ENTROPY GENERATION ANALYSIS OF MIXED CONVECTION WITH CONSIDERING MAGNETOHYDRODYNAMIC EFFECTS IN AN OPEN C-SHAPED CAVITY

by

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This paper studies the effect of a constant magnetic field on the mixed convection heat transfer and the entropy generation of CuO-water nanofluid in an open C-shaped cavity with a numerical method. The governing equations are presented by control volume method and they are solved simultaneously by the SIMPLE algorithm. This study examines the effect of the Hartman number, aspect ratio, Reynolds number, and Richardson number parameters for different solid volume fraction of nanoparticles. Also Nusselt number, entropy generation, thermal performance criteria and coefficient of performance is studied in this research. The calculated parameters are the Hartman number, aspect ratio, Reynolds number, Richardson number, nanofluid solid volume fraction, Nusselt number, and coefficient of performance. The results show that increasing the Hartmann number reduces the entropy generation. However, the thermal performance increases. Increasing the aspect ratio raises heat transfer and thermal performance. The effects of nanofluid solid volume fraction on mixed convection heat transfer and entropy generation are also investigated and discussed.

Key words: nanofluid, MHD mixed convection, aspect ratio, entropy generation, thermal performance

Introduction

Mixed convection heat transfer has been widely investigated in the literature due to its application in a wide range of engineering problems such as, electronic cooling machines, chemical processing equipment and solar energy collection. Heat transfer enhancement is a very important challenge in many industries [1, 2]. Dispersion of nanoscale particles in pure fluids, called nanofluids, can be one of new techniques for heat transfer enhancement. It allows performing heating or cooling with better efficiency while reducing energy consumption [3, 4]. Some studies focused on the heat transfer of nanofluid in the various shapes of cavities [5]. Mahmoudi et al. [5] studied the mixed convection heat transfer of nanofluid. Showed that at high Reynolds number and Richardson number adding the nanoparticles to pure fluid leads to improving heat transfer performance. Numerical studies of mixed convection heat transfer of nanofluid in a ventilated square cavity were investigated by [6, 7]. Shahi et al. [6] and Soortij et al. [7] showed that the average Nusselt number is a function of increasing the Reynolds number.

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Richardson number, and solid volume fraction, $\phi$, of the nanoparticles. Mansour et al. [8] investigated natural convection inside a C-shaped nanofluid-filled cavity with localized heat sources. The results showed that the Nusselt number increased with an increase in Rayleigh number, and $\phi$ of Cu nanoparticles, regardless of the aspect ratio (AR), of the cavity. A numerical study on mixed convection heat transfer in a porous L-shaped cavity was conducted by Mojumder et al. [9]. Kasaeipour et al. [10] investigated the effect of a magnetic field on mixed convection heat transfer of nanofluids in a T-shaped cavity. They reported that heat transfer was enhanced via increased nanofluid solid $\phi$. Natural and forced convection heat transfer of $\text{Al}_2\text{O}_3$-water nanofluid were achieved by some researchers [11-21]. Natural convection heat transfer of $\text{Al}_2\text{O}_3$-water nanofluid in oblique C-shaped cavity under the magnetic field was studied by Makulati et al. [22]. The results showed that, with increasing the Hartmann number, the influence of the nanofluid on the average Nusselt number, $\text{Nu}_{m}$, is reduced. Also, with an increasing the AR of the cavity, the effect of the inclination angle on the heat transfer is reduced. Rahman et al. [23] studied the mixed convection heat transfer in a horizontal channel with an open cavity for different values of Rayleigh number, Reynolds number, and Hartmann number. Various characteristics such as streamlines, isotherms and heat transfer rate in terms of the $\text{Nu}_{m}$, the drag force, $D$, and average dimensionless temperature, $\theta_{av}$, was investigated. The results indicate that the mentioned parameters strongly affect the flow phenomenon and temperature, $T$, field inside the cavity whereas, in the channel, these effects are less significant. One of the main purposes of using nanofluid in engineering devices is to enhance convection heat transfer [24-26]. However, to reach an optimization configuration and design, not only has the heat transfer to be maximized but the minimization of entropy generation is significant. Bejan [27-29] introduced this subject and focused on the different reasons behind entropy generation in applied thermal engineering. Many investigators focused on the entropy generation and heat transfer of nanofluid in some engineering problems [30-33]. A parametric analysis and entropy generation minimization of unsteady MHD flow over a stretch rotating disk was completed by Rashidi et al. [34], using an artificial neural network and particle swarm optimization algorithm. Al-Zamily and Ruhul Amin [35] investigated natural convection and entropy generation in nanofluid-filled semi-circular cavity with a heat flux source. Chamkha et al. [36] studied the entropy generation and natural convection of CuO-water nanofluid in a C-shaped cavity under magnetic field. A new criterion for thermal performance, as $\varepsilon$ defined by the ratio between global entropy and $\text{Nu}_{m}$, was introduced by Ismael et al. [37]. Particle shapes can affect the thermal conductivity of nanofluids. Nanoparticles with large AR are shown to enhance the thermal conductivity more. Das et al. [38], Patel et al. [39], Chon et al. [40], Beck et al. [41] and Moghadassi et al. [42] described that the thermal conductivity augments with the reduction of nanoparticle diameters. Ganguly et al. [43] considered aluminum oxide nanoparticles spread in deionized water to process stable nanofluids. They studied the effects of $\phi$ and $T$ on the actual electrical conductivity of $\text{Al}_2\text{O}_3$-water nanofluid and indicated that the electrical conductivity of alumina nanofluid is considerably more than the base fluid. The intensification in electrical conductivity is dependent on both $\phi$ of nanoparticles and bulk temperature of the suspension. The electrical conductivity of the nanofluid augments almost linearly with augmentation in the $\phi$ of nanoparticles and bulk temperature. A two-factor linear regression analysis founded on the $T$ and $\phi$ has been applied to develop an empirical relationship for the dimensionless enhancement factor at different $\phi$ as a function of $T$. Also, Armaghani et al. [44] studied the numerical investigation of $\text{Al}_2\text{O}_3$-water nanofluid natural convection heat transfer and entropy generation in a baffled L-shaped cavity. They used the thermal performance criteria for introducing the best AR and a baffled length of an L-shaped cavity.
This paper will discuss the numerical study of mixed convection heat transfer of nano-
fluids and entropy generation in an open C-shaped cavity under magnetic field. In this regard,
the effects of different parameters such as Reynolds number, Hartmann number, Richardson
number, and the AR of the cavity and the \( \phi \) of nanoparticles on the rate of heat transfer and
entropy generation is investigated.

A schematic of a C-shaped open cavity under the magnetic field is displayed in fig. 1(a)
and corresponding grid shown in fig 1(b). This is a 2-D C-shaped cavity with entrance and exit
of CuO-water nanofluid and length and height of \( L \). The cavity is under a constant magnetic
field. Nanofluid with cold temperature of \( T_c \) and velocity of \( u_c \) enters from the top right corner of
the cavity and exits from the bottom right corner of the cavity. The vertical wall of the left side
is subjected to hot and constant temperature of \( T_h \). The other walls are insulated and \( H \) is length
of heat source. The AR is defined as \( AR = H/L \). The governing conservation equation of mass,
momentum and energy in a 2-D Cartesian co-ordinate system was consider of the Mahmoudi
et al. [45] and eq. (1) of Chamkha et al. [36]. To be concise, this cavity is considered for an elec-
tronic device, it has a C-shaped space that can be used to analyze heat transfer phenomena for
different effective parameters. It should be mentioned that this cavity is subject to the magnetic
field effect.

![Figure 1. C-shaped open cavity under the magnetic field, (a) schematic (b) grid presentation](image_url)

The local entropy generation equation for the problems under magnetic field is given
by Bejan [27] and Batti and Rashidi [46] and can be adopted for nanofluid as eq. (2) of Cham-
kha et al. [36]. The non-dimensional form of conservation and local entropy generation equa-
tions can be written as eqs. (3)-(6) of Chamkha et al. [36]. The boundary conditions include the
condition of non-slipping on the cavity wall, \( U, V = 0 \), \( T \) of hot walls equal to \( \theta = 1 \) and uniform
inlet velocity and temperature \( U = 1, V = 0 \) and \( \theta = 0 \), respectively. Also, conditions of devel-
opment \( (U = 0, \partial U/\partial X = 0 \) and \( \partial \theta/\partial X = 0 \) \) in the outlet of the cavity are considered. The rate of
heat transfer is indicated in the form of a Nusselt number, which the local Nusselt number on
the hot wall is defined as eq. (16) of Chamkha et al. [36].

The average Nusselt number is also obtained through integration over the hot walls
and the distribution of volumetric entropy generation calculated by integral over the whole
domain V, to yield the Global Entropy Generation (GEG) rate show by \( S_m \). The thermophysical
properties of nanofluid can be calculated by Brinkman [47] and from eqs. (8)-(10) of Chamkha
et al. [36]. The dynamic viscosity of nanofluids is also calculated by the Brickman relation [47]
and Patel et al. [48] have proposed a model for thermal conductivity of nanofluids. Also, ther-
mal conductivity of nanofluids, \( k_{nf} \), for two independent components of spherical particles sus-
pension model show in eq. (8) of Santra et al. [49]. The thermophysical properties of water and CuO are shown in tab. 1 of Chamkha et al. [36].

**Numerical study and validation**

To model the desired geometry, a program was written in FORTRAN language for eq. (4) of Chamkha et al. [36] along with the designated boundary conditions for solving algebraic equation through finite difference method based on control volume. The solution field is meshed through displaced network method. For solving the algebraic equations at the same time, the SIMPLE algorithm is used; full details are given in Patankar [50]. The convergence criterion is from Chamkha et al. [36]. Figure 2(a) show the variation of Nuₘ with ϕ in a C-shaped cavity with three internal walls in cold temperature and three external walls in hot temperature, compared with the results of Mahoodi and Hashemi [51] that used Cu-water nanofluid in their research. This comparison is completed for Ra = 10⁵ as a function of two different AR via variation of ϕ. According to fig. 2(a), it is clear that result of the present study is in agreement with the numerical results of Mahmoddi and Hashemi [51]. To verify the performance of the present numerical research, the code was validated with the numerical results of Shahi et al. [6] in mixed convection heat transfer as a function of Richardson number at Re = 100 in fig. 2(b) and shows very good agreement.

![Figure 2. Variation of Nuₘ with ϕ as function of (a) AR in Ra = 10⁵, (b) Richardson number in Re = 100; this study is compare with Mahmoodi and Hashemie [51] and Shah et al. [6], respectively](image)

Figure 3(a) shows grid sensivity study in consideration the variation of Nuₘ of the hot wall as a function of Reynolds number. The results are obtained at AR = 0.3, Ri = 1, Ha = 20, and ϕ = 0.04. According to this figure, it is specified that almost for networks bigger than 100 × 100, the answers have remained the same. Also, some researche [52, 53] studied numerically mixed convection in the different form of channel and cavity by cuo-water nano-fluid.

**Results and discussion**

The effects of different parameters such as AR (0.1 ≤ AR ≤ 0.7), Hartmann number (0 ≤ Ha ≤ 60), (0 ≤ ϕ ≤ 0.06), and Richardson number (0.1 ≤ Ri ≤ 100) on heat transfer and entropy generation are studied at a fixed Re = 100.
In fig. 3(b), the horizontal velocity profile $U$ ($x = 0.5$) is plotted for different Reynolds number for nanofluid fluid-flow $\phi = 0.04$ at $AR = 0.3$ and $Ri = 1$. The results show that in the vicinity of the inlet and outlet flow from the cavity, increasing the Reynolds number so increases the velocity. However, in the middle of the C-shaped cavity, the process is reversed and with increase in the Reynolds number, the velocity reduces.

Figure 4 streamlines, isothermal and entropy in different AR for pure fluid (water) and nanofluids with $\phi = 0.04$ is drawn. The results in fig. 4 show that, increasing the AR of the cavity limits the space available for streamline decrease, which causes a closer streamline.

Figure 3. Variation of (a) $\text{Nu}_m$ with number of grids per unit area (b) $U$ with $Y$ as a function of Reynolds number, $AR = 0.3$, $Ri = 1$, $Ha = 20$, $X = 0.5$ and $\phi = 0.04$. 

Figure 4. Variation of streamline, isotherms, and isentropic lines for pure water (––) and nanofluid (-----) $\phi = 0.04$ as a function of AR at $Ri = 1$ and $Ha = 20$. 
The thermal boundary-layer is developed when AR increases. Pure water conduction is noticed within the cavity channels. This is because of the diminishing vertical distance between the horizontal hot and cold walls. The entropic lines show that the entropy generation rate has a minor increase with increase in the AR. However, the dashed contours of nanofluid with $\phi = 0.04$ indicate slight effect of nanoparticles on the nanofluid-flow, heat transfer, and entropy generation field. In fig. 5(a), the influence of different AR for nanofluid flow $\phi = 0.04$ on the local Nusselt number is plotted. The results indicate that with increase in AR, the local Nusselt number decreases along the heating area. The reason for this is the limitation of the cavity space and the approach of the flow to the hot wall. Also, the results indicate that the greatest influence of AR on the local Nusselt number is on the top of the hot wall. Figure 5(b) shows the effect of AR and $\phi$ of nanofluid flow on the $Nu_m$. Results indicate that with an increase in the AR of the cavity, the heat transfer increases. Also the addition of nanofluid results in an increase of the Nusselt number, which indicates that we may increase the $\phi$ for nanofluid flow to increase the cooling of the cavity. With an increase in the AR, the nanofluid puts a higher influence on increasing the Nusselt number.

![Figure 5](image-url)

**Figure 5.** Variation of the (a) local Nusselt number with $Y$ (b) average Nusselt number with $\phi$ (c) $S_m^*$ with $\phi$ (d) $\varepsilon^*$ with $\phi$ as a function of AR for $\phi = 0.04$ in $Ri = 1$ and $Ha = 20$

In conclusion, this shows that increasing nanofluid or increasing the entering nanofluid flow in the cavity space increases heat transfer and results in a better cooling condition for an electronic component and a better cooling condition results in increased efficiency fig. 5(c) indicates the effect of $\phi$ nanofluid flow on heat transfer in various AR. As shown in fig. 5(c) the $S_m^*$ is decreased via increasing the $\phi$ in low concentration. Also, this figure shows the $S_m^*$ is less
than unit and therefore adding nanoparticle to pure fluid leads to decreasing the $S_m^*$ in all ranges of AR. The effect of AR and $\phi$ on thermal performance $\varepsilon^*$ is shown in fig. 5(d). The results shows, increasing the AR leads to better $\varepsilon^*$ such as adding the $\phi$ so that the best $\varepsilon^*$ is seen at AR = 0.7 and $\phi = 0.06$. With increasing nanofluid flow or reduction of $\phi$ results in better cooling condition in electronic component and increasing efficiency of the system.

Figure 6(a) indicates variation of GEG $S_m^* = S_m / S_m(\phi=0)$ with $\phi$ as a function of Hartmann number at Ri = 1. As shown in this figure, at low concentration, the $S_m$ decreases via increasing the $\phi$ but it increases at high concentration increasing via the $\phi$. Generally, increasing the Hartmann number leads to enhancing the $S_m^*$, therefore, at Ha = 60 and $\phi = 0.06$ the $S_m$ is more than others.

![Figure 6. Variation of (a) $S_m^*$ with $\phi$ (b) $\varepsilon^*$ with $\phi$, as a function of Hartmann number at Ri = 1](image)

The augmentation in $\varepsilon^*$ is seen with adding the nanoparticle to pure fluid and increasing the $\phi$ in all values of Hartmann number. But increasing the Hartmann number leads to decreasing the $\varepsilon^*$ as shown in fig. 6(b). For example it can be seen that if the magnetic field in the electronic board increases, the performance of the electronic component is reduced. In fig. 7 variation of streamline, isotherms and isentropic lines via Richardson number pure water (——) and nano fluid $\phi = 0.04$ (- - -) as a function of Richardson number at AR = 0.3 is drawn. The results show that increase with the Richardson number stems increases buoyancy. Thus, the lines have been drawn to the right and left cavities. Richardson number increases; the effect of nanofluids increases the streamline. However, by increasing Richardson number, the temperature gradient increased and is projected to increase heat transfer. It also shows that the increase in the entropy contours of the Richardson number entropy lines are near the bottom wall of the warm and dense container and this is expected to increase entropy. The isothermal lines show that by increasing the isothermal lines the Richardson number dumped closer the heat source.

Figure 8(a) shows variation of Nu$_m$ with Hartmann number at various AR for pure fluid and nanofluid. The results show, in all ranges of AR and Hartmann number, the heat transfer of nanofluid is more important than the pure fluid.

The rate of increasing the Nusselt number with enhancing the Hartmann number for AR = 0.1 is more than others therefore the maximum Nusselt number is seen for nanofluid at AR = 0.1 and Ha = 60. In fig. 8(b), variation of $S_m$ with AR as a function of Richardson number for $\phi = 0.04$ is seen. In this figure observed that increasing the AR decreases average entropy $S_m$. Richardson number also increased, $S_m$ decreases and the maximum influential was AR = 0.1 and the minus influence occurred at AR = 0.5.
the cavity AR on the $\varepsilon$ for pure fluids with $\phi = 0$ and nanofluids with $\phi = 0.04$ be seen. The results show a consistently better $\varepsilon$ of nanofluids than pure fluids. With the increase in AR, the impact of nanofluids increases the $\varepsilon$. For $Ri = 0.1, 1$, the best $\varepsilon$ occurs in $AR = 0.7$, however for $Ri = 10, 100$, the best $\varepsilon$ is in $AR = 0.1$. Richardson number also increases by increase in value of $\varepsilon$. As shown in fig. 8(d), at $Ri = 0.1-10$ the entropy generation ratio is less than unit for $\phi = 0-0.06$ but in $Ri = 100$ the $S_m^*$ is more than unit at $\phi \geq 0.02$. Finally, adding the nanoparticles to pure fluid leads to increasing the $\varepsilon$ in all ranges of Richardson number and in $Ri = 0.1$ experiences more $\varepsilon$ increment via increasing the $\phi$ as appeared in fig. 8(e). In conclusion increasing Richardson number does not affect the cooling procedure for an electronic component.

**Conclusion**

In this paper, entropy generation due to MHD mixed convection heat transfer of nanofluids CuO-water in an open C-shaped cavity was investigated numerically. The governing equations were solved by SIMPLE algorithm. The Rayleigh number, the Hartmann number, Richardson number, AR, and $\phi$ of nanoparticles were changed.

The results show that Hartmann number increases leads to increasing entropy generation and with an increase in the AR of the cavity, the heat transfer increases. However, increasing the AR leads to better $\varepsilon^*$ caused by the adding of the $\phi$ and the best $\varepsilon^*$ is seen at $AR = 0.7$ and $\phi = 0.06$. Therefore, adding the nanoparticles to pure fluid leads to increasing the $\varepsilon^*$ in all ranges of Richardson number. Otherwise in all ranges of AR and Hartmann number the heat transfer of nanofluid is more than the pure fluid.

Nomenclature

\( B_0 \) – magnetic field strength, [T]
\( g \) – gravitational acceleration, [ms\(^{-2}\)]
\( \text{Gr} \) – Grashof number, \( = g \beta (T_h - T_c) L^3 / \rho \nu ) \), [-]
\( \text{Ha} \) – Hartmann number, \( = B_0 / (\sigma / \mu \nu ) \), [-]
\( k \) – thermal conductivity, [Wm\(^{-1}\)K\(^{-1}\)]
\( \text{Nu}_{\text{m}} \) – average Nusselt number, \( = (1/L) (\text{Nu}_{\text{m}}) \), [-]
\( \text{Re} \) – Reynolds number, \( = u_0 L / \mu \), [-]
\( \text{Ri} \) – Richardson number, \( = \text{Gr} / \text{Re}^2 \), [-]
\( \varepsilon^* \) – dimensionless local entropy generation, [-]
\( S^m \) – volumetric entropy generation, \( = \int_S \nabla \cdot \dot{\varepsilon} \) d\( v \)
\( T \) – temperature, [K]
\( u \) – inlet flow velocity, [ms\(^{-1}\)]
\( U, V \) – dimensionless velocity components, \( = u / U_0 = v / V_0 ) \), [-]
\( u, v \) – velocity components in x-, y-directions, [ms\(^{-1}\)]

Greek symbols

\( \varepsilon \) – dimensionless temperature, \( = \left( \frac{\text{Nu}_{\text{m}}}{S^m} \right) \times 100 \)
\( \phi \) – solid volume fraction
\( \theta \) – dimensionless temperature, \( = (T - T_c) / (T_h - T_c) \), [-]
\( \mu \) – dynamic viscosity, [Nsm\(^{-2}\)]
\( \nu \) – kinematic viscosity, [m\(^2\)s\(^{-1}\)]
\( \rho \) – density, [kgm\(^{-3}\)]
\( \sigma \) – effective electrical conductivity, [\( \mu \)Scm\(^{-1}\)]

Subscripts

c – cold
f – pure fluid
h – hot wall
nf – nanofluid

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