A COMPARISON OF TWO STATISTICAL NARROW BAND MODELS FOR NON-GRAY GAS RADIATION IN PLANAR PLATES

by

Huaqiang CHU*, Fei REN, and Yan WEI

School of Energy and Environment, Anhui University of Technology, Ma'anshan, China

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Non-gray gas radiation analysis and comparison are conducted by combining a ray tracing method and two statistical narrow band (SNB) spectral models, namely the Goody SNB model and the Malkmus SNB model. In this paper, gas radiation in real gas containing H_2O, H_2O/N_2, or $H_2O/CO_2/N_2$ mixtures at 1 atm in planar plates was studied. Comparisons between these models are performed using the latest narrow-band database. The present computations are validated by reproducing the published results in the literature. The radiative source term, the wall fluxes, the narrow-band radiation intensities along a line-of-sight and the computing time are all compared. From the comparisons, it is found that the Malkmus SNB model is somewhat superior to the Goody SNB model and the former is preferred in engineering application.

Key words: *non-gray gas, SNB, Goody, Malkmus*

Introduction

Radiative transfer plays a key role in high temperature equipment or processes such as boilers, industrial furnaces, and combustors. Accurate analysis of those radiation problems is closely linked with the radiative properties of gases because of the existence of water vapor, CO_2, CO or a mixture of these gases in these problems. The radiative properties of these gases exhibit very strong spectral dependence in the near-infrared region at temperatures relevant to combustion. The grey gas assumption, however, is often made in fundamental and applied combustion and flame studies. In many practical applications, it has been well established that the commonly used grey gas model for systems containing combustion gas can not provide reliable predictions [1, 2]. Thus, we must consider the non-grey radiation properties of real gases.

The complex spectral dependence of the gas radiation makes the determination of spectral radiation properties very difficult. However, significant progress has been made in the development of the spectral models. Nowadays, there are many methods or models to describe the non-grey radiative properties of gases. These approximate methods may be loosely put into four groups as described by Modest [1]: line by line (LBL) model, narrow band models, wide band models, and global models.

The radiative heat transfer in absorbing-emitting gas mixtures can be most accurately predicted by the LBL approach, but this model requires large computer resources and computational time and relies on the high resolution transmission molecular database [3] and its extension HITEMP [4]. At present, LBL models are used only for benchmark solutions to validate the approximate methods. The SNB model, one of the narrow band models, can lead to

* Corresponding author, e-mail: hqchust@163.com

results that agree closely with LBL model with good accuracy. Although the last two models (spectral line wighted and full-spectrum k-distribution) still belong to global models, they can produce fairly accurate results provided the reference temperature that is carefully selected for the problem at hand [5-10].

In the last ten years, the SNB models have received renewed attention due to the rapid development in computers and interest for accurate analyses of radiation. There are two well known SNB models: the Goody SNB model and the Malkmus SNB model [11-19]. In order to analyze radiative heat transfer, many numerical methods have been developed to solve radiative transfer equation, examples being the discrete ordinate method and the Monte Carlo method (MCM). These methods have both advantages and disadvantages, so readers should choose suitable methods according to their problems. Detailed descriptions of these methods are available in the literature, *e. g*. Modest [1] and Siegal and Howell [20]. The ray-tracing method along with the SNB models was employed by Liu *et al*. [16] Chu *et al*. [21-24] and Marakis [25]. In the present work, it was used to solve the RTE.

This study concerns non-grey gas radiation in parallel plates using the Goody SNB model and the Malkmus SNB model. Although a similar work was conducted by Marakis [25] using out-of-date narrow-band databases for H_2O, the present study employed the latest available narrow-band database, Soufiani and Taine [18], and considered mixtures containing H_2O and CO_2. In addition, we also compared spectrally resolved radiation intensities from the two SNB models.

Non-gray gas radiation methods

Narrow band Formulation. The spectral RTE for an absorbing, emitting, but non-scattering medium can be written as given by Siegel and Howell [20]:

$$\frac{\partial I_v}{\partial s} = -\kappa_{av} I_v + \kappa_{av} I_{bv} \tag{1}$$

The corresponding boundary spectral radiation intensity at a diffuse wall is given:

$$I_v(s_w,\Omega) = \varepsilon_{wv} I_{bwv} + \frac{1-\varepsilon_{wv}}{\pi}\int_{\hat{n}\cdot\Omega'} |\hat{n}\Omega'| I_v(s_w,\Omega')\mathrm{d}\Omega' \text{ , for } |\hat{n}\Omega'| > 0 \tag{2}$$

After the spectral radiation intensity is calculated, the total (spectrally integrated) net radiative flux can be obtained from the relation:

$$q(x_i) = \sum_{\text{all}\,\Delta v}\left(\sum_{n=1}^{N} \mu \bar{I}_{v,n,i} w_n\right)\Delta v \tag{3}$$

The radiative source term or the divergence of heat flux, $-\mathrm{d}q/\mathrm{d}x$, of this 1-D problem in the medium is then given:

$$-\frac{\mathrm{d}q}{\mathrm{d}x} = -\frac{q_{i+1}-q_i}{x_{i+1}-x_i} \tag{4}$$

The *SNB models.* For an isothermal and homogeneous path-length, *L,* at total pressure, *p,* and molar fraction, *f*, the narrow band averaged transmittance is given by the Goody SNB model and the Malkmus SNB model, Ludwig *et al*. [26], respectively:

$$\bar{\tau}_{Gv}(L) = \exp - \frac{SL}{\sqrt{1+\dfrac{SL}{\pi B}}} \tag{5}$$

and

$$\overline{\tau}_{M\nu}\left(L\right)=\exp\left[-\frac{\pi B}{2}\left(\sqrt{1+\frac{4SL}{\pi B}}-1\right)\right] \tag{6}$$

where L is the optical length, $B = 2\overline{\beta}_\nu/\pi^2$, $S = k_\nu fp$, and $\overline{\beta}_\nu = 2\pi\overline{\gamma}_\nu/\overline{\delta}_\nu$. The mean narrow band parameters $\overline{\gamma}_\nu$, $\overline{\delta}_\nu$, and $\overline{k}_\nu$ for H_2O and CO_2 have been given by Ludwig *et al*. [26]. Soufiani and Taine [18], Soufiani *et al*. [27], but these databases are out-of-date and may yield inaccurate results. More recently, an updated data set of these parameters has been made available [18, 28] for CO_2, H_2O and CO for the much wider temperature range. Further details of this data set can be found in [18, 28].

For a non-isothermal and/or inhomogeneous path, the Curtis-Godson approximation [29] is commonly used to obtain equivalent band parameters. Equivalent band parameters $\overline{k}_\nu$ and $\overline{\beta}_\nu$ are given by averaging k and β over the optical path U of the column in [29]. Following Kim *et al*. [14], the overlapping band is treated as a new band.

Results and discussion

In order to compare the two SNB models, four different cases for the concentration and temperature distributions between two infinite planar plates are tested. Three problems for H_2O/N_2 mixture were first employed by Kim *et al*. [14] and the last one was calculated by Liu *et al*. [30] using the SNB-CK model for a $H_2O/CO_2/N_2$ mixture. In the four cases, the wall surfaces were assumed to be black and the medium was at a uniform total pressure of 1 atm.* The same spatial and angular discretization have been adopted for all the test cases. The planar geometry was subdivided into 20 sublayers and the polar angle into 20 intervals. The physical explanation for the four cases were given in Kim *et al*. [14], Liu *et al*. [16], and Liu *et al*. [30]. Reference results for the problems presented are those given in Kim *et al*. [14], Liu *et al*. [16], Chu *et al*. [21], and Liu *et al*. [30].

Assessment of the radiative source term

Isothermal homogeneous medium (Case 1). In the first case, the two walls were held at 0 K. The medium between the two planar plates is filled with 100% water vapor at a uniform temperature of 1000 K. Two wall separation distances, of 0.1 m and 1 m, were used.

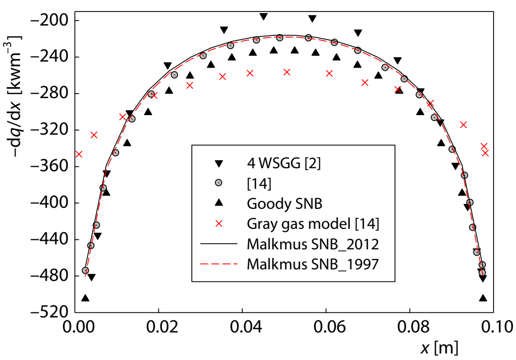

Figure 1. Distributions of the radiative source term for the homogeneous and isothermal case with the separation distance of 0.1 m (Case 1)

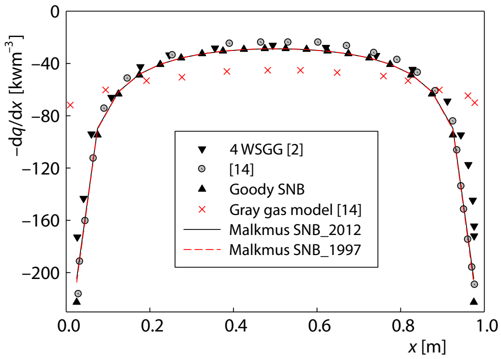

Figure 2. Distributions of the radiative source term for the homogeneous and isothermal case with the separation distance of 1 m (Case 1)

* 1 atm = 101325 kPa

Figures 1 and 2 show the predicted radiative source distributions using the Goody and the Malkmus SNB models for the two different separation distances. The discrepancies between the results obtained from the two SNB models are found to be large for the smaller separation distance of 0.1 m, while the predictions are in better agreement with each other for the larger separation distance of 1 m. In these figures, the weighted-sum-of-gray-gases model was that developed by Song [2] based on the exponential wide band model, Edwards and Balakrishnan [5], while the SNB model and the gray gas model were adopted by Kim *et al*. [14]. The WSGG model also gives reasonable results, whereas the results of the gray gas model are far off from the reference solution, Kim *et al*. [14]. Note that the results of the Malkmus SNB model are in excellent agreement with those of the reference solution for both optical lengths. The magnitude of the heat source becomes smaller as the optical thickness of the medium increases.

Isothermal inhomogeneous medium (Case 2). For this case, the gas is maintained at a uniform temperature of 1000 K, but the medium is a non-uniform mixture of H_2O/N_2 with a parabolic H_2O concentration profile given by $f_{H_2O} = 4(1 - x/L)x/L$ the separation distance is 1 m.

Figure 3 presents the results of the source term from different gas models. The Goody SNB model result does not show significant difference from that of the Malkmus SNB model in the middle of the medium or close to the walls. However, there are relatively large differences around the two minima. The W-shaped profile of the source term is well captured by the two SNB models. The predictions of the WSGG model and the gray gas model are much worse than those of the two SNB models, especially for the gray gas model which even fails to capture the W-shaped distribution.

Non-isothermal homogeneous medium (Case 3). The third problem analyzed has a boundary-layer type temperature profile. The medium is again pure water vapor, the left plate is at 1500 K and the right plate is at 300 K. The distance between the plates is 0.2 m. The calculated source term distributions are compared in fig. 4. From fig. 4, a similar observation to that in Case 1 can be made. Both SNB models predict the sharp rise of the radiative source term near the left wall with the results of the Malkmus model in better agreement with the reference solution. However, the WSGG model and the gray gas model cannot resolve the rapid change in the source term distribution.

Comparison of the net wall heat fluxes and computing time

The net heat fluxes at the boundary and computing time for all the cases are summarized in tab. 1. The discrepancy between the two SNB models is evident, although it is quite small, from the table.

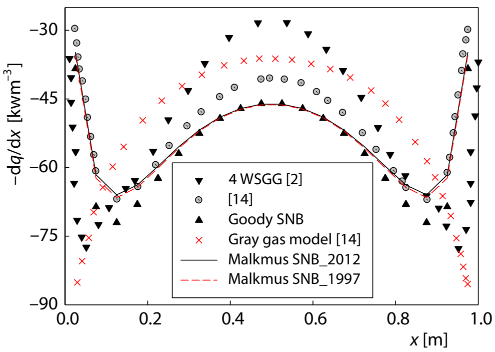

Figure 3. Distributions of the radiative source terms for the isothermal case with parabolic water vapor concentration profile (Case 2)

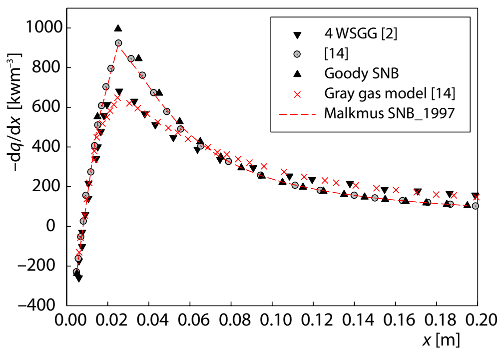

Figure 4. Distributions of the radiative source term for the boundary-layer type temperature case with pure water vapor (Case 3)

Table 1. Net wall heat fluxes and computing time for the four cases

Cases	Length, [m]	Kim *et al.* [14]		Malkmus SNB		Goody SNB	
		Heat flux, [kWm^{-2}]	Time, [s]	Heat flux, [kWm^{-2}]	Time, [s]	Heat flux, [kWm^{-2}]	Time, [s]
Case 1	0.1	–14.3	/	–14.2	3.14	–15.1	3.25
	1.0	–28.2	/	–30.3	3.20	–31.3	3.34
Case 2	1.0	–25.2	/	–27.0	3.20	–28.1	3.34
Case 3	0.2	277.4	/	271.7	3.16	271.2	3.25

Narrow band intensities along a line-of-sight

Figures 5-7 compare narrow band intensities along a line-of-sight (at $x = L$ and along the positive x-direction) for the four cases aforementioned. From these figures, we can see that there are not significant differences in the narrow band intensities obtained using the Goody SNB model and Malkmus SNB model, however, results of the former are in general slightly higher than those of the latter. This is consistent with the somewhat higher radiative source term and heat flux from the Goody SNB model shown earlier. Figure 5 shows that the narrow band intensities with a greater optical length vary more than those with a shorter length. It is found that variations of the narrow band intensities also become significant when the medium of pure water is replaced by a mixture of H_2O, CO_2 and N_2 as shown in fig. 7. The treatments for the

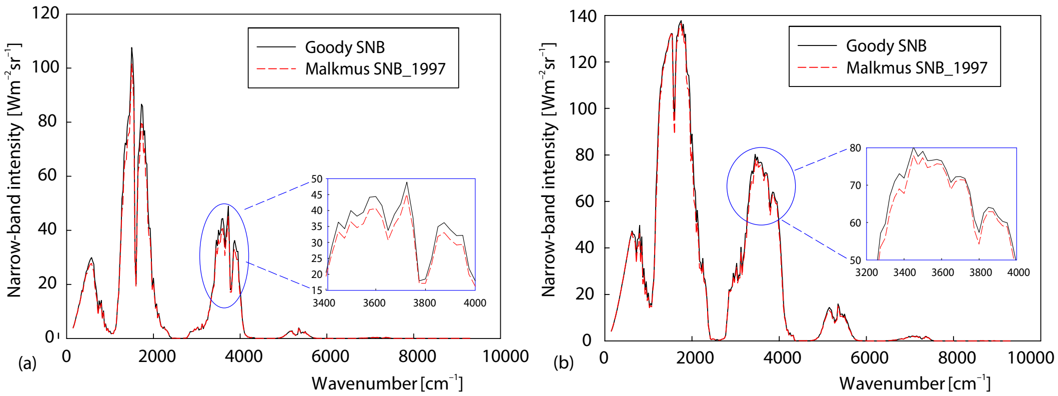

Figure 5. Comparisons of narrow-band intensities for Case 1 with two optical paths; (a) 0.1 m and (b) 1 m

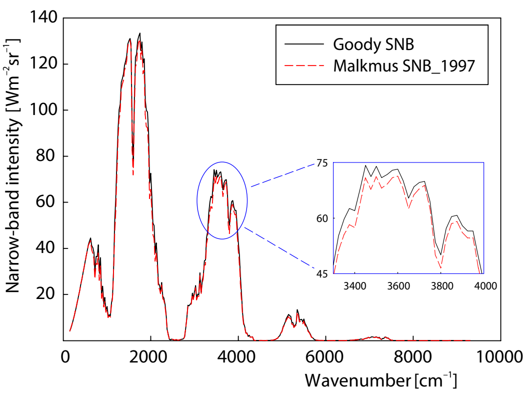

Figure 6. Comparisons of narrow-band intensities for Case 2

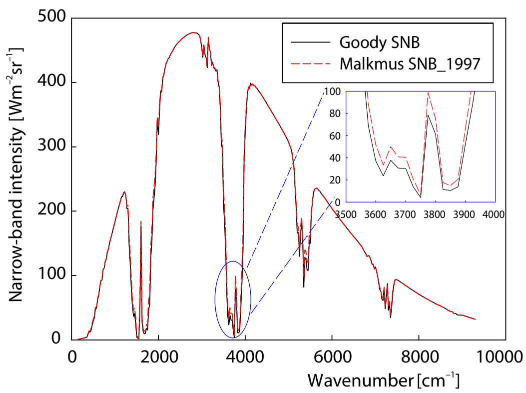

Figure 7. Comparisons of narrow-band intensities for Case 3

overlapping bands (such as 450-1200, 1950-2450, and 3300-3800) of H_2O and CO_2 have been discussed by Liu *et al*. [31]. In the present paper, we employed the same treatment as that of Kim *et al*. [14]. In future work, we will investigate the treatment of the overlapping bands to improve the accuracy of the model.

Conclusions

Four test problems have been investigated to compare the Goody SNB model and the Malkmus SNB model using the recently available narrow-band database in the literature. The radiative source term, the wall fluxes, the narrow band intensities along a line-of-sight and the computing time were investigated. In general the differences between the two SNB models are fairly small. The following observations were made from the results of the present study.

- For the isothermal homogeneous medium, Case 1, results of the Malkmus model exhibit relatively large differences from those of the Goody model when the optical length is small. However, the differences become smaller with the increase of the optical length.
- For the isothermal inhomogeneous medium, Case 2, results of the two SNB models depart from the literature solution, with those of the Malkmus model in better agreement with the benchmark solution.
- For the non-isothermal cases, Cases 3, the Malkmus SNB model is again more accurate than the Goody model, judged by the closer agreement between the results of the Malkmus model and solutions from the literature.
- For the CPU time required for the four problems, the Malkmus model is slightly more efficient than the Goody model.
- The differences in the narrow-band radiation intensities from both SNB models are fairly small for the four cases considered.

More recently, the other methods have been proposed [32, 33] for gray and non-gray media. By this method, the intensity with high directional resolution at any point can be obtained with high precision. This method will be extended for non-gray gas radiation in the near future.

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Nomenclature

f – species molar fraction
I – radiation intensity, [Wm^{-2}sr^{-1}]
I_ν – spectral radiation intensity, [Wm^{-2}sr^{-1}cm^{-1}]
k_ν – mean line-intensity to spacing ratio, [cm^{-1}atm^{-1}]
$\bar{k}_\nu$ – equivalent mean line-intensity to spacing ratio, [cm^{-1}atm^{-1}]
L – separation distance between parallel walls, [m]
p – pressure, [atm]
q – heat flux density, [kWm^{-2}]
s, s' – position variables, [m]
x – cartesian co-ordinates, [m]

Greek symbols

$\bar{\beta}_\nu$ – mean line-width to spacing ratio
$\bar{\gamma}_\nu$ – ean half-width of an absorption line, [cm^{-1}]
$\bar{\delta}_\nu$ – equivalent line spacing, [cm^{-1}]
μ – direction cosines
$\Delta\nu$ – wavenumber interval, [cm^{-1}]
ν – wavenumber, [cm^{-1}]
τ_ν – spectral transmittance

Subscripts

b – blackbody
i – spatial discretization (along a line of sight) index
n – angular discretization index
w – wall

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