input current frequency.

The fluid-flow through the test section and discharge into knockout drum for gas-liquid separation. After coal fines particles are filtered from the waste slurry, the fresh water will flow into the pure water tank, and circulate again by the transmission of centrifugal pump. The coal fines particles will be washed at the end of the cycle in the filter tank, which will be used to circulate.

Flow parameters measurement system

There are several flowmeters installed on each pipeline before fluid entering Loop 2, including a magnetic flowmeter for the slurry flow with a measurement range of 0-1.0 m³/h, a gas mass flowmeter with a measurement range of 15-1000 L per minute, and a turbine flowmeter for the water flow with a measurement range of 0.5-1.5 m³/h. Four pressure gauges with a measurement range of 0-500 kPa are installed on the undulating pipeline (test line), and they are placed at the entrance of the test line, the entrance of V-shaped elbow, the outlet of the V-shaped elbow and the outlet of the test line.

A conductivity method for the liquid holdup measurement in a gas-liquid flow was proposed by Fossa [17]. Compared with other methods, the conductivity method is relatively, simple, and can monitor the gas-liquid flow in real time without disturbing the original flow. Three conductivity measurement units (CMU1-CMU3) are installed the upstream of elbow section (CMU1), the outlet of elbow section (CMU2), and the outlet of uphill section (CMU3), respectively, fig. 3. The

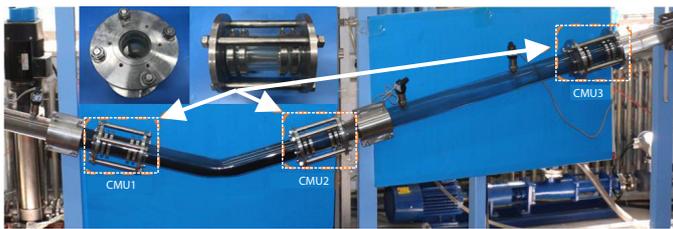


Figure 3. Conductivity measurement unit

relatively homogeneous flow parameters at the decline section are obtained by CMU1. The CMU2 can measure the initial parameters of the slugs formed at the bottom of the elbow. By comparing the results of CMU2 and CMU3, the change of slug parameters during the flow along the upward pipe can be analyzed.

A conductivity measurement unit consists of a pair of conductivity sensors. There are two conductivity sensors in a measurement unit, and a conductivity sensor consists of two stainless steel rings 10 mm in width. The conductive rings are embedded in the pipe with the same internal diameter as the test line, without disturbing the flow.

In the measurement circuit, as shown in fig. 4(a), a 24 V DC power source is used, and each of the six branches connects a conductivity sensor (CS1-CS6) with a 100 Ω resistor. The resistance between the paired conductive rings increases as the liquid level decreases. Thus, according to the basic principles of electricity, the lower voltage amplitude across the series

resistor illustrates the lower value of the liquid at the corresponding conductivity sensor. The real-time voltage signals of the six resistors are converted into a digital signal and transferred to the PC by the voltage data acquisition card with an acquisition frequency of 1 kHz. Using the collected voltage signals, the local value of the liquid holdup at each conductivity sensors is obtained by a PC program. The relationship between the liquid holdup and voltage amplitude should be obtained in advance. In this work, the static calibration method for gas-liquid flow is used. Under the experimental conditions, the coal fines particles are dispersed in the liquid phase because of the turbulence produced by unsteady flow of the slug, and the experiments are conducted with a low particle volume concentration. Therefore, the concentration of particles in gas-liquid flow has a negligible influence on the liquid phase resistance between the conductance rings.

All the slug units are assumed to be composed by liquid slug section and liquid film section, and each section can be identified on the holdup curve by assigning a critical holdup value. The translational velocity of the slug is measured using the phase difference on the holdup curve of two conductivity sensors in one measuring unit, which is the time required for a slug to move the distance between two conductivity sensors. The length of each section in a slug unit can be calculated using the translational velocity and the consumed time for the entire section to pass through the measurement unit.

A software which can be used to monitor the flow parameters in real-time is developed to make the data analysis easier. Figure 4(b) shows the main page of this software where the entire experimental system can be controlled and all the flow parameters, such as pressure and liquid holdup, can be obtained.

Experimental program

The first step of the experiments is pouring the classified coal fines particles into the tank for coal-water slurry. Then the mixer and screw pump are turned on to start fluid movement through Loop 1. The fluid in Loop 1 is sampled every minute, and the solid-liquid separation was performed by filtration, thereby the particle volume concentration of the samples could be measured. When

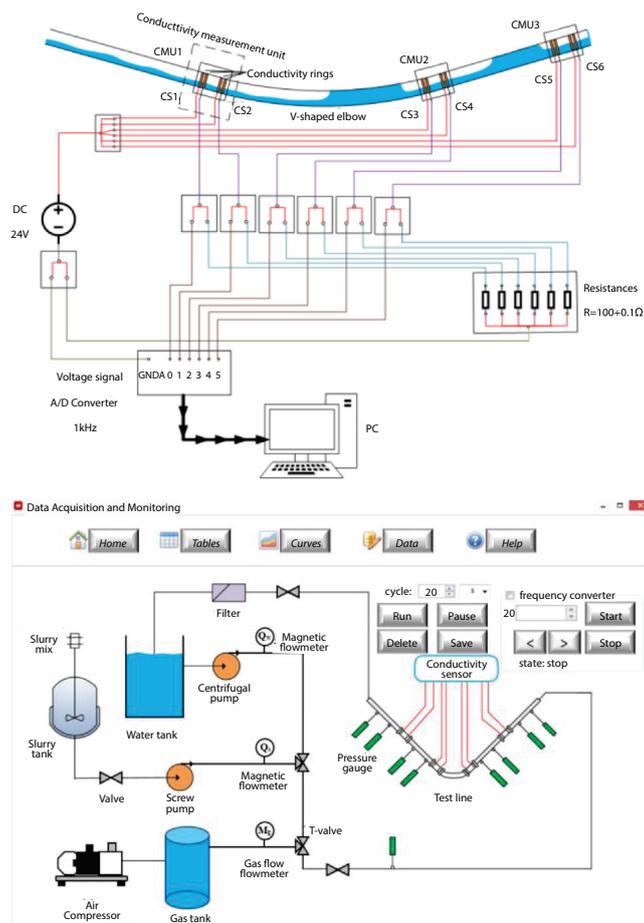


Figure 4. Schematic diagram of conductivity measurement circuit and sync software

the concentration of the samples is stabilized, Loop 1 is closed so that the screw pump would supply slurry for Loop 2 at a specified rate. By adjusting the frequency of the centrifugal pump and opening the solenoid valve, Loop 2 maintained constant gas and liquid flow velocities. The volume flow rate of the liquid-solid mixture in Loop 2 is $Q_1 = Q_w + Q_s$, and the concentration of coal fines in a liquid phase is calculated from the expression $C_p = C_v Q_s / (Q_w + Q_s)$, where Q_w is the pure water flow rate, Q_s – the coal-water slurry flow rate, and C_v – the concentration of solid particles in slurry. In each experiment, the gas liquid-flow rate, coal fines concentration and mean particle diameter are kept at a set value. The unsteady flow and migration of coal particles in the test line can be observed. The range of each operation parameter used in the experiment is given in tab.1.

Table 1. Experimental operating conditions

Variable	Range	Unit
Temperature	25	[°C]
Pipe inner diameter	50	[mm]
Particle volume concentration	0.001, 0.005, 0.01, 0.02	
Particle diameter	0.99, 0.58, 0.30	[mm]
Particle density	1340	[kgm ⁻³]
Superficial gas velocity	0.15-2.5	[ms ⁻¹]
Superficial liquid velocity	0.04-0.50	[ms ⁻¹]
Inclination angle	±5, ±15, ±25	[°]
Slurry injection rate	0.3-0.8	[m ³ h ⁻¹]

Real-time liquid holdup data are collected along with experiment. At the upstream of the test line, a small dose mixture of water and dyed coal fines particles is injected into the test line by a plunger injector. The injection rate and concentration are small enough to ensure that negligible influence on the original flow. The dyed materials on the surface of particles also has little effect on the particle properties, such as density or diameter. The dyed particles helped to determine whether the particles can be discharged from the test line within a certain period of time, which is determined as the time needed for 200 slugs passing through the outlet of the test line at current velocity. If there are dyed particles remaining at the elbow, it means that the present flow conditions cannot meet the requirements of particles discharge. Therefore, the experiments should be repeated at different flow rates. If the dyed particles are discharged in all experiments, it means the current flow rate has already reached the coal fines discharge requirement. Figure 5 shows the slug flow and the flow behavior of coal fines particles.

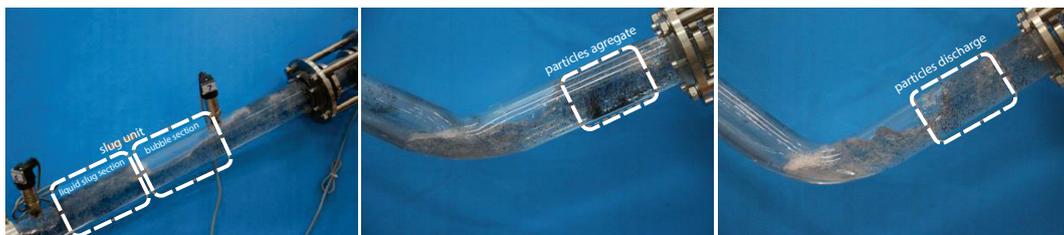


Figure 5. Flow behavior in the test line

Results and discussion

Based on the experimental study, the prediction models of critical flow velocity for particles discharge have been built on the basis of the correlation between gas-liquid velocity and the particle moving state. In undulating pipeline, the existence of slugs has a decisive influence on the motion state of solid particles. Through the physical experiments, the gas and liquid velocity conditions for coal fines particles to be completely discharged from the test line is obtained. Meanwhile, the influence of slug parameters on particle migration can be analyzed and the relation between flow velocity and particle transport state can be established.

Flow analysis

According to the previously mentioned experiments, the real-time curves of the liquid holdup at the place of each measurement unit are obtained. There are three measurement units installed in present experiments could obtain six curves of liquid holdup at the same time. Figure 6 shows the liquid holdup data collected from the conductivity sensors (CS) of each measurement unit in a period of time at the velocity condition $u_{sg} = 0.87$ m/s, $u_{sl} = 0.052$ m/s and inclination angle $\theta = 25^\circ$, where u_{sg} is the superficial gas velocity and the u_{sl} is the superficial gas velocity.

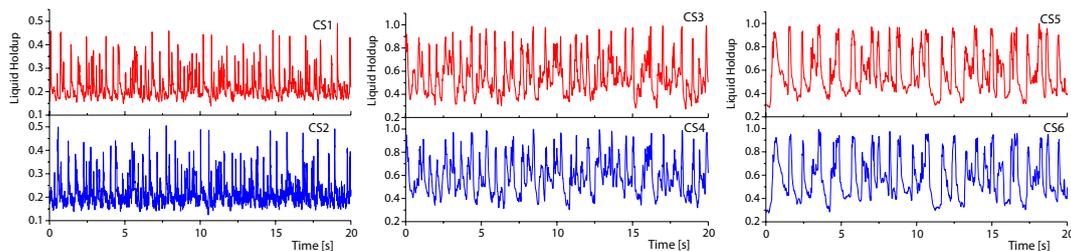


Figure 6. Real-time curves of liquid holdup ($u_{sg} = 0.87$ m/s, $u_{sl} = 0.052$ m/s, $\theta = 25^\circ$)

Due to the force of gravity, the gas-liquid flow pattern in the whole downward section of pipeline is stratified flow, which in accordance with Taitel and Dukler's model [18]. As CS1 and CS2 shown in fig. 6, most of the liquid holdup data on the curves fluctuate around 0.2, and the low level of liquid layer could be observed in downward pipeline at the experimental velocity condition. The transition from stratified flow to slug flow and the generation of slugs occurred at the bottom of the elbow. The vibration of the newly generated slugs under the CMU1 leads to the fluctuation of gas-liquid interface, and makes the instantaneous liquid holdup of CS1 and CS2 up to 0.4.

Compared with the liquid holdup curves of CMU2 and CMU3 in fig. 6, it is obvious to find the reduction of slug frequency during the upward flow. This is because the slugs generated at the bottom of the elbow is unstable, and the number of slugs reduced due to the coalescence and dissipation of temporary slugs during the uphill flow. The flow velocity of fluid and the inclination angle of pipeline will influence the formation and development of the slug in the upward section. Experimental observation shows that the number of slugs generated at the elbow changes with the gas-liquid velocity. At a condition of high flow velocity ($u_{sg} = 1.72$ m/s, $u_{sl} = 0.10$ m/s), as many as 330 slugs are identified at CMU2 during the acquisition time of 153 seconds. When the flow velocity is low ($u_{sg} = 0.87$ m/s, $u_{sl} = 0.10$ m/s), only 209 slugs are identified in 151 seconds at CMU2. The high velocity condition leads to flow fluctuation more easily, and more slugs form at the bottom of the elbow because of stochastic fluctuation.

According to the experimental observation, the temporary short slugs coalesce and become more stable in the upward movement. The slugs become more homogeneous in characteristics and closer to the fully developed slug. No matter what inclination angles are used in the experiment, the number of slugs at CMU3 is less than that at CMU2.

At the same flow rate, there are more slugs when the inclination angle is 25° than 15° . The larger inclination angle of the pipeline comes with larger change in the flow direction at the bottom of the elbow. As a result, the gas-liquid flow in the pipe of larger inclination angle is more unstable, and the small bubbles form easier. Meanwhile, the difference in the number of slugs between CMU2 and CMU3 is 99 ($\theta = 15^\circ$), which is less than 135 ($\theta = 25^\circ$). The experimental results illustrate that in the process of upward flow, more slugs dissipate in the pipe with a larger inclination angle.

Critical velocity for discharging the coal fines particles

At the gas and liquid velocities condition in present experiments, the gas- and liquid-flow pattern in the decline section of the test line is stratified flow, and the slugs generated at the bottom of the elbow. The experimental observation showed the particles are driven upward by the liquid slug in the upstream position. The backflow water of the previous slug entered the liquid slug body, and the large velocity change produced a violent eddy at the front of the liquid slug, causing the bubble to merge with the liquid slug. The coal fines particles dispersed in the liquid phase because of the turbulent kinetic energy [19], and the transportation tracks of the particles are rotatory in the liquid slug. When the front of the bubble passed through the location of particles, the particles have the largest velocity moving forward. The liquid flow velocity in the liquid film section decreased rapidly to negative. The particles in the liquid film section moved backward until next slug passed through, and the coal fines regained upward velocities. The particles keep repeating the movement process of up and down due to the existence of slugs. The final result is that the particles may not be discharged from the elbow. Enough length and rising velocity of liquid slug are required to ensure that the distance of particles moving forward more than that of backflow in the liquid film section. The particle discharge condition of gas-liquid velocity is that the dyed coal fines can be discharged from the V-shaped pipe. For a fixed liquid velocity, the critical gas velocity is the lower limit of gas velocity that satisfied the particles discharge and vice versa. Figure 7 shows the critical gas velocities vs. liquid velocities for different operational conditions.

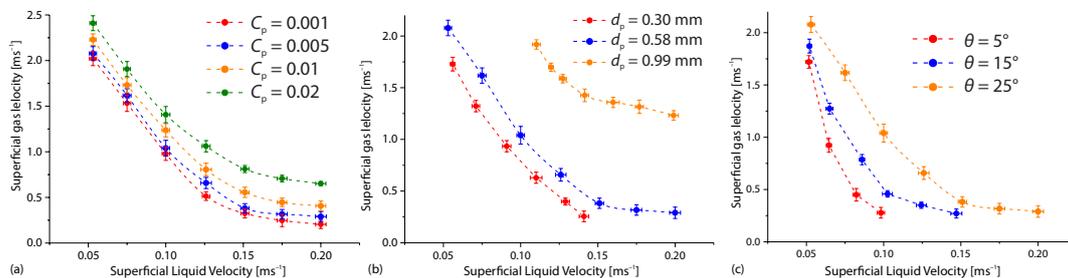


Figure 7. Critical discharge velocity at different experimental conditions; (a) critical discharge velocity at different particle concentrations ($d_p = 0.58$ mm, $\theta = 25^\circ$), (b) critical discharge velocity at different particle diameters ($C_p = 0.005$, $\theta = 25^\circ$), and (c) critical discharge velocity at different inclination angles ($d_p = 0.58$ mm, $C_p = 0.005$)

Figure 7(a) illustrates the critical discharge velocity increased with the increase of particle concentration. When the concentration is low, the interaction between particles is limited. The particles movement can be considered as the motion of individual particle. At a high concentration, the interaction between particles increases, and in the liquid slug section, more particles locate near the bottom of the pipe cross-section, where the local velocity of liquid phase is relatively lower. Many particles keep moving up and down, and have a smaller probability to be discharged from the test line. Therefore, the concentration at the lowest part of the pipeline would increase constantly. The influence of concentration becomes smaller at high velocity condition, most likely because the slugs translate quickly when the mixture velocity is high. The particles distribution on the pipe cross section in liquid slug is more uniform as a result of the more severe flow vibration.

Figures 7(b) and 7(c) show the critical discharge velocity of different particle sizes and different pipeline inclination angles, respectively. Compared to the particle concentration, the influence of particle diameter and inclination angle on the particle transportation are more obvious in the experimental concentration range. The critical discharge velocities increase for larger coal fines particles, or at a larger inclination angle condition, because the particles need to overcome greater gravity to maintain upward movement.

Prediction model of critical velocity

From the previously mentioned experiments, it has been known that the coal fines particles in slug flow rise with the liquid slug and return in the liquid film. Therefore, the differences of flow characteristic parameters between liquid slug section and liquid film section determine whether the particles can get the forward displacement during the repeated processes of moving up and down, and finally discharge from the elbow. In the studies of slug flow, the translational velocity of liquid slug is related to the mixture velocity of gas and liquid [20]. The differences of flow characteristics of the liquid slug and liquid film section are related to the difference of gas- and liquid-flow rate. In order to regress an empirical model of the critical velocity from the previous experimental data, a dimensionless analysis is carried out. The definition of dimensionless variables are:

$$U_D = \frac{u_m}{\sqrt{gd_p \left(\frac{\rho_p}{\rho_l} - 1 \right) \sin \theta}} \quad (1)$$

$$r = \frac{u_{sl}}{u_m} \quad (2)$$

$$s = \frac{d_p}{D} \quad (3)$$

$$\text{Re}_p = \frac{D \sqrt{gd_p \left(\frac{\rho_l}{\rho_g} - 1 \right)}}{v_l} \quad (4)$$

where the dimensionless critical velocity, U_D , represents the ability of particles to overcome their own gravity and move forward with the liquid slug, and the u_m is the critical mixture velocity. The dimensionless variables r and $(1 - r)$ reflect the different flow conditions of the liquid slug and the liquid film section. The ratio of characteristic length s is used to reflect the effect of the

pipeline wall on the movement of the particles. The particle Reynolds number takes into account the effect of the density difference of the gas and liquid on the slug flow. Turian *et al.* [21] presented an empirical correlation to calculate the critical velocity in the single liquid phase flow. Based on this model, considering the influence of multi-phase flow and inclination angle, the calculation model of critical discharge velocity of coal fines particles in the pipeline is given:

$$U_D = 1.833C_p^{0.060} r^{-0.526} (1-r)^{0.295} s^{-0.133} Re_p^{0.119} (\sin\theta)^{-0.232} \quad (5)$$

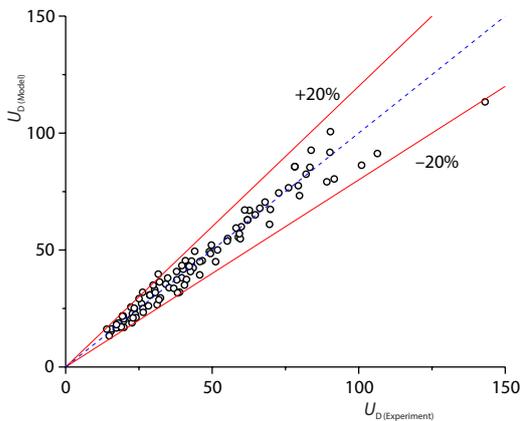


Figure 8. Difference between the experimental and calculated values of U_D

Figure 8 shows the results of the dimensionless critical velocity, U_D , calculated using eq. (5) vs. the value of U_D calculated using the data collected from the experiments. As shown in the graph, the mean of the relative error is 7.37%, and the maximum error is 25.47%. The calculation results of eq. (5) are approximately consistent with the experimental results, and there are only 11 of 113 data points with errors greater than 15%, of which three data errors are more than 20%. Equation (5) is the implicit equation of critical gas-liquid mixture velocity, u_m . The iteration method can solve the value of u_m , like Newton iteration, and the critical gas velocity can be calculated using the equation $u_{sg} = u_m - u_{sl}$.

Conclusions

In present study, the experiments of gas-liquid-coal fines three-phase flow in a visualized V-shaped pipeline were conducted. The flow behavior of coal fines particles in slug flow was investigated, and the critical velocity of gas and liquid to continuously discharge coal fines particles from the undulating pipeline were obtained. The conclusions are as follows.

- A visualized physical experiment device for simulating gas-liquid-solid three-phase flow in undulating wellbore is designed, which can be used to simulate three-phase flow with different types of fluids, inclination angles and flow patterns. The conductivity method is used to monitor the flow status of fluid in the strategic locations in real time and obtain corresponding curves of liquid holdup. The flow characteristics of fluid and the change law of liquid holdup of each measurement unit at different positions and time are analyzed. To make the analysis more convenient a software is developed.
- The multiphase slug flow in V-shaped pipe is studied, based on which the transport mechanism of solid particles in the slug flow is analyzed. The movement of particles in liquid slug section and liquid film section of a slug unit has different characteristics. In the liquid slug, coal fines particles disperse in the liquid phase, and move forward with the translation of the slug body. The particles in liquid film would move backward with the liquid flow until the next slug passed. The previously described process repeated all the time, and may lead to the accumulation of particles at the elbow.
- The critical discharge velocity of gas and liquid is measured in the different experimental conditions of inclination angle, particle size and concentration. Meanwhile, the correlation to predict the dimensionless critical discharge velocity is obtained, and there is a good agreement with the experimental results.

Acknowledgment

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Nomenclature

C – volume concentration
 D – internal diameter of pipeline, [mm]
 d – coal particle diameter, [mm]
 g – gravitational acceleration, [ms^{-2}]
 Q – volume flow rate, [m^3h^{-1}]
 Re – Reynolds number
 r, s – dimensionless variables in eqs. (2) and (3)
 U, u – velocity, [ms^{-1}]

Greek Letters

θ – Inclination angle, [$^\circ$]

ρ – density, [kgm^{-3}]
 ν – kinematic viscosity, [m^2s^{-1}]

Subscripts

D – dimensionless
 g, sg – gas, superficial liquid velocity
 l, sl – liquid, superficial liquid velocity
 m – critical mixture velocity
 p – particle
 w – pure water

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