

THERMOSOLUTAL INSTABILITY IN A HORIZONTAL FLUID LAYER AFFECTED BY ROTATION

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Thermosolutal convective instability in a horizontal layer affected by rotation is studied. Stationary convection and over-stability cases are considered for different boundary conditions. Analytical solutions were obtained when both boundaries are free and numerical results were obtained for the cases of free and rigid boundaries. The numerical computations of this problem were performed using the method of expansion of Chebyshev polynomials. This method is better suited to the solution of hydrodynamic stability problems than expansions in other sets of orthogonal polynomials. This method not only has high accuracy but also allows stationary and over-stable modes to be treated simultaneously, which is important if perchance the critical eigenvalue flits between the different modes in response to changing parameter values. The results obtained show that the effect of both solute concentration and rotation is to stabilize the system for stationary convection case and for the over-stability case when both boundaries are free. However when both boundaries are rigid some unexpected behavior are obtained in the case of over-stability.

Key words: Thermal instability, Thermosolutal convection, Rotation, Thermal Rayleigh number; Concentration Rayleigh number.

1. Introduction

Thermal instability theory has attracted considerable interest and has been recognized as a problem of fundamental importance in many fields of fluid dynamics. Rayleigh [1] provided a fundamental theoretical basis for the thermal instability in a fluid layer heated from below. The instability of a layer of fluid heated from below and subjected to Coriolis forces was first studied by Chandrasekhar [2] and Chandrasekhar & Elbert [3] for stationary convection and over-stability respectively. They showed that the effect of Coriolis forces on the instability of the fluid layer is to inhibit the onset of instability. Several other authors discussed the instability of fluids and the effect of rotation (Abdullah [4], Julien *et al.* [5], Chun *et al.* [6], Prosperetti [7], Geurts and Kunnen [8], Horn and Shishkina [9], Khan and Shafie [10], Sharma *et al.* [11]).

Previous studies (Stern [12], Walin [13], Veronis [14], Nield [15]) showed that the presence of salinity in convection problems usually has the effect of inhibiting the development of instabilities when the solute concentration decreases upwards and the fluid is heated from below. However it has an opposite effect when the solute concentration increases upwards. Moreover if the fluid is heated from above and soluted from below, then the temperature at the bottom of the fluid will be relatively high comparing to the other parts of the fluid. A comprehensive review of literature concerning thermosolutal convection can be found in Nield and Bejan [16]. Further studies in thermosolutal convection are presented in [17-24].

This work studies thermosolutal convective instability in a horizontal layer affected by rotation. Stationary convection and overstability cases are considered for different boundary conditions. Analytical solutions were obtained when both boundaries are free and

numerical results were presented for the cases of free and rigid boundaries. Many flows in nature are driven by buoyant convection and subsequently modulated by rotation. This problem can be expected to exist near coastal regions where tidal effects could carry salty warm water under fresher cold water, or in areas such as the eastern Atlantic where salty Mediterranean water flows out through the strait of Gibraltar into the open ocean. The problem is of great importance because of its application to atmospheric physics, oceanography, limnology and geophysical flows driven by rotation. Salt gradient solar ponds are a good application of this problem.

2. Formulation of the problem

Consider an infinite horizontal layer occupied by an incompressible, viscous, soluted fluid. The fluid is subject to a constant gravitational acceleration in the negative x_3 direction and is affected by rotation about the x_3 axis with a constant angular velocity Ω . The governing equation for this problem are

$$v_{i,i} = 0, \quad (1)$$

$$\dot{v}_i = -\left(\frac{P}{\rho_0}\right)_{,i} + \nu \nabla^2 v_i - g[1 - \alpha(T - T_0) + \alpha'(c - c_0)]\delta_{i3} + 2\epsilon_{ijk}v_j\Omega_k, \quad (2)$$

$$\frac{\partial T}{\partial t} + v_j \frac{\partial T}{\partial x_j} = \kappa \frac{\partial^2 T}{\partial x_j^2}, \quad (3)$$

$$\frac{\partial c}{\partial t} + v_j \frac{\partial c}{\partial x_j} = \kappa' \frac{\partial^2 c}{\partial x_j^2} \quad (4)$$

where v_i are the velocity components, P is the pressure, ν is the kinematic viscosity, g is the acceleration due to gravity, T is the temperature, c is the solute concentration, T_0, ρ_0, c_0 are Temperature, density and solute concentration at $x_3 = 0$ respectively, α is the coefficient of volume expansion, α' is the coefficient of solute expansion κ is the coefficient of heat conduction and κ' is the coefficient of solute diffusion. These equations are supplemented by an equation of state which depends on the Boussinesq approximation which state that density is constat everywhere except in the body force term in the equation of motion (2) where the density is linearly proptional to temperature T and solute mass concentration c , i.e.

$$\rho = \rho_0[1 - \alpha(T - T_0) + \alpha'(c - c_0)], \quad (5)$$

The fluid is confined between the planes $x_3 = 0$ and $x_3 = d$ and on these planes, we need to specify mechanical, thermal and solute conditions.

3. The Linearized equations

Equations (1)-(4) have a steady state solution in which the fluid is at rest and the pressure, temperature and solute concentration are functions of x_3 alone. i.e.

$$v_i = 0, \quad P = P(x_3), \quad T = T(x_3), \quad c = c(x_3) \quad (6)$$

Now, let the steady state solution be slightly perturbed such that

$$v_i = 0 + \epsilon \hat{v}_i, \quad T = T_0 - \beta x_3 + \epsilon \hat{\theta}, \quad c = c_0 - \beta' x_3 + \epsilon \hat{c}, \quad P = P + \epsilon \hat{P},$$

where $\hat{v}_i, \hat{\theta}, \hat{P}, \hat{c}$ are respectively the linear perturbation of velocity, temperature, solute concentration and pressure. So the linearized form of equations (1)-(4) are:

$$\hat{v}_{i,i} = 0. \quad (7)$$

$$\frac{\partial \hat{v}_i}{\partial t} = -(\hat{P}/\rho_0)_{,i} + v\nabla^2 \hat{v}_i + g[\alpha \hat{\theta} - \alpha' \hat{c}] \delta_{i3} + 2\epsilon_{ijk} \hat{v}_j \Omega_k, \quad (8)$$

$$\frac{\partial \hat{\theta}}{\partial t} - \beta \hat{v}_3 = \kappa \nabla^2 \hat{\theta}, \quad (9)$$

$$\frac{\partial \hat{c}}{\partial t} - \beta' \hat{v}_3 = \kappa' \nabla^2 \hat{c}. \quad (10)$$

Now, we use the non-dimensional variables $x_i^*, t^*, v_i^*, c^*, \theta^*, P^*$ such that

$$x_i = dx_i^*, t = \frac{d^2}{\nu} t^*, \hat{v}_i = \frac{\kappa}{d} v_i^*, \hat{c} = \frac{\kappa}{d} \sqrt{\frac{\nu|\beta'|}{\kappa'\alpha'g}} c^*, \hat{\theta} = \frac{\kappa}{d} \sqrt{\frac{\nu|\beta|}{\kappa\alpha g}} \theta^*, \hat{P} = \frac{\rho_0 \kappa \nu}{d^2} P^*,$$

then equations (7)-(10) become

$$v_{i,i} = 0, \quad (11)$$

$$\frac{\partial v_i}{\partial t} = -P_{,i} + \nabla^2 v_i + \sqrt{R_t} \theta \delta_{i3} - \sqrt{R_s} c \delta_{i3} + \sqrt{T_r} \epsilon_{ijk} v_j \delta_{k3}, \quad (12)$$

$$P_r \frac{\partial \theta}{\partial t} + H \sqrt{R_t} v_3 = \nabla^2 \theta, \quad (13)$$

$$P_r' \frac{\partial c}{\partial t} + H' \sqrt{R_s} v_3 = \nabla^2 c, \quad (14)$$

where the (*) superscript has been dropped but all the variables are now non-dimensional and where the non-dimensional numbers R_t, R_s, P_r, P_r' and T_r are given by

$$R_t = \frac{\alpha g |\beta|}{\kappa \nu} d^4, R_s = \frac{g |\beta'| |\alpha'|}{\kappa' \nu} d^4, P_r = \frac{\nu}{\kappa}, P_r' = \frac{\nu}{\kappa'}, T_r = \frac{4\Omega^2}{\nu^2} d^4,$$

where R_t is the thermal Rayleigh number, R_s is the concentration Rayleigh number, P_r is the viscous Prandtl number, P_r' is the Schmidt number, and T_r is the Taylor number and where

$$H = -\frac{\beta}{|\beta|} = \begin{cases} 1 & \text{when heating from above,} \\ -1 & \text{when heating from below.} \end{cases}$$

$$H' = -\frac{\beta'}{|\beta'|} = \begin{cases} 1 & \text{when solute concentration increases upwards,} \\ -1 & \text{when solute concentration decreases upwards.} \end{cases}$$

4. The boundary conditions

The fluid is confined between the planes $x_3 = 0$ and $x_3 = d$ and on these planes, we need to specify mechanical, thermal and solutal conditions. Suitable mechanical conditions assume either a rigid boundary on which no slip occurs or a free boundary on which no tangential stresses act. Suitable thermal conditions assume either an insulating or a perfectly conducting boundary and suitable solutal conditions assume either a permeable or an impermeable boundary.

4.1 Mechanical conditions:

For a rigid boundary, the no slip condition implies that the horizontal components of the fluid velocity and all of x_1, x_2 partial derivatives of each component of the fluid velocity vanish. Thus if $A_{,i}$ denotes the derivative of A with respect to i then

$$v_1 = v_2 = 0 \quad \text{and} \quad v_{1,1} = v_{2,2} = 0.$$

and from the continuity equation (11) we obtain $v_{3,3} = 0$. If we introduce ξ to be the fluid vorticity then $\xi = \text{curl } \mathbf{v}$ and it is clear that $\xi_3 = 0$. For a free boundary no tangential stresses act, and we can show that $v_{3,33} = 0$. $\xi_{3,3} = 0$.

4.2 Thermal conditions:

For a perfectly conducting boundary the temperatures of the boundary and impinging fluid match, whereas on a perfectly insulating boundary no heat transfer can take place between the fluid and the surrounding, hence the normal derivative of temperature is zero. In mathematical terms, the thermal conditions are:

$$\theta = \theta_{ext}, \quad \text{on a conducting boundary,} \quad \theta_{,3} = 0, \quad \text{on an insulating boundary}$$

where θ_{ext} is the exterior non-dimensional temperature.

4.3 Solute conditions

The possible solute conditions are

$$c = c_{ext}, \quad \text{on a permeable boundary,} \quad c_{,3} = 0, \quad \text{on an impermeable boundary,}$$

where c_{ext} is the exterior non-dimensional concentration.

5. Normal mode analysis

In many convection problems, the vector components parallel to the direction of gravity (i.e. the x_3 direction) play a central role and so we introduce the variables w and ξ such that $w = v_3$, $\xi = \xi_3$. When we take the curl of equation (12), we obtain

$$\frac{\partial \xi_i}{\partial t} = \nabla^2 \xi_i + \sqrt{R_t} \epsilon_{ijk} \frac{\partial \theta}{\partial x_j} \delta_{k3} - \sqrt{R_s} \epsilon_{ijk} \frac{\partial c}{\partial x_j} \delta_{k3} + \sqrt{T_r} v_{i,3}. \quad (15)$$

Taking the curl again we obtain

$$\frac{\partial}{\partial t} \nabla^2 v_i = \nabla^4 v_i - \sqrt{R_t} (\theta_{,3i} - \nabla^2 \theta \delta_{i3}) + \sqrt{R_s} (c_{,3i} - \nabla^2 c \delta_{i3}) - \sqrt{T_r} \xi_{i,3}. \quad (16)$$

The third components of equations (15), (16), (13) and (14) are

$$\frac{\partial \xi}{\partial t} = \nabla^2 \xi + \sqrt{T_r} w_{,3}, \quad (17)$$

$$\frac{\partial}{\partial t} \nabla^2 w = \nabla^4 w + \sqrt{R_t} (\theta_{,11} + \theta_{,22}) - \sqrt{R_s} (c_{,11} + c_{,22}) - \sqrt{T_r} \xi_{,3}, \quad (18)$$

$$P_r \frac{\partial \theta}{\partial t} + H \sqrt{R_t} w = \nabla^2 \theta, \quad (19)$$

$$P_r' \frac{\partial c}{\partial t} + H' \sqrt{R_s} w = \nabla^2 c. \quad (20)$$

Now, we consider a solution of the form

$$\varphi = \varphi(x_3) \exp[i(nx_1 + mx_2) + \sigma t]$$

where n, m are the wave numbers for harmonic disturbance and σ is the growth rate. Thus equations (17)-(20) become

$$\sigma\xi = L\xi + \sqrt{T_r}Dw, \quad (21)$$

$$\sigma Lw = L^2w - a^2\sqrt{R_t}\theta + a^2\sqrt{R_s}C - \sqrt{T_r}D\xi, \quad (22)$$

$$\sigma P_r\theta = L\theta - H\sqrt{R_t}w, \quad (23)$$

$$\sigma P_r'C = LC - H'\sqrt{R_s}w, \quad (24)$$

where D is the operator $\frac{\partial}{\partial x_3}$, $a = \sqrt{(n^2 + m^2)}$ is the wave number and $L = (D^2 - a^2)$. We may ξ, C and θ from equation (22) to obtain

$$L^5w + L^4w[-\sigma P_r - 2\sigma - \sigma P_r'] + L^3w[2\sigma^2 P_r + \sigma^2 + \sigma^2 P_r' P_r + 2\sigma^2 P_r'] + L^2w[-\sigma^3 P_r - 2\sigma^3 P_r P_r' - \sigma^3 P_r' - a^2 R_t H + a^2 R_s H'] + Lw[\sigma^4 P_r P_r' + \sigma a^2 R_t H - \sigma a^2 R_s H' P_r - \sigma a^2 R_s H'] + \sigma a^2 R_t P_r' Hw - \sigma^2 a^2 R_t H P_r' w + \sigma^2 a^2 R_s H' P_r w - \sigma T_r [P_r' + P_r] L D^2 w + \sigma^2 T_r P_r P_r' D^2 w + T_r L^2 D^2 w = 0 \quad (25)$$

which is a tenth order ordinary differential equation to be satisfied by w . This problem has exact solutions only in the case of two free boundaries. However for the cases of two rigid boundaries or mixed boundaries it is not possible to obtain exact solutions and we have to rely on numerical methods. In the following section we discuss the analytical method when both boundaries are free, however in the result section we shall present numerical results for free and rigid boundaries.

6. The free boundary problem

In the following analysis, we shall consider both boundaries to be free. For the free boundary value problem $w = D^2w = 0$ on $x_3 = 0, 1$. If we suppose that $w = A \sin(l\pi x_3)$, is a suitable solution where A is a constant and l is an integer, then $Dw = A l \pi \cos l\pi x_3$, $D^2w = -A l^2 \pi^2 \sin l\pi x_3$, $Lw = -\lambda w$ where $\lambda = l^2 \pi^2 + a^2$ and (32) becomes

$$\sigma^4 P_r P_r' + \sigma^3 (P_r + P_r' + 2P_r P_r') \lambda + \sigma^2 \{ [1 + P_r P_r' + 2(P_r + P_r')] \lambda^2 + (a^2 R_t H P_r' - a^2 R_s H' P_r + T P_r P_r' l^2 \pi^2) \lambda^{-1} \} + \sigma \{ (2 + P_r + P_r') \lambda^3 + a^2 R_t H (1 + P_r') - a^2 R_s H' (1 + P_r) + T_r l^2 \pi^2 (P_r + P_r') \} + \lambda^4 + a^2 R_t H \lambda - a^2 R_s H' \lambda + T_r l^2 \pi^2 \lambda = 0 \quad (26)$$

The solutions of (26) are functions of $P_r, P_r', T_r, R_s,$ and R_t and we have to examine how the nature of these solutions depends on these variables by considering the following cases. Note that because of the complexity of equation (26) it is not possible to discuss analytically the effect of overstability but later on we shall discuss its effect from the numerical results.

Case (1): when the fluid is heated from above and the solute concentration decreases upwards.

Here $H = 1, H' = -1$. So we need to discuss the roots of the polynomial equation $f(\sigma) = 0$ where

$$f(\sigma) = \sigma^4 P_r P_r' + \sigma^3 (P_r + P_r' + 2P_r P_r') \lambda + \sigma^2 \{ [1 + P_r P_r' + 2(P_r + P_r')] \lambda^2 + (a^2 R_t P_r' - a^2 R_s P_r + T P_r P_r' l^2 \pi^2) \lambda^{-1} \} + \sigma \{ (2 + P_r + P_r') \lambda^3 + a^2 R_t (1 + P_r') - a^2 R_s (1 + P_r) + T_r l^2 \pi^2 (P_r + P_r') \} + \lambda^4 + a^2 R_t \lambda + a^2 R_s \lambda + T_r l^2 \pi^2 \lambda = 0. \quad (27)$$

To obtain the critical thermal Rayleigh number for the onset of stationary convection case we set $\sigma = 0$ in (27). Thus

$$R_t = - \left[\frac{\lambda^3}{a^2} + \frac{T_r l^2 \pi^2}{a^2} + R_s \right], \quad l^2 = 1. \quad (28)$$

Since R_t is negative then no stationary convection happened.

Case (2): when the fluid is heated from above and the solute concentration increases upwards.

Here $H = 1$, $H' = 1$. To obtain the critical thermal Rayleigh number for the onset of stationary convection case we set $\sigma = 0$ in (27). Thus

$$R_t = - \left[\frac{\lambda^3}{a^2} + \frac{T_r l^2 \pi^2}{a^2} - R_s \right]. \quad (29)$$

Clearly stationary stability is possible provided $R_s > \frac{1}{a^2} (\lambda^3 + T_r l^2 \pi^2)$. Moreover

$$\frac{dR_t}{dR_s} = 1, \quad \frac{dR_t}{dT_r} = - \frac{l^2 \pi^2}{a^2}. \quad (30)$$

So the solute concentration has a stabilizing effect on the system, but the rotation has a destabilizing effect on the system in this case.

Case (3): when the fluid is heated from below and the solute concentration increases upwards.

Here $H = -1$, $H' = 1$. So from equation (27) the thermal Rayleigh number for the onset of stationary convection has the form

$$R_t = \frac{\lambda^3}{a^2} + T_r \frac{l^2 \pi^2}{a^2} - R_s. \quad (31)$$

Clearly stationary stability is possible provided $\frac{\lambda^3}{a^2} + T_r \frac{l^2 \pi^2}{a^2} > R_s$. Moreover

$$\frac{dR_t}{dR_s} = -1, \quad \frac{dR_t}{dT_r} = \frac{\pi^2 l^2}{a^2}. \quad (32)$$

So it is clear in this case that the solute concentration has a destabilizing effect on the system but the rotation has a stabilizing effect on the system.

Case (4): when the fluid is heated from below and the solute concentration decreases upwards.

Here $H = -1$, $H' = -1$. So from equation (27) the thermal Rayleigh number for the onset of stationary convection has the form $R_t = \frac{\lambda^3}{a^2} + R_s + \pi^2 l^2 \frac{T_r}{a^2}$. From which we can see that

$$\frac{dR_t}{dR_s} = 1, \quad \frac{dR_t}{dT_r} = \frac{l^2 \pi^2}{a^2}. \quad (33)$$

It is clear that the solute concentration and rotation have a stabilizing effect on the system.

7. Numerical Method

The numerical computations of this problem are performed using the method of expansion of Chebyshev polynomials. This method is better suited to the solution of hydrodynamic stability problems than expansions in other sets of orthogonal polynomials. Chebyshev method has been used to obtain numerical solutions of thermal stability problems by several authors (Orszag [25], Hassanien *et al.* [26], Abdullah and Lindsey [27], Hassanien *et al.* [28], Straughan [29], Banjer and Abdullah [30]). The eigenvalue problem (21)-(24) together with the boundary conditions are to be solved numerically for the case when the fluid layer is heated from below and the solute concentration decreases upwards which corresponds to case (4) in the previous section. This eigenvalue problem can be solved using free or rigid boundaries for the two cases of solute conditions. For free boundaries the conditions are $w = \theta = \phi = D\xi = 0$, $x_3 = 0, 1$ and for rigid boundaries the conditions are $w = \theta = Dw = \xi = 0$, $x_3 = 0, 1$. The interval (0,1) is first mapped into the interval (-1,1) by the transformation $z = 2x_3 - 1$. Thereafter, the variables of the problem are assigned the Chebyshev spectral expansion

$$y_r(z) = \sum_{k=0}^{N-1} a_{rk} T_k(z). \quad (34)$$

where N is a user-specified number of Chebyshev polynomials. The system of ordinary differential equations (21) – (24) and boundary conditions lead to a generalized eigenvalue problem of type $AY = \sigma BY$ in which B is a singular matrix. The eigenvalues, σ , and corresponding eigenvectors are calculated using a specialized routine.

8. Results and discussion

8.1 Both boundaries are free

The relation between the concentration Rayleigh number, R_s , and the critical thermal Rayleigh number, R_{ct} , for the stationary convection case is displayed in fig. 1 for different values of the Taylor number T_r . It is clear from this figure that as R_s increases, R_{ct} increases which indicates that the solute concentration has a stabilizing effect on the system. Moreover the figure shows that as T_r increases R_{ct} increases which indicates that rotation has a stabilizing effect on the system. These results coincide with the results obtained in equation (33) in the analytical solution of the problem.

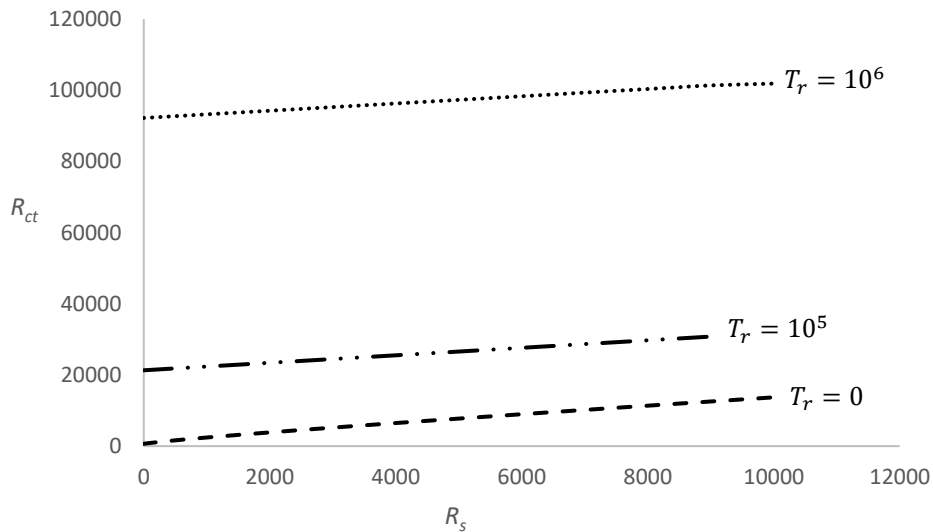


Fig. 1. The relation between R_s and critical R_{ct} for the stationary convection case when both boundaries are free for different values of T_r .

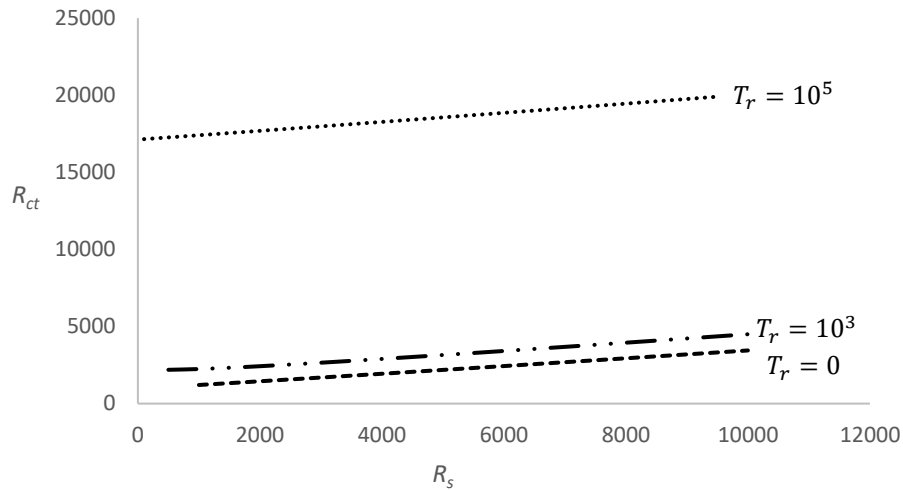


Fig. 2. The relation between R_s and critical R_{ct} for the over-stability case when both boundaries are free for different values of T_r when $P_r = 0.5$.

In the case of over-stability, the relation between the concentration Rayleigh number, R_s , and the critical thermal Rayleigh number, R_{ct} for different values of the Taylor number, T_r , is displayed in fig. 2 when $P_r = 0.5$, $P_r' = 0.025$. It is clear from this figure that as R_s increases R_{ct} increases which indicates that the solute concentration has a stabilizing effect on the system for the over-stable case also. Moreover the figure shows that as T_r increases R_{ct} increases which indicates that rotation has a stabilizing effect on the system. The numerical results in this case show that there is a condition for over-stability to ensue. This condition depends on the values of the Taylor number, T_r , and the concentration Rayleigh number, R_s . As T_r decreases over-stability is possible provided that R_s exceeds a certain value as shown in fig. 2.

Figure 3 shows a comparison between stationary convection and over-stability cases. In this figure a relation between the concentration Rayleigh number, R_s , and the critical thermal Rayleigh number, R_{ct} , is displayed when $T_r = 0, 10^5$. When $T_r = 0$, stationary convection is the preferred mechanism if $R_s < 500$ otherwise over-stability is the preferred mechanism. When $T_r = 10^5$ over-stability is possible provided $R_s \geq 100$ and if this condition is satisfied then over-stability is always the preferred mechanism.

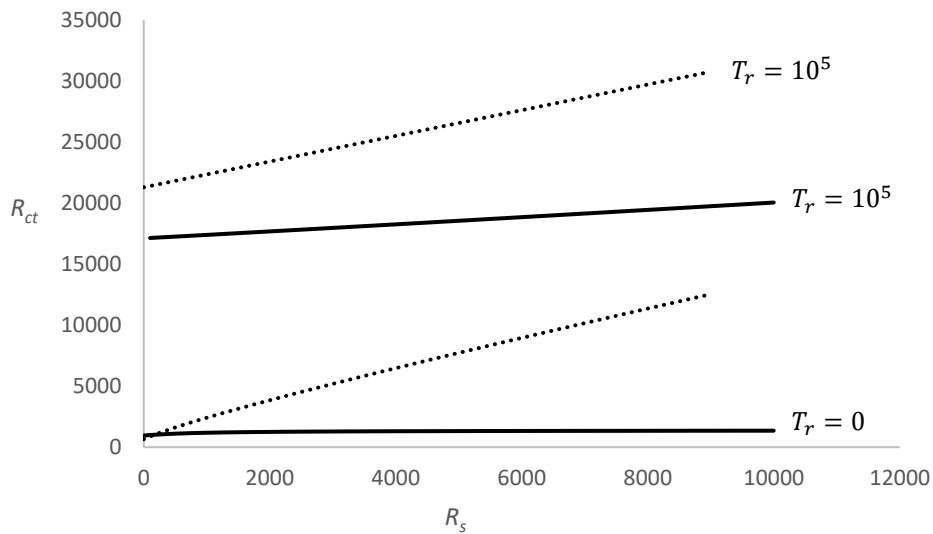


Fig. 3. A comparison between stationary convection and over-stability cases when both boundaries are free for different values of T_r .

8.2 Both boundaries are rigid

For the stationary convection case, the effect of the concentration Rayleigh number, R_s , on the critical Rayleigh number R_{ct} is similar to that of the free boundary case as shown in fig. 4. However the effect of the Taylor number T_r in this case is unexpected. The figure shows that as T_r increases the critical Rayleigh number R_{ct} decreases which indicates that rotation has a destabilizing effect in this case. This behavior appears when $R_s > 500$ but below this value we notice that as T_r increases the critical Rayleigh number, R_{ct} increases. In the case of over-stability, the effects of the concentration Rayleigh number, R_s , and the Taylor number, T_r , on the critical thermal Rayleigh number, R_{ct} , are displayed in fig. 5. These effects are similar to those of the stationary convection case.

A comparison between stationary convection and over-stability cases is displayed in fig. 6. In this figure, a relation between the concentration Rayleigh number, R_s , and the critical thermal Rayleigh number, R_{ct} , is displayed when $T_r = 0$. Here the preferred mechanism flits between stationary convection case and over-stability case.

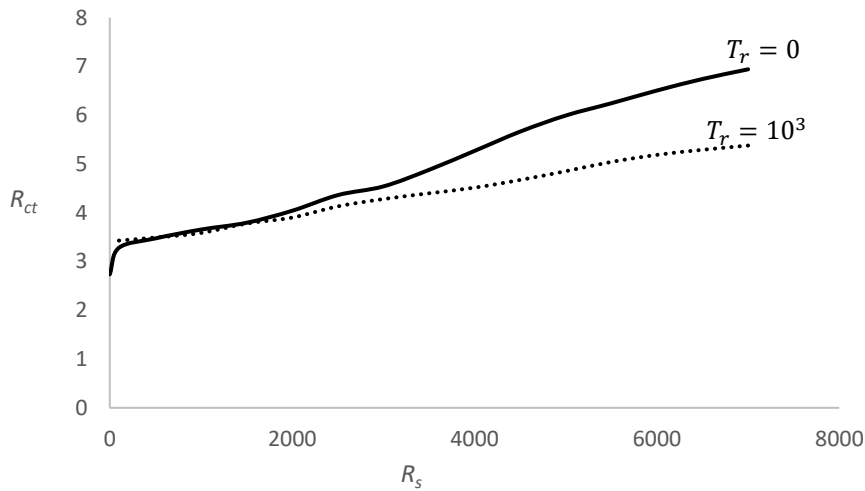


Fig. 4. The relation between R_s and critical R_{ct} for the stationary convection case when both boundaries are rigid for different values of T_r .

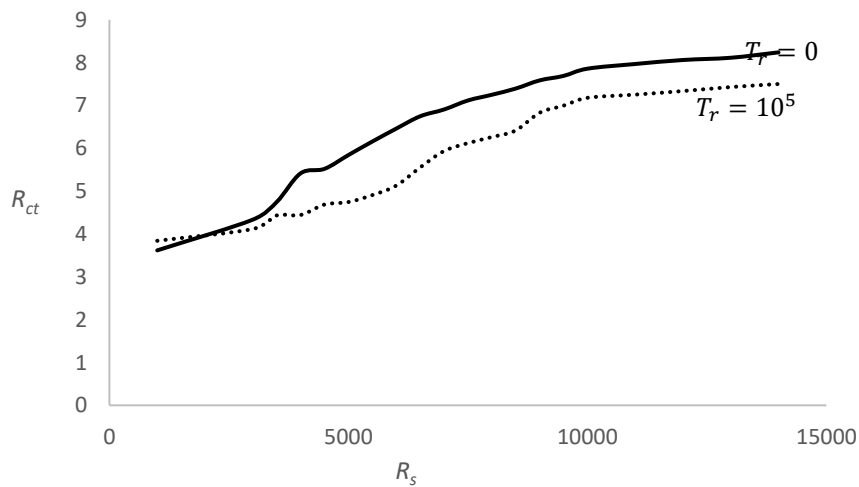


Fig. 5. The relation between R_s and critical R_{ct} for the over-stability case when both boundaries are rigid for different values of T_r .

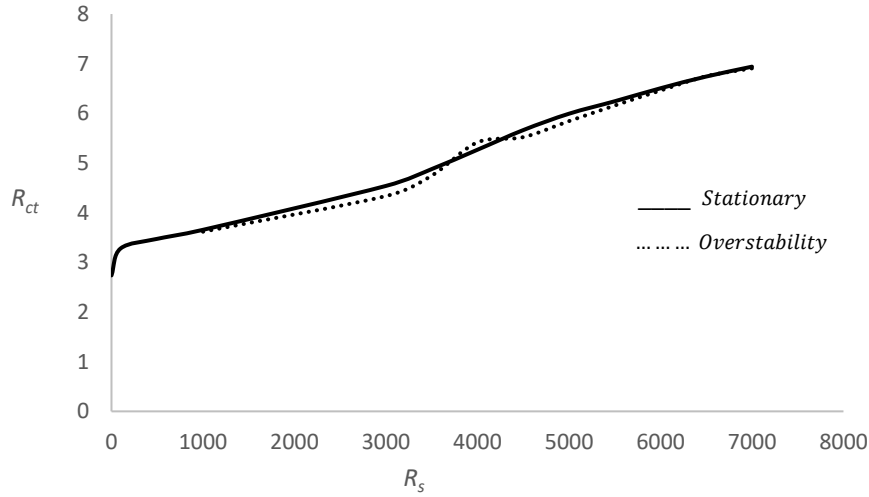


Fig. 6. A comparison between stationary convection and over-stability cases when both boundaries are rigid. Here $T_{r=0}$.

Conclusion

This work studies thermosolutal convective instability in a horizontal layer affected by rotation. Analytical solutions are obtained for the free boundary problem in the case of stationary convection. Typical cases are discussed analytically depending on heating and solute concentration in both boundaries. Numerical results are obtained using the method of expansion of Chebyshev polynomials for the case when the layer of fluid is heated from below and the solute concentration decreases upwards when both boundaries are free and when both boundaries are rigid. The results obtained for the free boundary case coincide with the analytical solutions obtained.

The effect of both solute concentration and rotation is discussed and results show that their effect is to stabilize the system for stationary convection case and for the over-stability case when both boundaries are free. However when both boundaries are rigid some unexpected behavior are obtained in the case of over-stability.

Acknowledgment

The authors would like to thank Deanship of Scientific Research at Umm Al-Qura University (Project ID 43405021) for the financial support.

Nomenclature

a	wave number [cm^{-1}]	β'	adverse concentration gradient [kg/m^4]
c	solute concentration [kg/m^3]	δ_{ij}	Kronecker delta
c_{ext}	exterior solute concentration [kg/m^3]	ϵ_{ijk}	permutation tensor
d	thickness of fluid layer [m]	κ	heat conduction coefficient [$\text{W}/\text{m K}$]
g	gravitational acceleration [m/s^2]	κ'	solute diffusion coefficient [m^2/s]
P	pressure [N/m^2]	ν	kinematic viscosity [$\text{kg}/\text{m s}$]
P_r	viscous Prandtle number ($=\nu/\kappa$)	Ω	angular velocity [rad/s]
P_r'	Schmidt number ($=\nu/\kappa'$)	ρ	density of fluid layer [kg/m^3]
R_t	thermal Rayleigh no. ($=\alpha g \beta d^4/\kappa\nu$)	σ	growth rate

R_s	concentration Ray. no. ($=\alpha g \beta' d^4/\kappa'\nu$)	θ	dimensionless temperature
T	Celsius temperature	θ_{ext}	Exterior dimensionless temperature
T_r	Taylor number ($=4\Omega^2 d^4/\nu^2$)	ξ	fluid vorticity [1/s]
t	Time [s]		
V	velocity [m/s]		
x_i	coordinates (i =1,2,3)		
Greek symbols		Superscripts	
α	coefficient of volume expansion [1/K]	\wedge	linear perturbation quantity
α'	coefficient of solute expansion [kg/m ³]	o	referential quantity
β	adverse temperature gradient [K/m]		
		Subscripts	

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